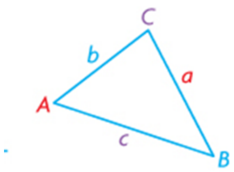
# Trigonometry Summary AND 'The AMBIGUOUS Case'...




sine law
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

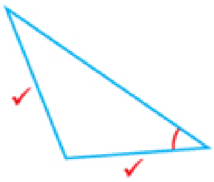
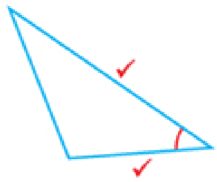
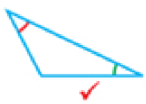
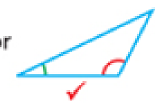
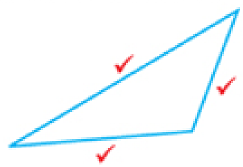
cosine law
$$a^2 = b^2 + c^2 - 2bc \cos A$$

**oblique triangle**

A triangle that does not contain a 90° angle.

## Need to Know

- The sine law and cosine law are used with obtuse triangles in the same way that they are used with acute triangles.

| Use the sine law when you know ... | Use the cosine law when you know ... |
|---|---|
| - the lengths of two sides and the measure of the angle that is opposite a known side  | - the lengths of two sides and the measure of the contained angle  |
| - the measures of two angles and the length of any side  or  | - the lengths of all three sides  |

Ambiguous Case

- Be careful when using the sine law to determine the measure of an angle. The inverse sine of a ratio always gives an acute angle, but the supplementary angle has the same ratio. You must decide whether the acute angle, $\theta$, or the obtuse angle, $180° - \theta$, is the correct angle for your triangle.

# Notes - Ambiguous Case.pdf

## In Summary

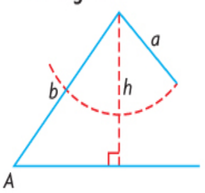
### Key Idea

- The ambiguous case of the sine law may occur when you are given two side lengths and the measure of an angle that is opposite one of these sides. Depending on the measure of the given angle and the lengths of the given sides, you may need to construct and solve zero, one, or two triangles.

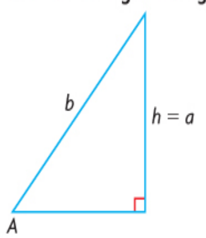
### Need to Know

- In $\triangle ABC$ below, where $h$ is the height of the triangle, $\angle A$ and the lengths of sides $a$ and $b$ are given, and $\angle A$ is acute, there are four possibilities to consider:

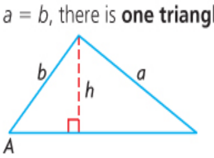
If $\angle A$ is acute and $a < h$, there is **no triangle.**



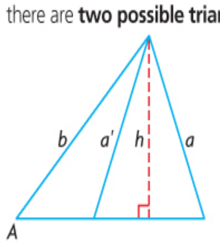

If $\angle A$ is acute and $a = h$, there is **one right triangle.**




If $\angle A$ is acute and $a > b$ or $a = b$, there is **one triangle.**

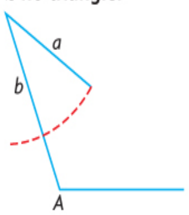



If $\angle A$ is acute and $h < a < b$, there are **two possible triangles.**




- If $\angle A$, $a$, and $b$ are given and $\angle A$ is obtuse, there are two possibilities to consider:

If $\angle A$ is obtuse and $a < b$ or $a = b$, there is **no triangle.**

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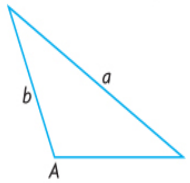
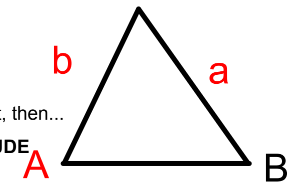



## Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b




*** If ALL 3 criteria are met, then...

**CALCULATE THE ALTITUDE**

$\boxed{\text{alt} = b \sin A}$

**CASE 1**: a < altitude; there is NO SOLUTION

**CASE 2**: a = altitude; there is ONE SOLUTION [Right Triangle]

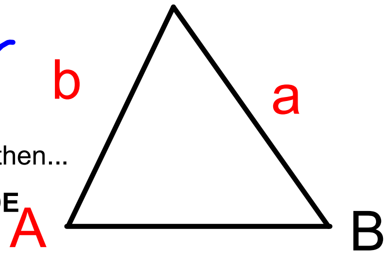
**CASE 3**: a >altitude; this is the 'AMBIGUOUS CASE'...TWO SOLUTIONS

1) Acute Triangle (angle, θ, is found with Law of Sines)
2) Obtuse Triangle (angle is 180° - θ)

**MUST MEMORIZE THESE NOTES IN ORDER TO KNOW AMBIGUOUS CASE**

## Criteria for the Ambiguous Case...

- Must be given SSA ✓
- Given angle is acute ✓
- a < b ✓

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1) Acute Triangle (angle, θ, is found with Law of Sines)
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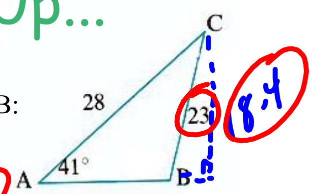
## Back to the Warm-Up...

