

# HOMWORK???

p. 225: #1 & 2

p. 235: #2, 5 & 6

5. For each system of linear inequalities,

- a) graph the solution set.
- b) describe the solution region.
- c) determine a solution. Check its validity.
  - i)  $\{(x, y) \mid x + y > 5, x \in \mathbb{R}, y \in \mathbb{R}\}$   
 $\{(x, y) \mid y \leq 4, x \in \mathbb{R}, y \in \mathbb{R}\}$
  - ii)  $\{(x, y) \mid x + y \geq 1, x \in \mathbb{N}, y \in \mathbb{N}\}$   
 $\{(x, y) \mid 2 > x - 2y, x \in \mathbb{N}, y \in \mathbb{N}\}$
  - iii)  $\{(x, y) \mid 4x + 11y > 44, x \in \mathbb{W}, y \in \mathbb{W}\}$   
 $\{(x, y) \mid 2x - 6 \leq 3y, x \in \mathbb{W}, y \in \mathbb{W}\}$

$$4x + 11y = 44$$

$$\frac{11y}{11} = \frac{-4x + 44}{11}$$

$$y = -\frac{4}{11}x + 4$$

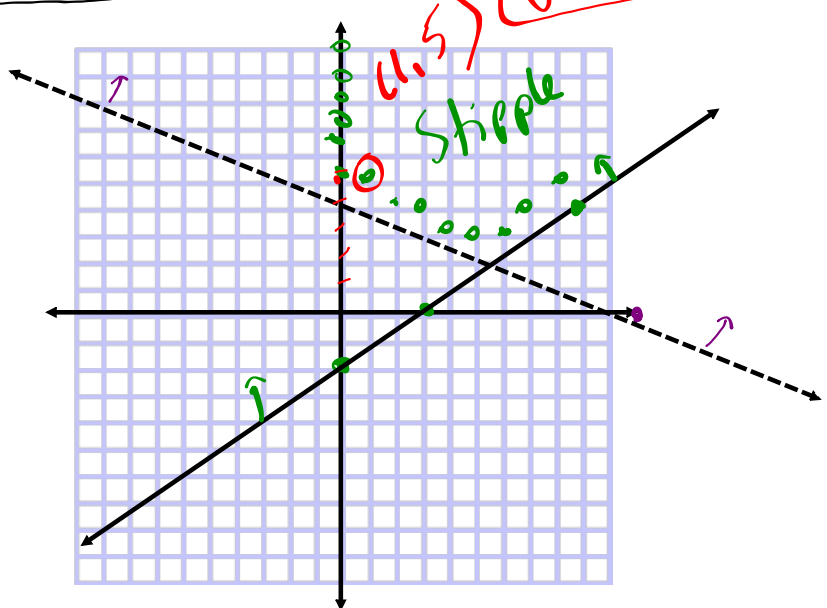
Test (0,0)  $0 > 44$  No

Test (0,0)  $0 \leq 6$  yes

$$\frac{2x - 6}{3} = \frac{3y}{3}$$

$$\frac{2x - 6}{3} = y$$

$$y = \frac{2x - 6}{3}$$



6. The staff in a cafeteria are making two kinds of sandwiches: egg salad, and ham and cheese:

- A maximum of 450 sandwiches are needed.
- Based on previous demand, there should be at least twice as many ham and cheese sandwiches as egg salad sandwiches.

- Define the variables and write a system of inequalities that models this situation.
- Describe the restrictions on the variables in this situation.
- Graph the system to determine the solution set.
- Suggest two combinations of numbers of sandwiches that the cafeteria staff could make.

