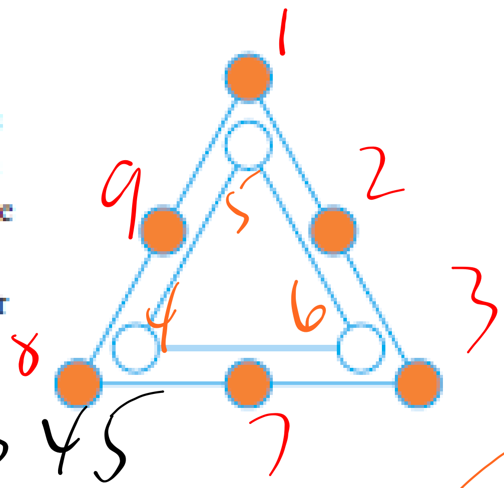
HOMEWORK...

2. Copy this diagram. Place the digits 1 through 9 in the circles so that the sum of the numbers on the outside triangle is double the sum of the numbers on the inside triangle. Explain whether more than one solution is possible.



Sum → 45

30

15

1 3 4 9 5 8

2 6 7

6. A farmer wants to get a goat, a wolf, and a bale of hay to the other side of a river. His boat is not very big, so it can only carry him and one other thing. If the farmer leaves the goat alone with the bale of hay, the goat will eat the hay. If he leaves the wolf alone with the goat, the wolf will eat the goat. When the farmer is present, the goat and the hay are safe from being eaten. How does the farmer manage to get everything safely to the other side of the river?

1st 2nd

goat

Hay Wolf

go

1st 2nd

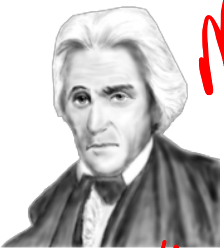
7. Determine the unknown term in this pattern: 17, 22, <u>28</u>, 35, 43.
Explain your reasoning.

5 6 7 8

Kurt → married

9. Bob, Kurt, and Morty are football players. One is a quarterback, one is a receiver, and one is a kicker. The kicker, who is the shortest of the three, is not married. Bob, who is Kurt's father-in-law, is taller than the receiver. Who plays which position?

Short | Tall


Morty
K
Kicker


Kurt
B
Receiver


Bob
M
Quarterback

APPLY the Math p. 28

EXAMPLE 2 Using deductive reasoning to generalize a conjecture

In Lesson 1.3, page 19, Luke found more support for Steffan's conjecture from Lesson 1.1, page 9—that the difference between consecutive perfect squares is always an odd number.

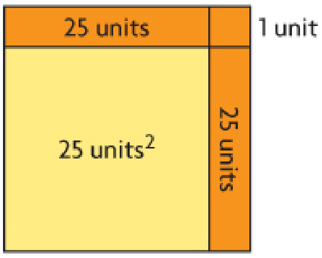
Determine the general case to prove Steffan's conjecture.

Back to previous lesson...

Gord's Solution

The difference between consecutive perfect squares is always an odd number.

Steffan's conjecture has worked for consecutive perfect squares with sides of 1 to 7 units.



$$26^2 - 25^2 = 2(25) + 1$$
$$26^2 - 25^2 = 51$$

I tried a sample using even greater squares: 26^2 and 25^2.

The difference is the two sets of 25 unit tiles, plus a single unit tile.

Let x be any natural number.
Let D be the difference between consecutive perfect squares.
$D = (x + 1)^2 - x^2$

Since the conjecture has been supported with specific examples, I decided to express the conjecture as a general statement. I chose x to be the length of the smaller square's sides. The larger square's sides would then be $x + 1$.

$$D = x^2 + x + x + 1 - x^2$$
$$D = x^2 + 2x + 1 - x^2$$
$$D = 2x + 1$$

I expanded and simplified my expression. Since x represents any natural number, $2x$ is an even number, and $2x + 1$ is an odd number.

Steffan's conjecture, that the difference of consecutive perfect squares is always an odd number, has been proved for all natural numbers.

Prove: The difference between consecutive perfect squares will always be odd.

Deductively ... use algebra

Let $x \to 1^{st}$ #

$x+1 \to 2^{nd}$ #

$(x+1)^2 - x^2$

$\cancel{x^2} + 2x + 1 - \cancel{x^2}$

$2x+1 \Rightarrow$ ODD

even

Squaring a Binomial

$(x+1)^2$

$(x+1)(x+1)$

$x^2 + x + x + 1$

$x^2 + 2x + 1$

3 Step Rule

1. $(1^{st})^2$
2. $1^{st} \times 2^{nd} \times 2$
3. $(2^{nd})^2$

Review

$(x-5)(x-5)$

$x^2-5x-5x+25$

① $(x-5)^2$

$x^2-10x+25$

② $(2x+3)^2$

$4x^2+12x+9$

Let's do one together...

2 digit #

Let $a \to 1^{st}$ digit
$b \to 2^{nd}$ digit

① $\boxed{10a + b}$

② $a + b$

③ $\boxed{9a}$ ← multiple of 9

p. 30

EXAMPLE 5 Communicating reasoning about a divisibility rule

The following rule can be used to determine whether a number is divisible by 3:

Add the digits, and determine if the sum is divisible by 3. If the sum is divisible by 3, then the original number is divisible by 3.

Use deductive reasoning to prove that the divisibility rule for 3 is valid for two-digit numbers.

Lee's Solution

3 digit #

$100a + 10b + c$

Expanded Number Forms		
Number	Expanded Form (Words)	Expanded Form (Numbers)
9	9 ones	9(1)
27	2 tens and 7 ones	2(10) + 7(1)
729	7 hundreds and 2 tens and 9 ones	7(100) + 2(10) + 9(1)
ab	*a* tens and *b* ones	*a*(10) + *b*(1)

Let *ab* represent any two-digit number.

I let *ab* represent any two-digit number.

$ab = 10a + b$

Since any number can be written in expanded form, I wrote *ab* in expanded form.

$ab = (9a + 1a) + b$

$ab = 9a + (a + b)$

sum of digits ÷ 3

÷ 3

I decomposed 10*a* into an equivalent sum. I used 9*a* because I knew that 9*a* is divisible by 3, since 3 is a factor of 9.

The number *ab* is divisible by 3 only when $(a + b)$ is divisible by 3.

From this equivalent expression, I concluded that *ab* is divisible by 3 only when both 9*a* and $(a + b)$ are divisible by 3. I knew that 9*a* is always divisible by 3, so I concluded that *ab* is divisible by 3 only when $(a + b)$ is divisible by 3.

The divisibility rule has been proved for two-digit numbers.

In Summary p. 31

Key Idea

- Deductive reasoning involves starting with general assumptions that are known to be true and, through logical reasoning, arriving at a specific conclusion.

Need to Know

- A conjecture has been proved only when it has been shown to be true for every possible case or example. This is accomplished by creating a proof that involves general cases.
- When you apply the principles of deductive reasoning correctly, you can be sure that the conclusion you draw is valid.
- The transitive property is often useful in deductive reasoning. It can be stated as follows: Things that are equal to the same thing are equal to each other. If $a = b$ and $b = c$, then $a = c$.
- A demonstration using an example is *not* a proof.

HOMEWORK...

p. 31: #1, 2
#4, 5
#7, 8
#10, 11
#15, 17