Determine the measure of the obtuse angle B:




$\text{alt} = 28 \sin 41°$

$\text{alt} = 18.4$

$\boxed{a > \text{alt}}$ ∴ ambiguous case

$$\frac{\sin B}{\cancel{28}} = \frac{\sin 41°}{23}$$

$\sin^{-1} \sin B = (0.7987) \sin^{-1}$

$\angle B = 53°$ ← acute

∴ ambiguous

or $\angle B = 180 - 53$

$\boxed{127°}$

**EXAMPLE 1** Connecting the *SSA* situation to the number of possible triangles

p.177

- SSA ✓
- angle given is acute ✓
- $a < b$ ✓

Given each *SSA* situation for $\triangle ABC$, determine how many triangles are possible.

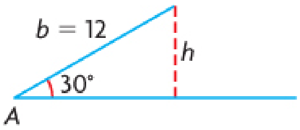
**a)** $\angle A = 30°$, $a = 4$ m, and $b = 12$ m
**b)** $\angle A = 30°$, $a = 6$ m, and $b = 12$ m
**c)** $\angle A = 30°$, $a = 8$ m, and $b = 12$ m
**d)** $\angle A = 30°$, $a = 15$ m, and $b = 12$ m

d) one solution
$a > b$

$h = b \sin A$
$h = 12 \sin 30$
$h = 6$

Cases...
1) $a < h$ (a) no solution
2) $a = h$ (b) 1 right triangle
*ambiguous 3) $a > h$ (c) 2 solutions

### Saskia's Solution



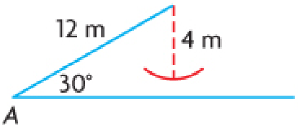

I drew the beginning of a triangle with a 30° angle and a 12 m side.

$$\sin 30° = \frac{h}{12}$$
$$12 \sin 30° = h$$
$$6 \text{ m} = h$$

I used the sine ratio to calculate the height of the triangle.

I can use this height as a benchmark to decide on side lengths opposite the 30° angle that will result in zero, one, or two triangles.

**a)** $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

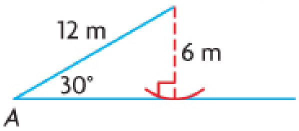



No triangles are possible.

Since $a < b$ and $a < h$, I knew that no triangles are possible.

I used a compass to be certain. I set the compass tips to represent 4 m. I placed one tip of the compass at the open end of the 12 m side and swung the pencil tip toward the other side. The pencil couldn't reach the base, so a 4 m side could not close the triangle.

**b)** $\angle A = 30°$, $a = 6$ m, and $b = 12$ m

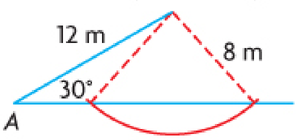



One triangle is possible.

Since $a < b$ and $a = h$, there is only one possible triangle, a right triangle.

A compass arc intersects the base at only one point.

**c)** $\angle A = 30°$, $a = 8$ m, and $b = 12$ m

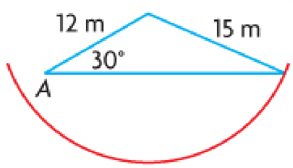



Two triangles are possible.

Since $a < b$ and $a > h$, there are two possible triangles.

A compass arc intersects the base at two points.

**d)** $\angle A = 30°$, $a = 15$ m, and $b = 12$ m



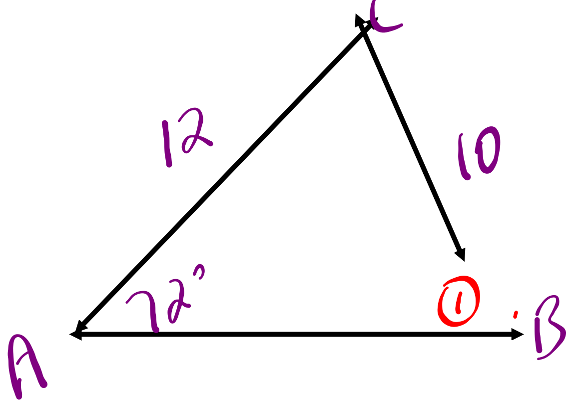

One triangle is possible.

Since $a > b$, only one triangle is possible.

A compass arc intersects the base at only one point.

