***ADD this one to your notes...

converse

A statement that is formed by switching the premise and the conclusion of another statement.

EXAMPLES...

Conjecture: If it is raining outside, then the grass is wet.

CONVERSE: If the grass is wet, then it is raining.

THEOREM: If you have parallel lines, then the corresponding angles are equal.

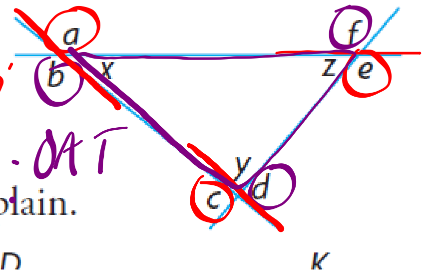
CONVERSE: If the corresponding angles are equal, then the lines are parallel.

8. Each vertex of a triangle has two exterior angles, as shown.

a) Make a conjecture about the sum of the measures of $\angle a$, $\angle c$, and $\angle e$. Add to 360°

b) Does your conjecture also apply to the sum of the measures of $\angle b$, $\angle d$, and $\angle f$? Explain. YES... OAT

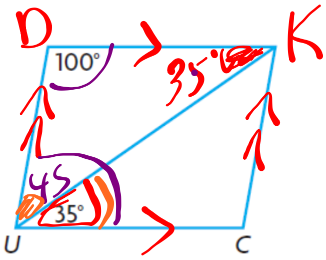
c) Prove or disprove your conjecture.



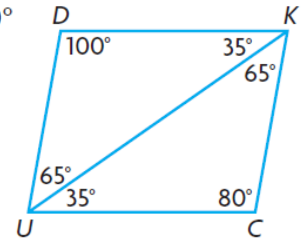
$$\angle a + \angle x = 180°$$
$$\angle c + \angle y = 180$$
$$\angle e + \angle z = 180°$$

$$\angle x + \angle y + \angle z = 180$$
$$(180° - \angle a) + (180° - \angle c) + (\cancel{180} - \angle e) = \cancel{180}\ 0$$
$$360° = \angle a + \angle c + \angle e$$

9. *DUCK* is a parallelogram. Benji determined the measures of the unknown angles in *DUCK*. Paula says he has made an error.



Benji's Solution

Statement	Justification
$\angle DKU = \angle KUC$	$\angle DKU$ and $\angle KUC$ are alternate interior angles. ✓
$\angle DKU = 35°$	
$\angle UDK = \angle DUC$ * $\angle UDK + \angle DUC = 180°$	✓ ~~$\angle UDK$ and $\angle DUC$ are corresponding an~~gles. CIA
$\angle DUK + \angle KUC = 100°$ X	$\angle DUK$ and $\angle UKC$ are alternate interior angles.
$\angle DUK = 65°$	
$\angle UKC = 65°$	
$\angle UCK = 180° - (\angle KUC + \angle UKC)$	The sum of the measures of the angles in a triangle is 180°.
$\angle UCK = 180° - (35° + 65°)$	
$\angle UCK = 80°$	
	I redrew the diagram, including the angle measures I determined.

a) Explain how you know that Benji made an error.
b) Correct Benji's solution.

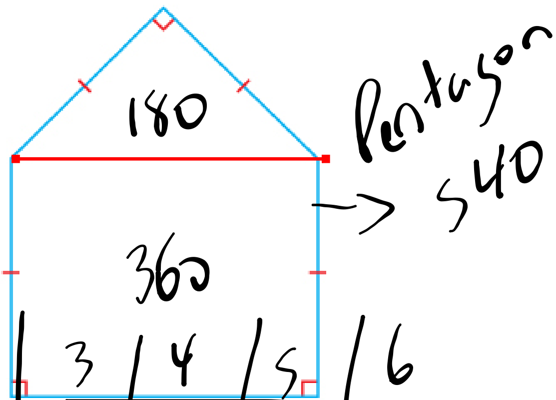
2.4 Angle Properties in Polygons

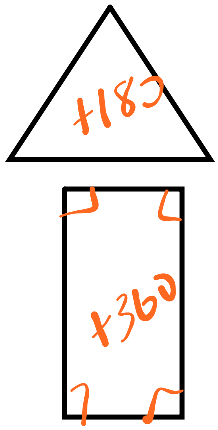
GOAL

Determine properties of angles in polygons, and use these properties to solve problems.

