

**MUST
MEMORIZE
THESE
NOTES
IN ORDER
TO KNOW
AMBIGUOUS
CASE**

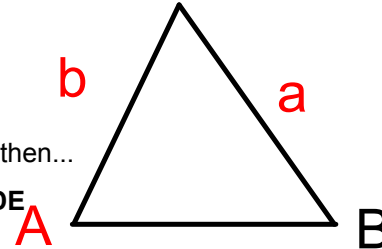
Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- $a < b$

*** If ALL 3 criteria are met, then...

CALCULATE THE ALTITUDE

$alt = b \sin A$



CASE 1: $a < alt$; there is NO SOLUTION

CASE 2: $a = alt$; there is ONE SOLUTION [Right Triangle]

CASE 3: $a > alt$; this is the 'AMBIGUOUS CASE'...TWO SOLUTIONS

- 1) Acute Triangle (angle, θ , is found with Law of Sines)
- 2) Obtuse Triangle (angle is $180^\circ - \theta$)

EXAMPLE 1

p.177

Connecting the SSA situation to the number of possible triangles

✓ SSA
✓ acute angle

✓ $a < b$
alt = $12 \sin 30^\circ$
alt = 6

Given each SSA situation for $\triangle ABC$, determine how many triangles are possible.

- a) $\angle A = 30^\circ$, $a = 4$ m, and $b = 12$ m c) $\angle A = 30^\circ$, $a = 8$ m, and $b = 12$ m
 b) $\angle A = 30^\circ$, $a = 6$ m, and $b = 12$ m d) $\angle A = 30^\circ$, $a = 15$ m, and $b = 12$ m

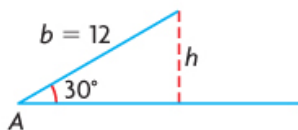
Case 1: $a < alt$
(a) no solution

Case 2: $a = alt$
(b) 1 Right Triangle

Case 3: $a > alt$
(c) 2 Solutions Ambiguous Case

$a > b \rightarrow 1$ solution

Saskia's Solution



$$\sin 30^\circ = \frac{h}{12}$$

$$12 \sin 30^\circ = h$$

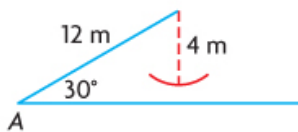
$$6 \text{ m} = h$$

I drew the beginning of a triangle with a 30° angle and a 12 m side.

I used the sine ratio to calculate the height of the triangle.

I can use this height as a benchmark to decide on side lengths opposite the 30° angle that will result in zero, one, or two triangles.

- a) $\angle A = 30^\circ$, $a = 4$ m, and $b = 12$ m

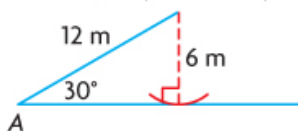


No triangles are possible.

Since $a < b$ and $a < h$, I knew that no triangles are possible.

I used a compass to be certain. I set the compass tips to represent 4 m. I placed one tip of the compass at the open end of the 12 m side and swung the pencil tip toward the other side. The pencil couldn't reach the base, so a 4 m side could not close the triangle.

- b) $\angle A = 30^\circ$, $a = 6$ m, and $b = 12$ m

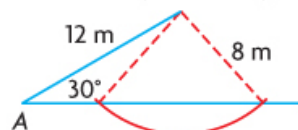


One triangle is possible.

Since $a < b$ and $a = h$, there is only one possible triangle, a right triangle.

A compass arc intersects the base at only one point.

- c) $\angle A = 30^\circ$, $a = 8$ m, and $b = 12$ m

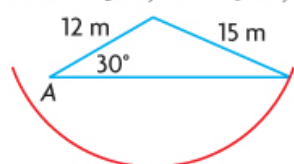


Two triangles are possible.

Since $a < b$ and $a > h$, there are two possible triangles.

A compass arc intersects the base at two points.

- d) $\angle A = 30^\circ$, $a = 15$ m, and $b = 12$ m



One triangle is possible.

