WARM-UP...

1. Grab a calculator. (you won't be able to do this one in your head)
2. Key in the first three digits of your phone number (NOT the area code)
3. Multiply by 80
4. Add 1
5. Multiply by 250
6. Add the last 4 digits of your phone number
7. Add the last 4 digits of your phone number again.
8. Subtract 250
9. Divide number by 2

Do you recognize the answer?

3 digit # $\Rightarrow$ $100a + 10b + c$

WHY??? Prove by deduction...

2 $80(100a + 10b + c)$

3 $250(8000a + 800b + 80c + 1)$

4

5 $\dfrac{2000000a + 200000b + 20000c + \cancel{250} + 2000d + 200e + 20f + 2g}{2}$

6

7 $1000000a + 100000b + 10000c + 1000d + 100e + 10f + g$

8

9

In Summary p. 31

Key Idea

- Deductive reasoning involves starting with general assumptions that are known to be true and, through logical reasoning, arriving at a specific conclusion.

Need to Know

- A conjecture has been proved only when it has been shown to be true for every possible case or example. This is accomplished by creating a proof that involves general cases.
- When you apply the principles of deductive reasoning correctly, you can be sure that the conclusion you draw is valid.
- The transitive property is often useful in deductive reasoning. It can be stated as follows: Things that are equal to the same thing are equal to each other. If $a = b$ and $b = c$, then $a = c$.
- A demonstration using an example is *not* a proof.

Questions

~~HOMEWORK...~~

p. 31: #1, 2
#4, 5
#7, 8
#10, 11
#15, 17

17. Simon made the following conjecture: When you add three consecutive numbers, your answer is always a multiple of 3. Joan, Garnet, and Jamie took turns presenting their work to prove Simon's conjecture. Which student had the strongest proof? Explain.

Inductive / Deductive STRONGEST

Deductive

Joan's Work Inductive		Garnet's Work Inductive	Jamie's Work
$1 + 2 + 3 = 6$	$3 \cdot 2 = 6$	$3 + 4 + 5$	Let the numbers be n, $n + 1$, and $n + 2$.
$2 + 3 + 4 = 9$	$3 \cdot 3 = 9$		
$3 + 4 + 5 = 12$	$3 \cdot 4 = 12$	The two outside numbers (3 and 5) add to give twice the middle number (4). All three numbers add to give 3 times the middle number.	$n + n + 1 + n + 2 = 3n + 3$
$4 + 5 + 6 = 15$	$3 \cdot 5 = 15$		$n + n + 1 + n + 2 = 3(n + 1)$
$5 + 6 + 7 = 18$	$3 \cdot 6 = 18$		
and so on ...			
Simon's conjecture is valid.		Simon's conjecture is valid.	Simon's conjecture is valid.

GCF

multiple of 3

11. Cleo noticed that whenever she determined the difference between the squares of consecutive even numbers or the difference between the squares of consecutive odd numbers, the result was a multiple of 4. Show inductively that this pattern exists. Then prove deductively that it exists.

Inductive

```
4²-2²
              12
6²-4²
              20
12²-10²
              44
```

```
7²-5²
              24
11²-9²
              40
23²-21²
              88
```

Deductive

$$x^2 - (x-2)^2$$

$$x^2 - (x^2 - 4x + 4)$$

$$\cancel{x^2} - \cancel{x^2} + 4x - 4$$

$$4x - 4$$

GCF

$$4(x-1)$$

Multiple of 4

Squaring Binomials

$(x-2)^2$

Distribute

$(x-2)(x-2)$

$x^2-2x-2x+4$

x^2-4x+4

"3 Step Rule"

① $(1^{st})^2$

② $1^{st} \times 2^{nd} \times 2$

③ $(2^{nd})^2$

x^2-4x+4

ex $(3x-4)^2$

$9x^2-24x+16$

$(3x-4)(3x-4)$

$9x^2-12x-12x+16$

$9x^2-24x+16$

APPLY the Math p. 28

EXAMPLE 2 Using deductive reasoning to generalize a conjecture

In Lesson 1.3, page 19, Luke found more support for Steffan's conjecture from Lesson 1.1, page 9—that the difference between consecutive perfect squares is always an odd number.

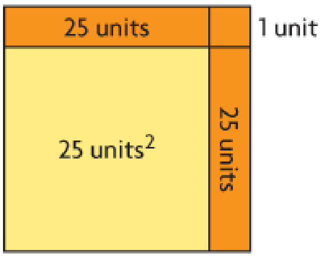
Determine the general case to prove Steffan's conjecture.

Back to previous lesson...

Gord's Solution

The difference between consecutive perfect squares is always an odd number.

> Steffan's conjecture has worked for consecutive perfect squares with sides of 1 to 7 units.



$$26^2 - 25^2 = 2(25) + 1$$
$$26^2 - 25^2 = 51$$

> I tried a sample using even greater squares: 26^2 and 25^2.
>
> The difference is the two sets of 25 unit tiles, plus a single unit tile.

Let x be any natural number.
Let D be the difference between consecutive perfect squares.
$D = (x + 1)^2 - x^2$

> Since the conjecture has been supported with specific examples, I decided to express the conjecture as a general statement. I chose x to be the length of the smaller square's sides. The larger square's sides would then be $x + 1$.

$D = x^2 + x + x + 1 - x^2$
$D = x^2 + 2x + 1 - x^2$
$D = 2x + 1$

> I expanded and simplified my expression. Since x represents any natural number, $2x$ is an even number, and $2x + 1$ is an odd number.

Steffan's conjecture, that the difference of consecutive perfect squares is always an odd number, has been proved for all natural numbers.

In Lesson 1.3, page 19, Luke found more support for Steffan's conjecture from Lesson 1.1, page 9—that the difference between consecutive perfect squares is always an odd number.

Determine the general case to prove Steffan's conjecture.

Inductive

9^2-8^2	7
21^2-20^2	17
5^2-4^2	41
	9

Deductive

$$x^2 - (x-1)^2$$

$$x^2 - (x^2 - 2x + 1)$$

$$\cancel{x^2} - \cancel{x^2} + 2x - 1$$

$$\underbrace{2x}_{\text{even}} - 1$$

even $-1 \to \boxed{\text{odd}}$

Revisit
~~Questions~~

~~HOMEWORK...~~

p. 31: #1, 2
#4, 5
#7, 8
#10, 11
#15, 17

p. 23 #17
#20

p. 32 #9

p. 35 #9

HOMEWORK