Example 2:

Solve the triangle ABC if $a = 10$, $b = 12$ and angle A = 72°.




Ambiguous ?

- SSA ✓
- acute ✓
- $a < b$ ✓

$\boxed{alt = b \sin A}$

$= 12 \sin 72°$

$alt = 11.4$

$a = 10$

$alt = 11.4$

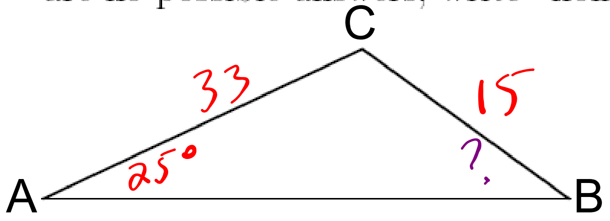
① $a < alt$ ② $a = alt$ ③ $a > alt$

✓

No solution

## Example 3:

Given that A $= 25°$, $a = 15$, and $b = 33$, find the measure of angle B to the nearest degree. If there are two answers, give both of them. If there are no possible answers, write "none".




- SSA ✓
- acute ✓
- $a < b$ ✓

$15 < 33$

Case #1: $a < alt$

Case #2: $a = alt$

Case #3: $a > alt$  $15 > 13.9$

$alt = b \sin A$

$= 33 \sin 25°$

$alt = 13.9$

ambiguous ⇒ 2 solutions

$$\cancel{33}\,\frac{\sin B}{\cancel{33}} = \frac{33 \sin 25°}{15}$$

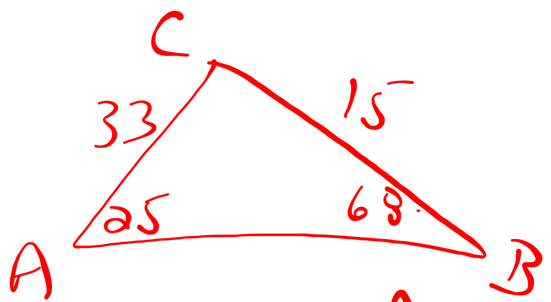
$$\cancel{\sin^{-1}}\ \sin B = \sin^{-1}(0.9298)$$

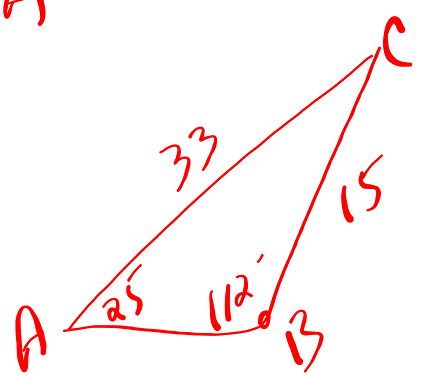
$$\boxed{\angle B = 68°}$$

OR

$\angle B = 180° - 68°$

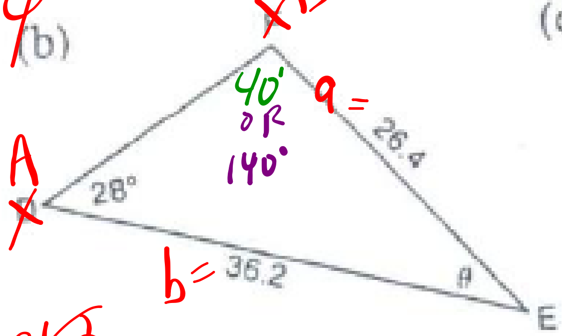
$$\boxed{\angle B = 112°}$$



HW

4 (b)




A, B, $a = 26.4$, $b = 36.2$, 40° or 140°

(c $a > alt$

$26.4 > 17.0$

- SSA ✓
- acute ✓
- $a < b$ ✓

$alt = 36.2 \sin 28°$

$alt = 17.0$

1) $a < alt$
2) $a = alt$
3) $a > alt$

* ambiguous 2 solutions

$$\frac{\sin F}{\cancel{36.2}} \cdot 36.2 = \frac{\sin 28°}{26.4} \cdot 36.2$$

$$\sin^{-1}(\sin F) = \sin^{-1}(0.6437)$$

$* \angle F = 40°$ —or→ $\angle F = 180 - 40°$

$\angle F = 140°$

$\angle E = 112°$ or $\angle E = 12°$

# HOMEWORK...

Worksheet - Ambiguous Case.pdf

Do questions #1, 2, 4, 5

**MEMORIZE!!!**

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- Given angle is acute
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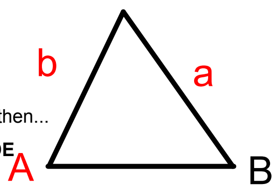
$\boxed{\text{alt} = b \sin A}$

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Attachments

Notes - Ambiguous Case.pdf

Worksheet - Ambiguous Case.pdf