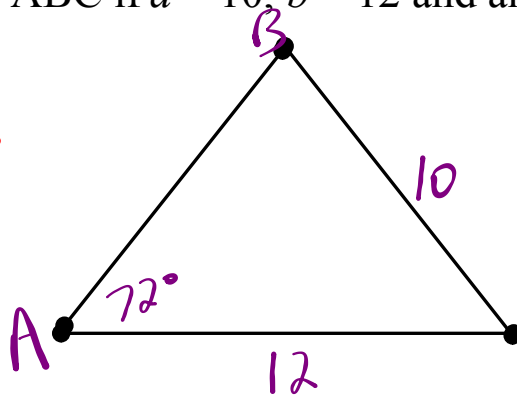
Since $a > b$, only one triangle is possible.

A compass arc intersects the base at only one point.

Example 2:

Solve the triangle ABC if $a = 10$, $b = 12$ and angle $A = 72^\circ$.

Find all angles/sides



No Solution

* SSA

✓ acute angle

✓ $a < b$

$$\text{alt} = b \sin A$$

$$\text{alt} = 12 \sin 72^\circ$$

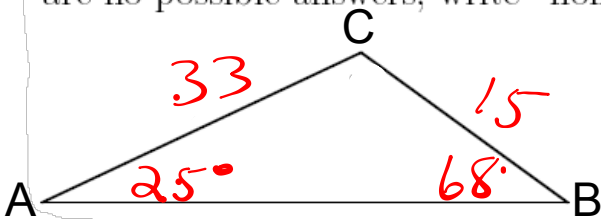
$$\text{alt} = 11.4$$

a vs alt

$$10 < 11.4$$

Example 3:

Given that $A = 25^\circ$, $a = 15$, and $b = 33$, find the measure of angle B to the nearest degree. If there are two answers, give both of them. If there are no possible answers, write "none".



$$\frac{33 \sin B}{33} = \frac{33 \sin 25^\circ}{15}$$

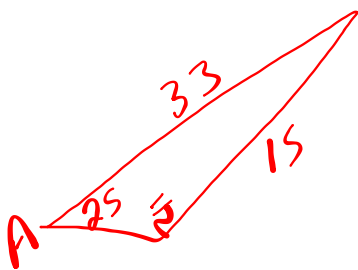
$$\sin B = \frac{33 \sin(25) / 15}{1}$$

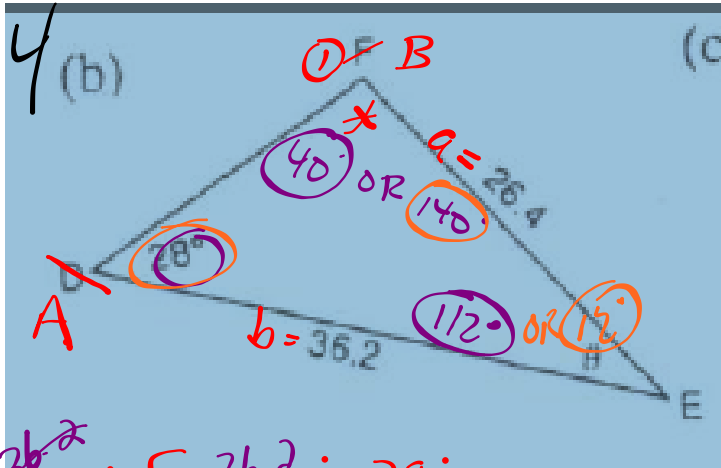
Calculator output:
 33sin(25)/15
 .9297601758
 sin⁻¹(Ans)
 68.39746161

$B = 68^\circ$ or $B = 180 - 68 = 112^\circ$

$B = 112^\circ$

* SSA
 ✓ acute angle
 ✓ $a < b$
 $alt = 33 \sin 25^\circ$
 $alt = 13.95$
 $a < alt$
 $15 > 13.95$
 Ambiguous
 2 solutions





* SSA
 ✓ acute angle
 ✓ $a < b$

$$\text{alt} = 36.2 \sin 28$$

$$\text{alt} = 17$$

a vs alt
 $26.4 > 17$
 ambiguous

$$\frac{\sin F}{36.2} = \frac{\sin 28}{26.4}$$

$$\sin F = \frac{36.2 \sin(28)}{26.4}$$

```
36.2sin(28)/26.4
.6437450975
sin^-1(Ans)
40.07165227
```

