

## In Summary

### Key Idea

- A single error in reasoning will break down the logical argument of a deductive proof. This will result in an invalid conclusion, or a conclusion that is not supported by the proof.

### Need to Know

- Division by zero always creates an error in a proof, leading to an invalid conclusion.
- Circular reasoning must be avoided. Be careful not to assume a result that follows from what you are trying to prove.
- The reason you are writing a proof is so that others can read and understand it. After you write a proof, have someone else who has not seen your proof read it. If this person gets confused, your proof may need to be clarified.

## HOMEWORK...

p. 42: #1 - 10  
?? (omit #8)

3, 4, 10

3. Mickey says he can prove that  $2 = 0$ . Here is his proof.

Let both  $a$  and  $b$  be equal to 1.

$$\begin{aligned}
 a &= b \\
 a^2 &= b^2 \\
 a^2 - b^2 &= 0 \\
 (a - b)(a + b) &= 0 \\
 \frac{(a - b)(a + b)}{(a - b)} &= \frac{0}{(a - b)} \\
 1(a + b) &= 0 \\
 a + b &= 0 \\
 1 + 1 &= 0 \\
 2 &= 0
 \end{aligned}$$

- Transitive property
- Squaring both sides
- Subtracting  $b^2$  from both sides
- Factoring a difference of squares
- Dividing both sides by  $a - b$
- Simplifying
- Substitution

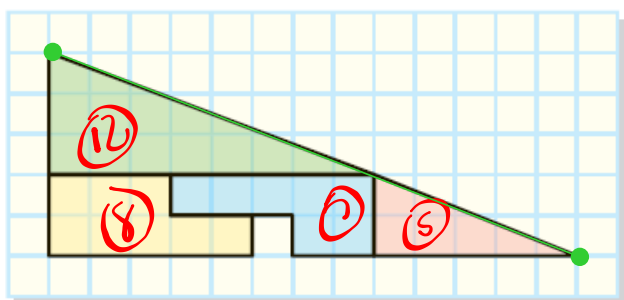
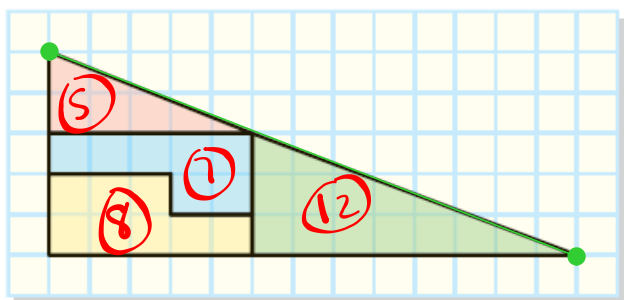
$a - b = 0$

zero

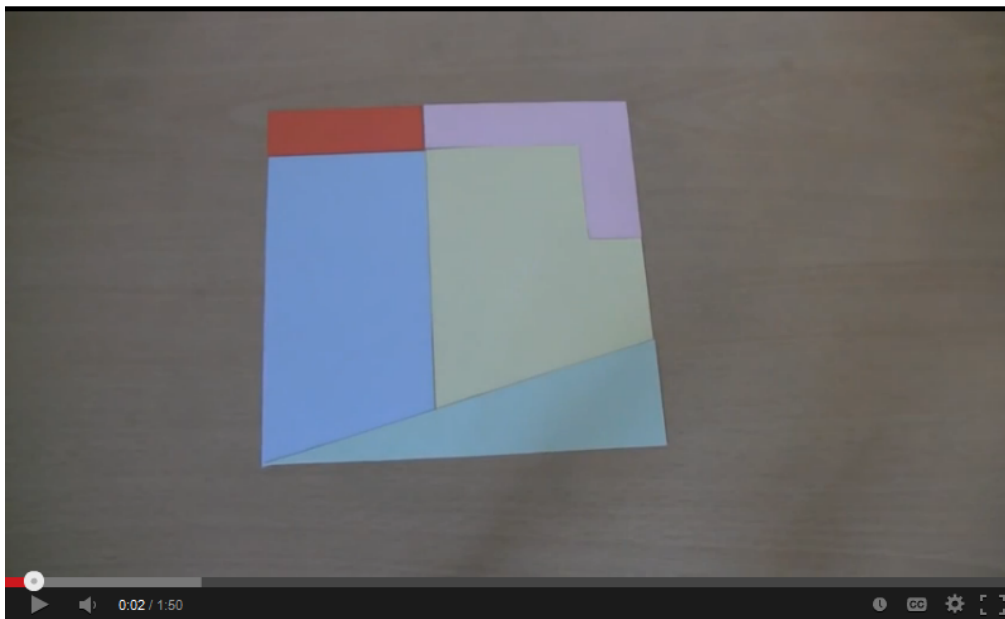
No good!

