HOMEWORK...

p. 236: #7 - 10

NOTE: Each question requires a graph to get possible solutions!

7. a) Graph the solution set for the following system of inequalities. Determine a solution. Check its validity.

$9x + 18y < 18$

$3x - 6y \leq 18$

b) Is each point below a possible solution to the system? How do you know?

i) (4, −1) **ii)** (−2, 2) **iii)** (−4, −2)

iv) (9, 1) **v)** (−2.5, −1.5) **vi)** (2, −2)

LS < RS

LS	RS
0	18

True

LS ≤ RS

LS	RS
0	18

True

$9x + 18y = 18$

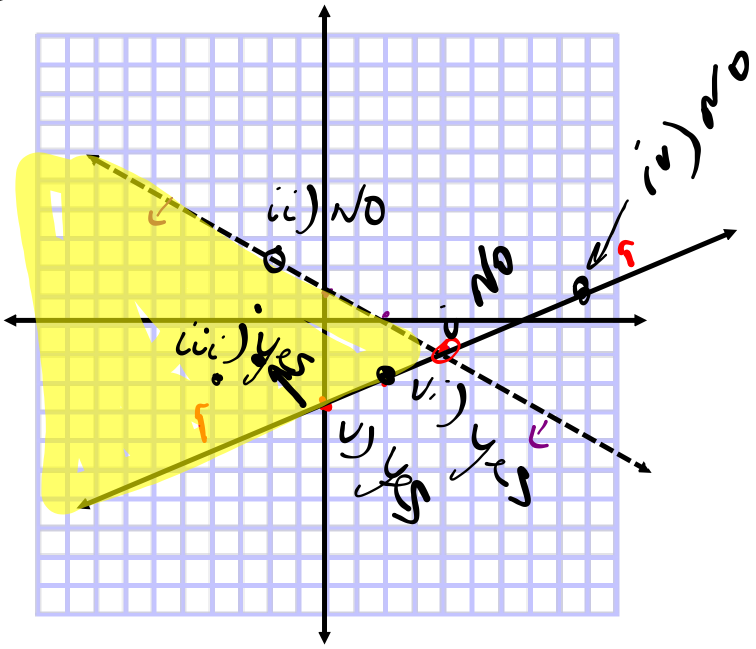
$\frac{18y}{18} = \frac{-9x}{18} + \frac{18}{18}$

$\boxed{y = -\frac{1}{2}x + 1}$

$3x - 6y = 18$

$\frac{-6y}{-6} = \frac{-3x}{-6} + \frac{18}{-6}$

$\boxed{y = \frac{1}{2}x - 3}$



8. Trish is setting up her social networking page:

- She wants to have no more than 500 friends on her new social networking page.
- She also wants to have at least three school friends for every rugby friend.

dependent (y)

independent (x)

a) Define the variables and write a system of inequalities that models this situation.

b) Describe the restrictions on the domain and range of the variables.

c) Graph the solution set to determine two possible combinations of school friends and rugby friends she could have.

Variables

$x \to$ # of rugby friends

$y \to$ # of school friends

$x \in W$

$y \in W$

$x + y \leq 500$

$y \geq 3x$

x int (500, 0)

y int (0, 500)

Test (100, 0)