OR
 $\angle F = 40^\circ$

$$\angle E = 180 - 28 - 40$$

$$\angle E = 112^\circ$$

$$\angle F = 180 - 40$$

$$\angle F = 140^\circ$$

$$\angle E = 180 - 140 - 28$$

$$\angle E = 12^\circ$$

Example 4: Solving a problem using the sine law

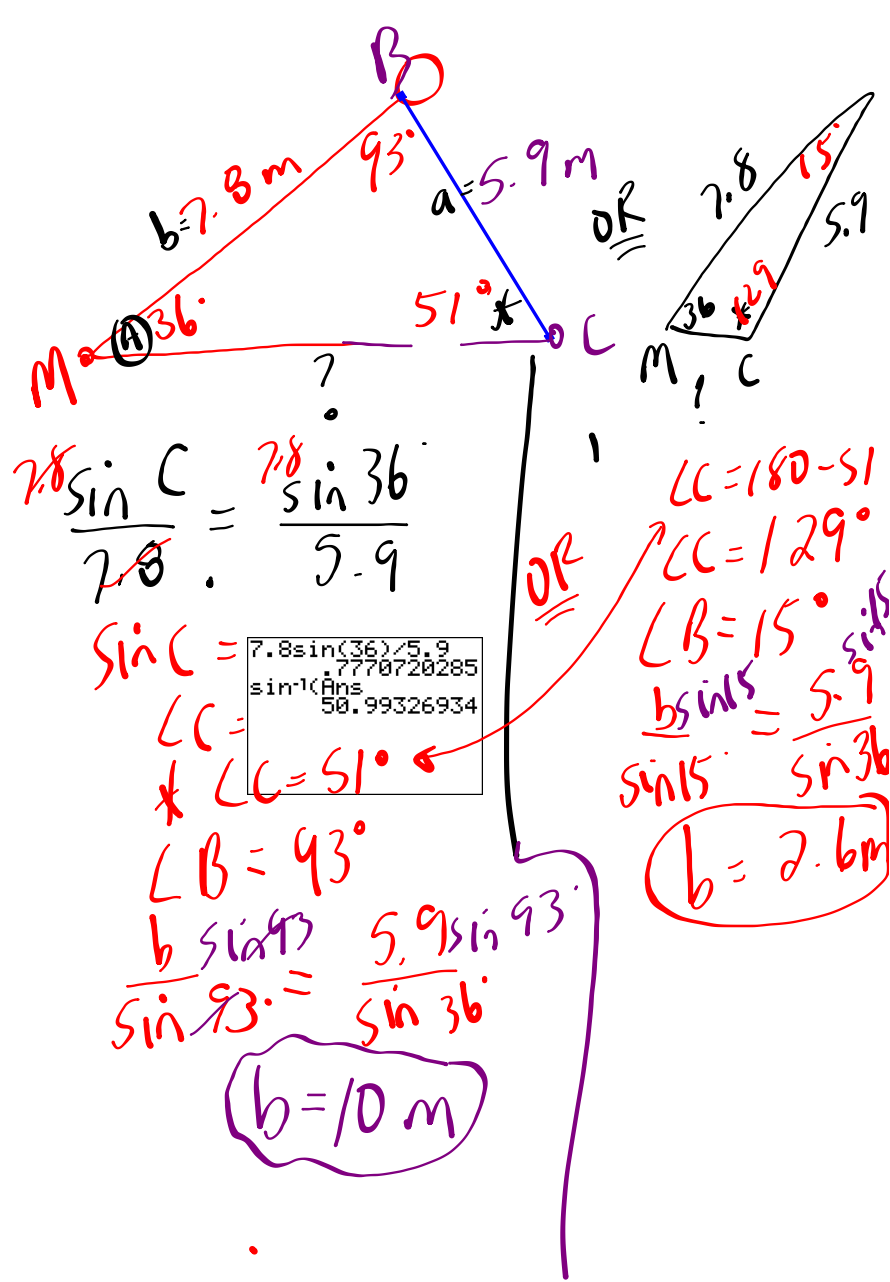
P. 178

Martina and Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of 36.0° with the ground. Carl's rope is 5.9 m long. Assuming that Martina and Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina and Carl, to the nearest tenth of a metre?



AMBIGUOUS???

SSA
 acute angle
 $a < b$
 $alt = 7.8 \sin 36^\circ$
 $alt = 4.6$
 a vs alt
 $5.9 > 4.6$
 Ambiguous



HOMEWORK...



Do questions #1, 2, 4ac, 5ac, 6, 7

MEMORIZE!!!

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Attachments

Worksheet - Ambiguous Case.pdf