HOMEWORK Questions (Wed)

Revisit ~~Questions~~

HOMEWORK...

p. 31: #1, 2
#4, 5
#7, 8
#10, 11
#15, 17

p. 23 #17
#20

p. 32 #9

p. 35 # 9

5. Prove that the product of an even integer and an odd integer is always even.

Even # $\rightarrow$ $2x$
Odd # $\rightarrow$ $2x+1$

$4 \times 3 = 12$
$8 \times 7 = 56$
$12 \times 1 = 12$

inductively

$2x(2x+1)$

$4x^2 + 2x \xrightarrow{\text{GCF}} 2(2x^2 + x)$

even

even + even $\rightarrow$ even

deductively

9. Recall Jarrod's number trick from Lesson 1.3, page 24:

* Test

• Choose a number.	11	1	10	x
• Double it.	22	2	20	$2x$
• Add 6.	28	8	26	$2(2x+6)$
• Double again.	56	16	52	$4x+12-4$
• Subtract 4.	52	12	48	$\frac{4x+8}{4}$
• Divide by 4.	13	3	12	$x+\cancel{2-2}$
• Subtract 2.	11	1	10	x

Get back the original #

Prove that any number you choose will be the final result.

inductive

deductive

1.5

Proofs That Are Not Valid

NOTE: Watch for...
- sentences that use the word ***all***
- division of zero

REMEMBER: Ask yourself does it make sense?

GOAL

Identify errors in proofs.

Logical Errors

Although deductive reasoning seems rather simple, it can go wrong in more than one way. Deductive reasoning based on incorrect premises leads to faulty conclusions. Similarly, a single error in reasoning will result in an invalid or unsupported conclusion, destroying a deductive proof.

Everyday situations are filled with examples of incorrect deductive reasoning, or **logical errors**.

Common logical errors include:

- A false assumption or generalizing
- An error in reasoning, like division by zero
- An error in calculation

Your Turn

Zack is a high school student. All high school students dislike cooking. Therefore, Zack dislikes cooking. Where is the error in the reasoning?

Answer

Communication *Tip*

Stereotypes are generalizations based on culture, gender, religion, or race. There are always counterexamples to stereotypes, so conclusions based on stereotypes are not valid.

EXAMPLE #1...

A **fallacy** is an incorrect conclusion arrived at by apparently correct, though flawed, reasoning. Such misleading or deceptive reasoning is called **specious reasoning**.

The most common example of a mathematical fallacy is the following specious proof that 1 = 2.

Let $a = b$.
Then: $ab = a^2$
$ab-b^2 = a^2-b^2$
$b(a-b) = (a+b)(a-b)$
$b = 2b$
$1 = 2$

Solution...

The error that makes this "proof" incorrect occurs in the following step, where each side is divided by (a-b). Since $a = b$ in this "proof," then a-b = 0, and dividing by zero is not permitted in algebra.

$b(a-b) = (a+b)(a-b)$
$b = 2b$

EXAMPLE 2 Using reasoning to determine the validity of a proof

$a + b - c = 0$

Bev claims he can prove that $3 = 4$.

Bev's Proof

Suppose that: $a + b = c$

This statement can be written as: $4a - 3a + 4b - 3b = 4c - 3c$

After reorganizing, it becomes: $4a + 4b - 4c = 3a + 3b - 3c$

Using the distributive property, $4(a + b - c) = 3(a + b - c)$

Dividing both sides by $(a + b - c)$, $4 = 3$

GCF ÷ $a+b-c$ [÷0] error

Show that Bev has written an **invalid proof**.

invalid proof A proof that contains an error in reasoning or that contains invalid assumptions.

Pru's Solution

Suppose that:

$$a + b = c \quad ✔$$

Bev's **premise** was made at the beginning of the proof. Since variables can be used to represent any numbers, this part of the proof is valid.

premise A statement assumed to be true

$$4a - 3a + 4b - 3b = 4c - 3c \quad ✔$$

Bev substituted $4a - 3a$ for a since $4a - 3a = a$. Bev substituted $4b - 3b$ for b since $4b - 3b = b$. Bev substituted $4c - 3c$ for c since $4c - 3c = c$.

$$4a + 4b - 4c = 3a + 3b - 3c \quad ✔$$

I reorganized the equation and I came up with the same result that Bev did when he reorganized. Simplifying would take me back to the premise. This part of the proof is valid.

$$4(a + b - c) = 3(a + b - c) \quad ✔$$

Since each side of the equation has the same coefficient for all the terms, factoring both sides is a valid step.

$$\frac{4(a + b - c)}{(a + b - c)} = \frac{3(a + b - c)}{(a + b - c)}$$

This step appears to be valid, but when I looked at the divisor, I identified the flaw.

$$a + b = c$$
$$a + b - c = c - c$$
$$a + b - c = 0$$

When I rearranged the premise, I determined that the divisor equalled zero.