EXPLORE...

- A pentagon has three right angles and four sides of equal length, as shown. What is the sum of the measures of the angles in the pentagon?

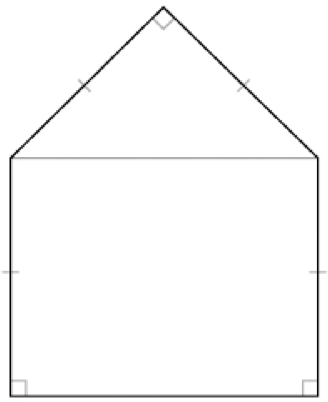




# of sides	3	4	5	6
Sum	180	360	540	720

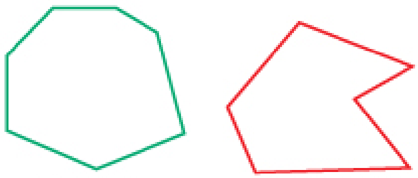
SAMPLE ANSWER

I drew a diagonal joining the two angles that are not right angles. This cut the pentagon into a rectangle and a triangle. I knew that the quadrilateral was a rectangle, not a trapezoid, because the two right angles share an arm, so their other arms must be parallel. As well, the other arms are equal length. I knew that the sum of the measures of the angles in a rectangle is 360° and the sum of the measures of the angles in a triangle is 180°, so the sum of the measures of the angles in the pentagon must be 540°.



convex polygon

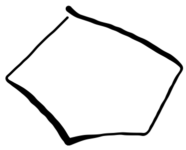
A polygon in which each interior angle measures less than 180°.



convex non-convex (concave)

$S(6) = 180°(6-2)$

$= 720°$

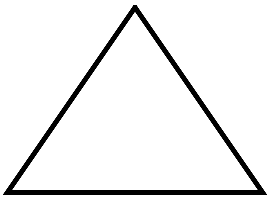


This is my conjecture: The sum of the measures of the interior angles in a polygon, $S(n)$, is:

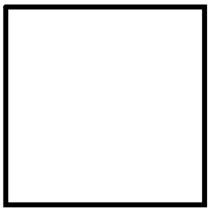
$S(n) = 180°(n - 2)$ OR $Sum = 180°(n-2)$

\# of sides

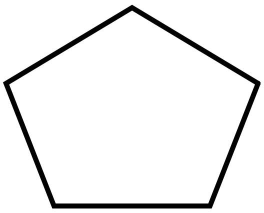
Regular Polygon → all angles / sides are equal



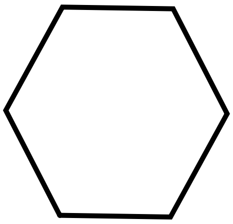
Equilateral Triangle



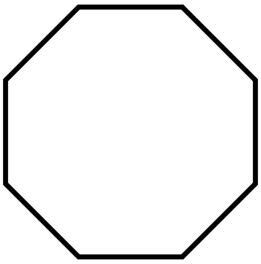
Square



Pentagon



Hexagon



Octagon

Undecagon

[11 sided]

EXAMPLE 2 Reasoning about angles in a regular polygon

Outdoor furniture and structures like gazebos sometimes use a regular hexagon in their building plan. Determine the measure of each interior angle of a regular hexagon.

Nazra's Solution

Let $S(n)$ represent the sum of the measures of the interior angles of the polygon, where n is the number of sides of the polygon.

$S(n) = 180°(n - 2)$ — A hexagon has six sides, so $n = 6$.

$S(6) = 180°[(6) - 2]$

$S(6) = 720°$

$\frac{720°}{6} = 120°$ — Since the measures of the angles in a regular hexagon are equal, each angle must measure $\frac{1}{6}$ of the sum of the angles.

The measure of each interior angle of a regular hexagon is 120°.

Hexagon vs Octagon

120°

135°

90

+

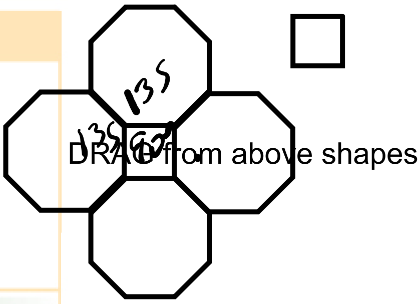
45

$$\frac{180^\circ(8-2)}{8}$$

Tiling Using Regular Polygons...