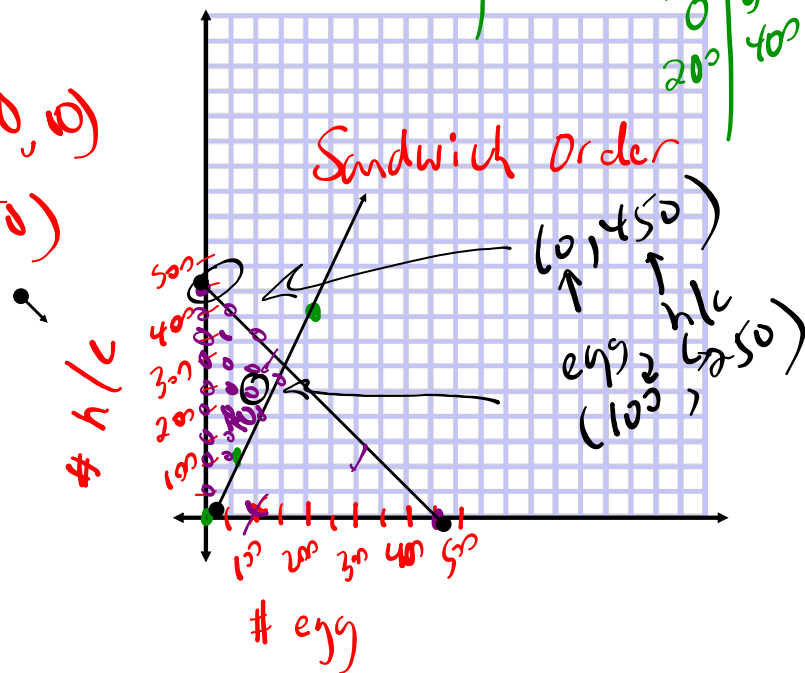
$y$   
h/c depends on egg  
 $x$

Variables  $\Rightarrow$   $x \rightarrow$  # of egg sandwiches  
 $y \rightarrow$  # of ham and cheese

Restrictions  $\Rightarrow$   $x \in W$   
 $y \in W$  } Quad I

Inequalities  $\Rightarrow$   $x + y \leq 450$  Test  $(0, 0) \{ 0 \geq 0 \}$  RS  
 $y \geq 2x$  No/200  
 $y = 2x$   $\frac{x}{y}$   
 $\frac{0}{200} \mid \frac{0}{400}$

$x + y = 450$   
 $x_{int}$   $\Rightarrow (450, 0)$   
 $y_{int}$   $\Rightarrow (0, 450)$



**LEARN ABOUT the Math** \*\*\* Can be found on p. 226

A company makes two types of boats on different assembly lines: aluminum fishing boats and fiberglass bow riders.



- When both assembly lines are running at full capacity, a maximum of 20 boats can be made in a day.
- The demand for fiberglass boats is greater than the demand for aluminum boats, so the company makes at least 5 more fiberglass boats than aluminum boats each day.

1 What combinations of boats should the company make each day?

**EXAMPLE 1** Solving a problem with discrete whole-number variables using a system of inequalities

**Mary's Solution: Using graph paper**

Let  $a$  represent the number of aluminum fishing boats.  
Let  $f$  represent the number of fiberglass bow riders.

$$a \in \mathbb{W} \text{ and } f \in \mathbb{W}$$

The relationship between the two types of boats can be represented by this system of inequalities:

$$a + f \leq 20$$

$$a + 5 \leq f$$

$a + f = 20$	$a$ -intercept:	$a + 0 = 20$
$f$ -intercept:	$a = 20$	
$0 + f = 20$		
$f = 20$		
$(0, 20)$		$(20, 0)$

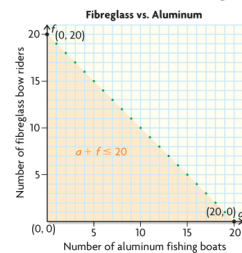
$a + 5 = f$	$a$ -intercept:	$a + 5 = 0$
$f$ -intercept:	$a = -5$	
$0 + 5 = f$		
$f = 5$		
$(0, 5)$		$(-5, 0)$

$(5, 10)$  is a point on this boundary.

Test  $(0, 0)$  in  $a + f \leq 20$ .