Explain whether each statement in Mickey's proof is valid.

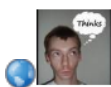
4. Noreen claims she has proved that  $32.5 = 31.5$ .



Is Noreen's proof valid? Explain.



### A Maths Puzzle: The Missing Square Solution



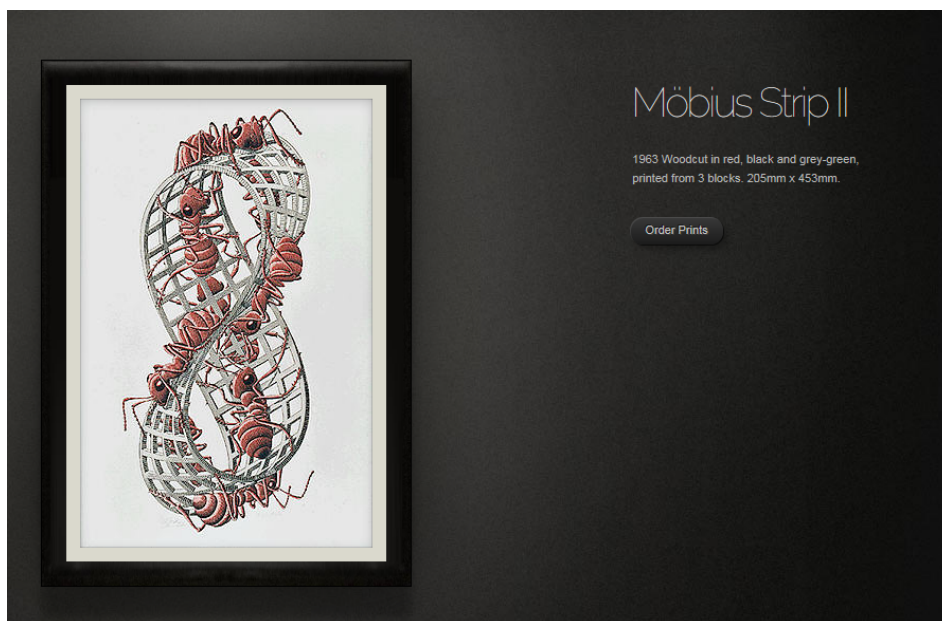
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9. Brittney said she could prove that a strip of paper has only one side. She took a strip of paper, twisted it once, and taped the ends together. Then she handed her friend Amber a pencil, and asked Amber to start at any point and draw a line along the centre of the paper without lifting the pencil. Does a strip of paper have only one side? Why or why not?



10. Brenda was asked to solve this problem:

Three people enjoyed a meal at a Thai restaurant. The waiter brought a bill for \$30. Each person at the table paid \$10.

Later the manager realized that the bill should have been for only \$25, so she sent the waiter back to the table with \$5.

The waiter could not figure out how to divide \$5 three ways, so he gave each person \$1 and kept \$2 for himself.

Each of the three people paid \$9 for the meal.

$$9 \cdot 3 = 27$$

The waiter kept \$2.

$$27 + 2 = 29$$



What happened to the other dollar?

Does the question make sense? How should Brenda answer it?



The image shows a YouTube video player interface. The video title is "Mystery of the Missing Dollar Puzzle" by the channel "AmazingMathGuy". The video has 3,015 views, 14 likes, and 1 dislike. The channel name "AmazingMathGuy" is followed by "14 videos". There is a "Subscribe" button and a notification bell icon with "206" next to it. The video player itself shows a black screen with white text that reads "THE MYSTERY OF THE VANISHING DOLLAR". The video progress bar indicates 0:02 / 4:30.