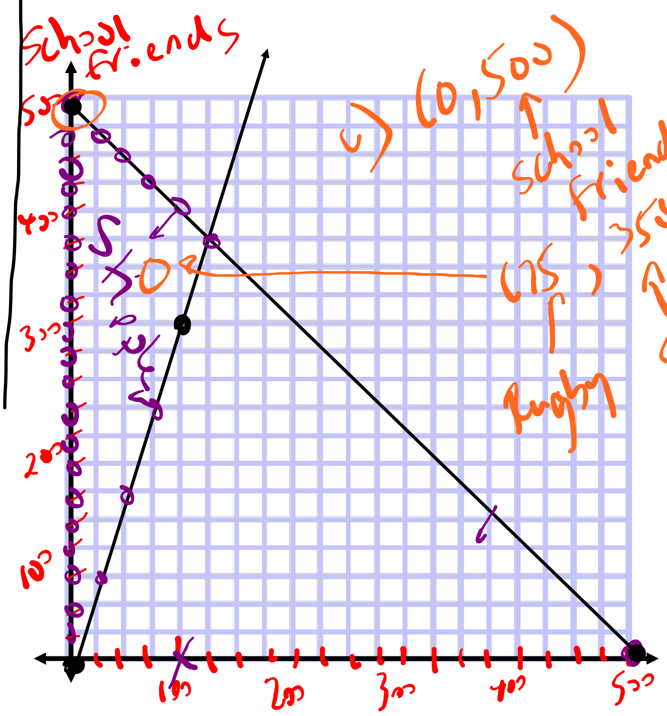
$0 \geq 3(100)$

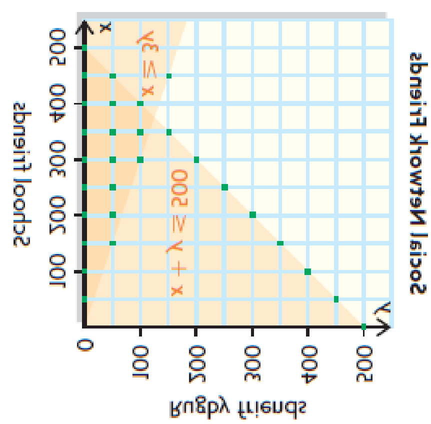
$0 \geq 300$

No

$y = 3x$

x	y
0	0
100	300





8. a) Let x represent the number of school friends. Let y represent the number of rugby friends.

$\{(x, y) \mid x + y \leq 500, x \in W, y \in W\}$

$\{(x, y) \mid x \leq 3y, x \in W, y \in W\}$

b) The variables must be whole numbers: $x \in W$, $y \in W$

c) e.g., 200 school friends and 50 rugby friends; 350 school friends and 100 rugby friends

Notes - Optimization Problems.pdf

5.4 Optimization Problems I: Creating the Model

linear inequality

overlap

optimization problem
A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

constraint
A limiting condition of the optimization problem being modelled, represented by a linear inequality.

objective function
In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized.

feasible region
The solution region for a system of linear inequalities that is modelling an optimization problem.

Need to Know GRAPH

- You can $\boxed{\text{create a model}}$ for an optimization problem by following these steps:

  Step 1. Identify the quantity that must be optimized. Look for key words, such as *maximize* or *minimize*, *largest* or *smallest*, and *greatest* or *least*.

  objective function

  Step 2. Define the variables that affect the quantity to be optimized. Identify any restrictions on these variables.

  Step 3. Write a system of linear inequalities to describe all the constraints of the problem. Graph the system.

  Step 4. Write an objective function to represent the relationship between the variables and the quantity to be optimized.

EXAMPLE of an OPTIMIZATION Problem...

Mick and Keith make MP3 covers to sell, using beads and stickers.

- At most, 45 covers with stickers and 55 bead covers can be made per day.
- Mick and Keith can make 45 or more covers, in total, each day.
- It costs \$0.75 to make a cover with stickers, \$1.00 to make one with beads.

Objective

Let x represent the number of covers with stickers and let y represent the number of bead covers.

Let C represent the cost of making the covers.

RESTRICTIONS: $x \in W$ $y \in W$

CONSTRAINTS: $x \le 45$ $y \le 55$ $x + y \ge 45$

$x = 45$ $y = 55$

LS ≥ RS	
0+0	45
0	

OBJECTIVE FUNCTION:

a) Graph the solution set.