Dividing both sides of the equation by $a + b - c$ is not valid. Division by zero is undefined.

EXAMPLE 3 p. 29 **Using reasoning to determine the validity of a proof**

Liz claims she has proved that $-5 = 5$.

Liz's Proof

I assumed that $-5 = 5$. X ← ERROR in your premise

Then I squared both sides: $(-5)^2 = 5^2$

I got a true statement: $25 = 25$

This means that my assumption, $-5 = 5$, must be correct.

Where is the error in Liz's proof?

Simon's Solution

I assumed that $-5 = 5$. — Liz started off with the false assumption that the two numbers were equal.

Then I squared both sides: $(-5)^2 = 5^2$

I got a true statement: $25 = 25$ — Everything that comes after the false assumption doesn't matter because the reasoning is built on the false assumption. Even though $25 = 25$, the underlying premise is not true.

$-5 \neq 5$ — Liz's conclusion is built on a false assumption, and the conclusion she reaches is the same as her assumption.

If an assumption is not true, then any argument that was built on the assumption is not valid. — **Circular reasoning** has resulted from these steps. Starting with an error and then ending by saying that the error has been proved is arguing in a circle.

circular reasoning
An argument that is incorrect because it makes use of the conclusion to be proved.

Your Turn

How is an error in a premise like a counterexample?

Answer

An error in a premise is like a counterexample because a single error invalidates the argument, just as a single counterexample makes a conjecture invalid.

EXAMPLE 4 p. 60 Using reasoning to determine the validity of a proof

Hossai is trying to prove the following number trick:
Choose any number. Add 3. Double it. Add 4. Divide by 2. Take away the number you started with.

Each time Hossai tries the trick, she ends up with 5. Her proof, however, does not give the same result.

Hossai's Proof

n	Choose any number. ✓
$n + 3$	Add 3. ✓
$2n + 6$	Double it. ✓
$2n + 10$	Add 4. ✓
$\cancel{2n + 5}$ n $\cancel{= n}$	Divide by 2. ✗
$\cancel{n +} \boxed{\cancel{5}}$	Take away the number you started with.

Where is the error in Hossai's proof?

Sheri's Solution

$1 \longrightarrow 5$
$10 \longrightarrow 5$

I tried the number trick twice, for the number 1 and the number 10. Both times, I ended up with 5. The math trick worked for Hossai and for me, so the error must be in Hossai's proof.

n ✔ — The variable n can represent any number. This step is valid.

$n + 3$ ✔ — Adding 3 to n is correctly represented.

$2n + 6$ ✔ — Doubling a quantity is multiplying by 2. This step is valid. Its simplification is correct as well.

$2n + 10$ ✔ — Adding 4 to the expression is correctly represented, and the simplification is correct.

$2n + 5$ ✘ — The entire expression should be divided by 2, not just the constant. This step is where the mistake occurred.

I corrected the mistake:

$$\frac{2n + 10}{2} = n + 5$$

$$n + 5 - n = 5$$

I completed Hossai's proof by subtracting n. I showed that the answer will be 5 for any number.

EXAMPLE 5 Using reasoning to determine the validity of a proof

Jean says she can prove that $\$1 = 1¢$.

Jean's Proof

$1 can be converted to 100¢. ✓

100 can be expressed as $(10)^2$. ✓

10 cents is one-tenth of a dollar. ✓

$(0.1)^2 = 0.01$ ✓

One hundredth of a dollar is one cent, so $\$1 = 1¢$. ✓

How can Jean's friend Grant show the error in her reasoning?

Grant's Solution

$1 can be converted to 100¢. ✔ — It is true that 100 cents is the same as $1.

100 can be expressed as $(10)^2$. ✔ — It is true that $(10)^2$ is $10 \cdot 10$, which is 100.

10 cents is one-tenth of a dollar. ✔ — It is true that 10 dimes make up a dollar.

$(0.1)^2 = 0.01$ ✔ — Arithmetically, I could see that this step was true. But Jean was ignoring the units. It doesn't make sense to square a dime. The units $¢^2$ and $\$^2$ have no meaning.

A dollar is equivalent to $(10)(\$0.10)$ or $10(10¢)$, not to $(10¢)(10¢)$ or $(\$0.10)(\$0.10)$.

$\$1 \neq 1¢$

In Summary

Key Idea

- A single error in reasoning will break down the logical argument of a deductive proof. This will result in an invalid conclusion, or a conclusion that is not supported by the proof.

Need to Know

- Division by zero always creates an error in a proof, leading to an invalid conclusion.
- Circular reasoning must be avoided. Be careful not to assume a result that follows from what you are trying to prove.
- The reason you are writing a proof is so that others can read and understand it. After you write a proof, have someone else who has not seen your proof read it. If this person gets confused, your proof may need to be clarified.

HOMEWORK...

p. 42: #1 - 10 (omit #8)

Logic Problems

Finish

Attachments

1s5e1 finalt.mp4

1s5e3 finalt.mp4