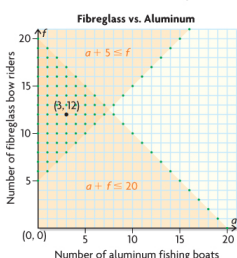
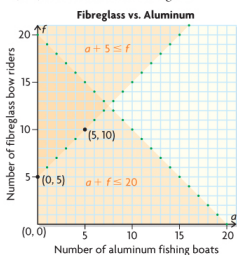
LS	RS
$a + f$	20
$0 + 0$	
0	

Since  $0 \leq 20$ ,  $(0, 0)$  is in the solution region.



Test $(0, 0)$ in $a + 5 \leq f$ .	
LS	RS
$a + 5$	$f$
$0 + 5$	0
5	

Since 5 is not less than or equal to 0,  $(0, 0)$  is not in the solution region.



$$\{(a, f) \mid a + f \leq 20, a \in \mathbb{W}, f \in \mathbb{W}\}$$

$$\{(a, f) \mid a + 5 \leq f, a \in \mathbb{W}, f \in \mathbb{W}\}$$

Any point with whole-number coordinates in the intersecting or overlapping region is an acceptable combination. For example, 3 aluminum boats and 12 fiberglass boats is an acceptable combination.

I knew I could solve this problem by representing the situation algebraically with a system of two linear inequalities and graphing the system.

Since only complete boats are sold, I knew that  $a$  and  $f$  are whole numbers and the graph would consist of discrete points in the first quadrant.

The two inequalities describe

- a combination of boats to a maximum of 20.
- at least 5 more fiberglass boats than aluminum boats.

To graph each linear inequality, I knew I had to graph its boundary as a stippled line, and then shade and stipple the correct half plane.

To graph each boundary, I wrote each linear equation and then determined the  $a$ - and  $f$ -intercepts so I could plot and join them.

For  $a + 5 = f$ , I knew  $(-5, 0)$  wasn't going to be a point on the boundary, because it's not in the first quadrant, so I chose another point by solving the equation for  $a = 5$ .

I tested point  $(0, 0)$  to determine which half plane to shade for  $a + f \leq 20$ .

I drew a green stippled boundary connecting  $(0, 20)$  and  $(20, 0)$  and shaded the half plane below it orange, because the solution region is discrete.

I tested  $(0, 0)$  to determine which half plane to shade for  $a + 5 \leq f$ .

I plotted the points  $(0, 5)$  and  $(5, 10)$  on the same coordinate plane. I used these points to draw a green stippled boundary for  $a + 5 \leq f$ .

I shaded the half plane above the boundary orange, since the test point  $(0, 0)$  is not a solution to the linear inequality and the solution region is discrete.

I knew that the solution set for the system of linear inequalities is represented by the intersection or overlap of the solution regions of the two inequalities. This made sense since points in this region satisfy both inequalities.

I knew that the triangular solution region included discrete points along its three boundaries, including the  $y$ -axis from  $y = 5$  to  $y = 20$ .

Since the solution set for the system contains only discrete points with whole-number coordinates, I stippled its solution region.

I knew that any whole-number point in the triangular solution region is a possible solution. For example,  $(3, 12)$  is a possible solution.

I knew that  $(3, 12)$  worked because this gives a total of 15 boats with 9 more fiberglass boats than aluminum boats.

**LEARN ABOUT the Math** \*\*\* Can be found on p. 226

A company makes two types of boats on different assembly lines: aluminum fishing boats and fiberglass bow riders.