Tile → Angles adding to 360° $\frac{180(n-2)}{n}$

	Regular Polygon	Measure of Interior Angle (degrees)
X	Equilateral Triangle	60
X	Square	90
	Pentagon	108
X	Hexagon	120
	Heptagon (7 sided)	128.3
X	Octagon	135
	Nonagon (9 sided)	140
	Decagon (10 sided)	144



EXAMPLE 3 Visualizing tessellations

A floor tiler designs custom floors using tiles in the shape of regular polygons. Can the tiler use congruent regular octagons and congruent squares to tile a floor, if they have the same side length?

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Vanessa's Solution

$S(n) = 180°(n - 2)$
$S(8) = 180°[(8) - 2]$
$S(8) = 1080°$

$\frac{1080°}{8} = 135°$

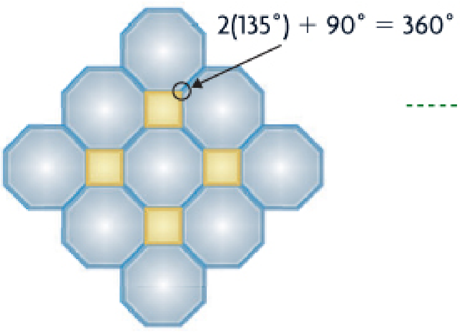
The measure of each interior angle in a regular octagon is 135°.
The measure of each internal angle in a square is 90°.

> Since an octagon has eight sides, $n = 8$.

> First, I determined the sum of the measures of the interior angles of an octagon. Then I determined the measure of each interior angle in a regular octagon.

Two octagons fit together, forming an angle that measures:
$2(135°) = 270°$.
This leaves a gap of 90°.
$2(135°) + 90° = 360°$
A square can fit in this gap if the sides of the square are the same length as the sides of the octagon.

> I knew that three octagons would not fit together, as the sum of the angles would be greater than 360°.



> I drew what I had visualized using dynamic geometry software.

The tiler can tile a floor using regular octagons and squares when the polygons have the same side length.

In Summary

Key Idea

- You can prove properties of angles in polygons using other angle properties that have already been proved.

Need to Know

- The sum of the measures of the interior angles of a convex polygon with n sides can be expressed as $180°(n - 2)$.
- The measure of each interior angle of a regular polygon is $\frac{180°(n-2)}{n}$.
- The sum of the measures of the exterior angles of any convex polygon is 360°.

HOMEWORK...

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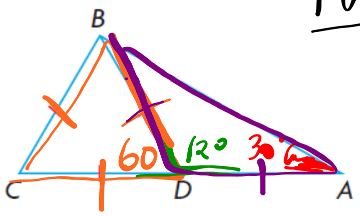
HISTORY on Buckyball Do A, B and C

Questions

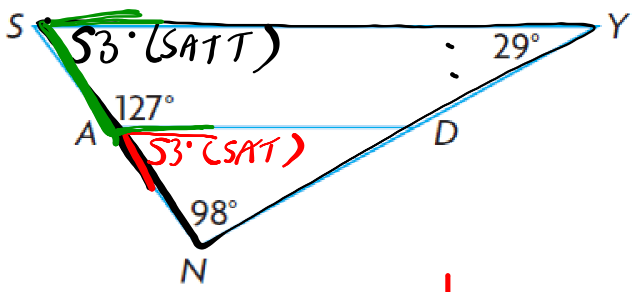
5. Prove: $\angle A = 30°$



$$\frac{180-120}{2} = 30°$$

Statement	Justification
$\angle BDC = 60°$	Equilateral
$\angle BDA = 120°$	SAT
$\angle BAD = 30°$	ITT & SATT

7. Prove: $SY \parallel AD$



S	J
$\angle SAD = 127°$	Given
$\angle ASY = 53°$	SATT
$\angle SAD + \angle ASY = 180°$	Adding
$\therefore SY \parallel AD$	CIA

S	J
$\angle NAD = 53°$	SAT
$\angle ASY = \angle NAD$	Transitive
$\therefore SY \parallel AD$	CA