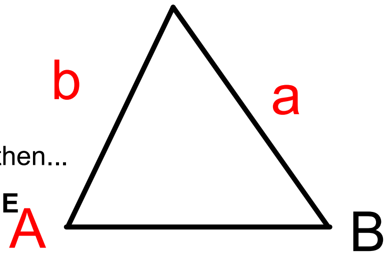
MUST MEMORIZE THESE NOTES IN ORDER TO KNOW AMBIGUOUS CASE

Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b

*** If ALL 3 criteria are met, then...



CALCULATE THE ALTITUDE

$\boxed{\text{alt} = b \sin A}$

CASE 1: a < altitude; there is NO SOLUTION

CASE 2: a = altitude; there is ONE SOLUTION [Right Triangle]

CASE 3: a >altitude; this is the 'AMBIGUOUS CASE'...TWO SOLUTIONS

1) Acute Triangle (angle, θ, is found with Law of Sines)
2) Obtuse Triangle (angle is $180^{\circ} - \theta$)

EXAMPLE 1 Connecting the *SSA* situation to the number of possible triangles

p.177

✓SSA
✓acute angle
✓$a < b$
aH $= 12\sin 30$
alt $= 6$

Given each *SSA* situation for $\triangle ABC$, determine how many triangles are possible.

a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m
b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m
c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m
d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m

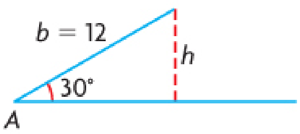
$a > b$ → 1 triangle

Case 1 → $a < \text{alt}$
(a) no solution

Case 2 → $a = \text{alt}$
(b) 1 Right triangle

Case 3 → $a > \text{alt}$
(c) 2 solutions

Saskia's Solution



I drew the beginning of a triangle with a 30° angle and a 12 m side.

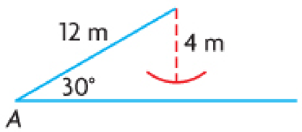
$$\sin 30° = \frac{h}{12}$$

I used the sine ratio to calculate the height of the triangle.

$$12 \sin 30° = h$$
$$6 \text{ m} = h$$

I can use this height as a benchmark to decide on side lengths opposite the 30° angle that will result in zero, one, or two triangles.

a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

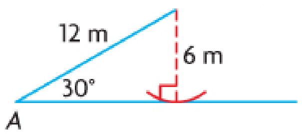


No triangles are possible.

Since $a < b$ and $a < h$, I knew that no triangles are possible.

I used a compass to be certain. I set the compass tips to represent 4 m. I placed one tip of the compass at the open end of the 12 m side and swung the pencil tip toward the other side. The pencil couldn't reach the base, so a 4 m side could not close the triangle.

b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m

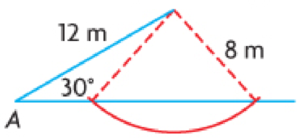


One triangle is possible.

Since $a < b$ and $a = h$, there is only one possible triangle, a right triangle.

A compass arc intersects the base at only one point.

c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m

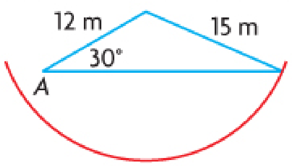


Two triangles are possible.

Since $a < b$ and $a > h$, there are two possible triangles.

A compass arc intersects the base at two points.

d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m



One triangle is possible.

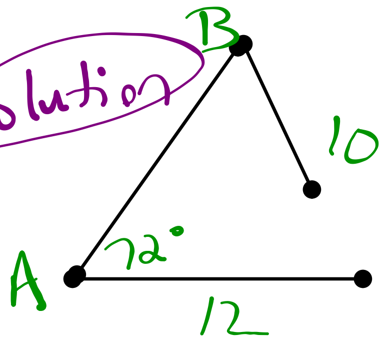
Since $a > b$, only one triangle is possible.

A compass arc intersects the base at only one point.

Example 2:

Solve the triangle ABC if $a = 10$, $b = 12$ and angle A = 72°.

No solution



✓ SSA *

✓ acute angle

✓ $a < b$

$h = 12 \sin 72°$

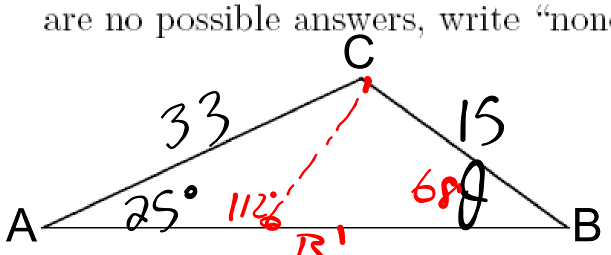
$h = 11.4$

a vs h

$10 < 11.4$

Example 3:

Given that A $= 25°$, $a = 15$, and $b = 33$, find the measure of angle B to the nearest degree. If there are two answers, give both of them. If there are no possible answers, write "none".



$$\frac{\sin B}{\cancel{33}} = \frac{\sin 25°}{15}$$

$\sin B =$

```
33sin(25)/15
      .9297601758
sin⁻¹(Ans
      68.39746161
```

$\angle B =$

$\angle B = 68°$

OR

$\angle B = 180° - 68°$

$\angle B = 112°$

✓ SSA
✓ acute angle
✓ $a < b$

$alt = 33 \sin 25°$
$alt = 13.9$

a vs alt
$15 > 13.9$

* Ambiguous
↳ 2 solutions

HOMEWORK...

Worksheet - Ambiguous Case.pdf

Do questions #1, 2 & 4

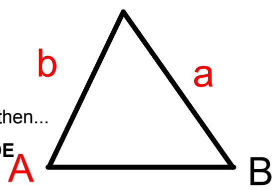
MEMORIZE...QUIZ MONDAY!

Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b

*** If ALL 3 criteria are met, then...

CALCULATE THE ALTITUDE



$\boxed{\text{alt} = b \sin A}$

CASE 1: a < altitude; there is NO SOLUTION

CASE 2: a = altitude; there is ONE SOLUTION [Right Triangle]

CASE 3: a >altitude; this is the 'AMBIGUOUS CASE'...TWO SOLUTIONS

1) Acute Triangle (angle, θ, is found with Law of Sines)
2) Obtuse Triangle (angle is $180^\circ - \theta$)

Attachments

Worksheet - Ambiguous Case.pdf