## 1.6

## Reasoning to Solve Problems

**GOAL**

Solve problems using inductive or deductive reasoning.

**EXPLORE...**

- Suppose that you are lost in the woods for hours and come upon a cabin. In the cabin, you find a lantern, a candle, a wood stove with wood in it, and a match. What do you light first?

**SAMPLE ANSWER**

I would light the match first. If I didn't, I couldn't light any of the other items. I would light the candle next, since it would stay lit for longer than the match and would allow me to light the other two items. Also, it's less likely that I would make an error or fail when lighting the candle. The lantern and the stove would be more difficult to light.



### INVESTIGATE the Math

Emma was given this math trick:

- Choose a number.
- Multiply by 6.
- Add 4.
- Divide by 2.
- Subtract 2.

Emma was asked to use inductive reasoning to make a conjecture about the relationship between the starting and ending numbers, and then use deductive reasoning to prove that her conjecture is always true. Here is her response to the problem:

**Inductive reasoning:**

#	×6	+4	÷2	−2
5	30	34	17	15
−3	−18	−14	−7	−9
0	0	4	2	0
24	144	148	74	72

I followed the steps to work through four examples.  
Conjecture: It is 3 times.

**Deductive reasoning:**

I chose  $d$ .

Then I multiplied, added, divided, and subtracted to get an expression.

$$\left(\frac{6d + 4}{2}\right) - 2$$

It worked.

It simplified to  $3d$ .

**? How can Emma's communication about her reasoning be improved?**

- With a partner, explain why Emma might have chosen the values she did.
- What details are missing from the deductive reasoning Emma used to arrive at the expression  $3d$ ?
- Improve Emma's conjecture, justifications, and explanations.

**Answers**

- Emma might have chosen the four values because each value represents a different attribute. One value is positive, another is negative, another is zero, and the last is a larger number. With this variety, Emma might have thought that she had sampled sufficiently from the range of possible values.
- The explanation does not include reasons for each step, nor does it show what each step looks like. It provides only a summary.
- Conjecture: The resulting value will always be three times the starting value. Justification and explanation:

Let $d$ represent any number.	$d$
Multiply by 6.	$6d$
Add 4.	$6d + 4$
Divide by 2.	$\frac{6d + 4}{2} = 3d + 2$
Subtract 2.	$3d + 2 - 2 = 3d$
The resulting value is three times the starting value.	$3d$

WARM UP PROBLEM: Need 4 gallons using only a 3 and 5 gallon jugs???



SOLUTIONS...

Sarah's

there is an alternate way to solve this:

1. fill the 3 gallon jug
2. pour that 3 gallons into the 5 gallon jug
3. refill the 3 gallon jug
4. fill the 5 gallon jug to the top, leaving 1 gallon in the 3 gallon jug
5. empty the 5 gallon jug
6. pour the 1 gallon from the 3 gallon jug into the 5 gallon jug
7. refill the 3 gallon jug
8. pour that 3 gallons into the 5 gallon jug which already has 1 gallon in it for a total of 4 gallons.

## Die Hard Solution

Step 1. Fill 5 gallon jug

Step 2. Pour 5 gallon jug into 3 gallon jug, leaving 2 remaining gallons in 5 gallon jug.

Step 3. Empty 3 gallon jug.

Step 4. Pour 2 gallons from 5 gallon jug into 3 gallon jug, leaving 1 gallon of empty space.

Step 5. Refill 5 gallon jug.

Step 6. Pour water from 5 gallon jug into 3 gallon jug, which already has 2 gallons in it, and only 1 gallon of empty space, leaving exactly 4 gallons in the 5 gallon jug.



**Die Hard III**

Maths problem-solving with Bruce Willis and Samuel L. Jackson

0:05 / 2:06

**Die Hard III - the water jug problem.**

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15 4

## In Summary

### Key Idea

- Inductive and deductive reasoning are useful in problem solving.

### Need to Know

- Inductive reasoning involves solving a simpler problem, observing patterns, and drawing a logical conclusion from your observations to solve the original problem.
- Deductive reasoning involves using known facts or assumptions to develop an argument, which is then used to draw a logical conclusion and solve the problem.

## **HOMEWORK...**

**p. 48: #10 - 16 (omit 14)**

**p. 55: #5, 7, 10**