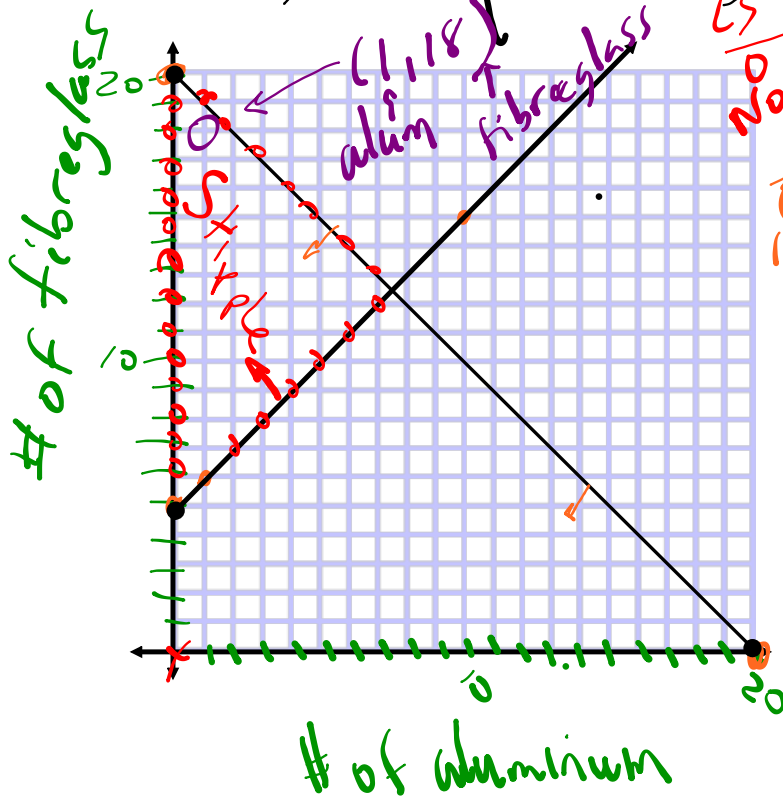
- When both assembly lines are running at full capacity, a maximum of 20 boats can be made in a day.
- The demand for fiberglass boats is greater than the demand for aluminum boats, so the company makes at least 5 more fiberglass boats than aluminum boats each day.

? What combinations of boats should the company make each day?

a) Variables  $\Rightarrow$   $x \rightarrow$  # of aluminum  
 $y \rightarrow$  # of fiberglass

b) Restrictions  $\Rightarrow$   $x \in \mathbb{W}$   
 $y \in \mathbb{W}$

c) Inequalities  $\Rightarrow$   $x + y \leq 20$



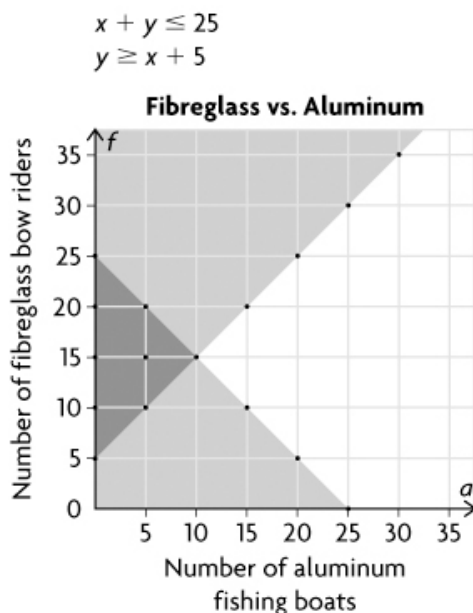
$x + y \leq 20$   
 $y \geq x + 5$   
 $y = \frac{1}{1}x + 5$   
 $x + y = 20$   
 $x$ -int  $(20, 0)$   
 $y$ -int  $(0, 20)$

### Reflecting

- A. Is every point on the boundaries of the solution region a possible solution? Explain.
- B. Are the three points where the boundaries intersect part of the solution region? Explain.
- C. How would the graph change if fewer than 25 boats were made each day?
- D. All points with whole-number coordinates in the solution region are valid, but are they all reasonable? Explain.

### Answers

- A. No. Only whole-number coordinate points on the boundaries are part of the solution region, because the variables represent numbers of boats and only whole numbers of boats make sense.
- B. Yes. Equality is possible for both inequalities, and all of these points have whole-number coordinates: (0, 5), and (0, 20).) Intersection not whole numbers.
- C.  $a + f \leq 20 \rightarrow a + f \leq 25$   
The solution region would be larger, because its boundary would move up.



- D. This would depend on the market. For example, if there was a high demand for boats, then points in the solution region with high coordinates, such as (7, 13), would probably make more sense. If there was a low demand for fishing boats, then points with low x-coordinates, such as (0, 20), would make more sense.

## **HOMEWORK...**

p. 236: #7 - 10

***NOTE:*** Each question requires a graph to get possible solutions!