$x + y = 45$

x-int (let $y = 0$)

$x + 0 = 45$

$x = 45$

$(45, 0)$

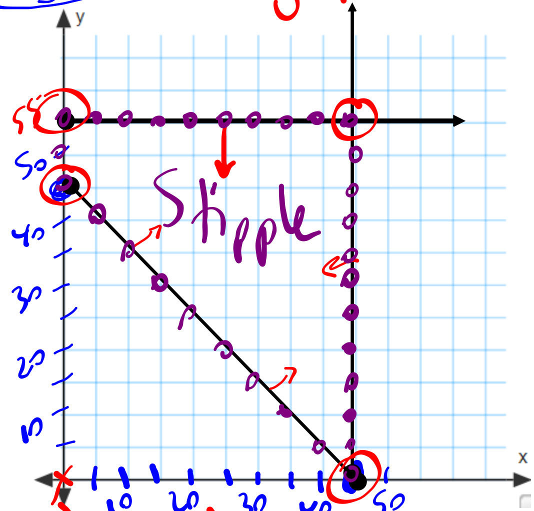
y-int

$0 + y = 45$

$y = 45$

$(0, 45)$

Bead Cover



Sticker Cover

b) What are the vertices of the feasible region?

Points of intersection

$(0, 45)$ $(45, 0)$ $(0, 55)$ $(45, 55)$

c) Which point would result in the maximum value of the objective function?

$(44, 55) \Rightarrow \$88.75$

d) Which point would result in the minimum value of the objective function?

$(45, 0) \Rightarrow \$33.75$

* Not graphed Objective Function (Make \$)

Use it to decide which solution is max/min (Vertices)

$$C = \$0.75x + 1.00y$$

Vertex	Cost
$(0,45)$	\$45
$(0,55)$	$0.75(0) + 1.00(55) = \$55$
$(45,0)$	$0.75(45) + 1.00(0) = \$33.75$
$(45,55)$	$0.75(45) + 1.00(55) = 88.75$

GOAL

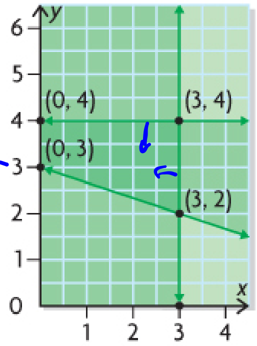
Solve optimization problems.

EXPLORE...

• The following system of linear inequalities has been graphed below:

System of linear inequalities:

$y \geq 0$
$x \geq 0$ Quad I
$y \leq 4$
$x \leq 3$
$3y \geq -x + 9$



a) For each objective function, what points in the feasible region represent the minimum and maximum values?
 i) $T = 5x + y$
 ii) $T = x + 5y$

b) What do you notice about the optimal points for the two objective functions? Why do you think this happened?

SAMPLE ANSWER

a) i) For $T = 5x + y$,

If (x, y) is...	Then...	
(3, 2)	$T = 5(3) + 2$ $T = 17$	
(3, 4)	$T = 5(3) + 4$ $T = 19$	maximum
(0, 3)	$T = 5(0) + 3$ $T = 3$	minimum
(0, 4)	$T = 5(0) + 4$ $T = 4$	

ii) For $T = x + 5y$,

If (x, y) is...	Then...	
(3, 2)	$T = 3 + 5(2)$ $T = 13$	minimum
(3, 4)	$T = 3 + 5(4)$ $T = 23$	maximum
(0, 3)	$T = 0 + 5(3)$ $T = 15$	
(0, 4)	$T = (0) + 5(4)$ $T = 20$	

b) I noticed that the values of the coefficients of the variables and the values of the variables themselves all contribute to the value of the objective function. For $T = 5x + y$, the x-value is multiplied by 5 and the y-value is multiplied by 1. For $T = x + 5y$, the x-value is multiplied by 1 and the y-value is multiplied by 5. In each case, the greater the coordinate that is multiplied by 5, the greater the value of the objective function is. The converse is true for the least values.

HOMEWORK...

Page 248: #1, #2, #3, #5

NOTE:

Create a model means graph the solution region

Attachments

Notes - Optimization Problems.pdf