HOMEWORK... QUESTIONS...

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4. A student council is ordering signs for the spring dance. Signs can be made in letter size or poster size.
   - No more than 15 of each size are wanted.
   - At least 15 signs are needed altogether.
   - Letter-size signs cost $9.80 each, and poster-size signs cost $15.75 each.

   Create a model that could be used to determine a combination of the two sizes of signs that would result in the lowest cost to the council.

Graph

$x \rightarrow$ # of letter size

$y \rightarrow$ # of poster size

$x \in W$ $y \in W$

$x \leq 15$

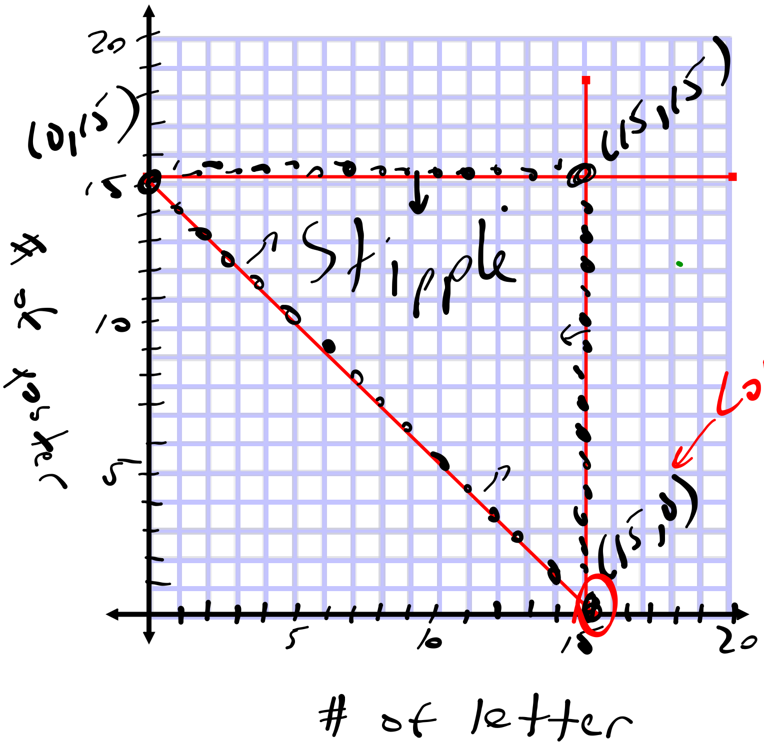
$y \leq 15$

$x + y \geq 15$

Constraints GRAPH

$C = 9.80x + 15.75y$

Objective MATH



$x + y = 15$

x int $(15, 0)$

y int $(0, 15)$

Lowest cost

$C = 9.80(15)$

$C = \$147.00$

6. Sung and Faith have weekend jobs at a marina, applying anti-fouling paint to the bottom of boats.
   - Sung can work no more than 14 h per weekend.
   - Faith is available no more than 18 h per weekend.
   - The marina will hire both of them for 24 h or less per weekend.
   - Sung paints one boat in 3 h, but Faith needs 4 h to paint one boat.

   The marina wants to maximize the number of boats that are painted each weekend.

   a) Create a model to represent this situation.

   b) Suppose that another employee, Frank, who can paint a boat in 2 h, replaced Faith for a weekend. How would your model change?

$x \rightarrow$ # of Sung's hours

$y \rightarrow$ # of Faith's hours

$x \in W$

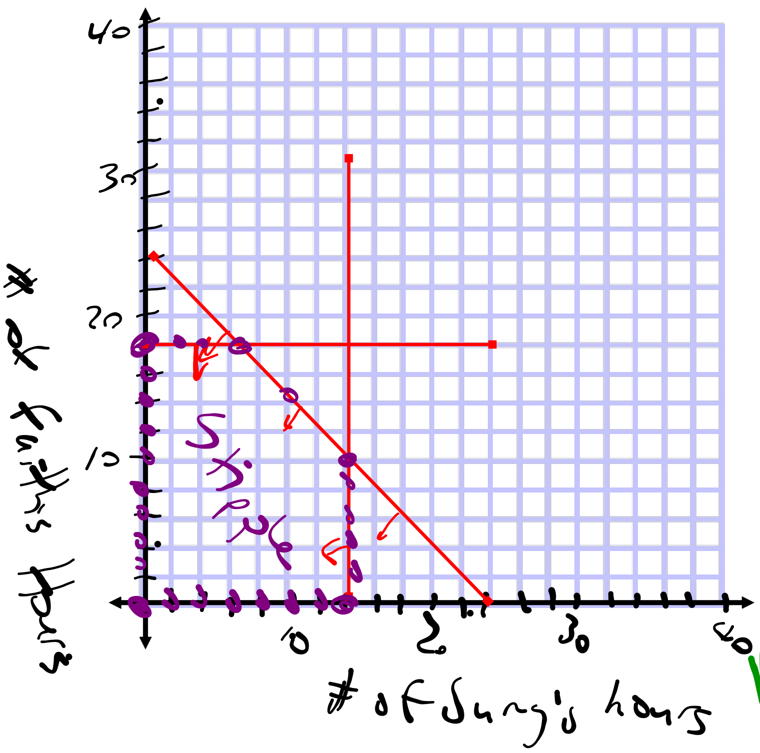
$y \in W$

$x \le 14$

$y \le 18$

$x + y \le 24$

$$B = \frac{x}{3} + \frac{y}{4}$$



$x + y = 24$

x-int $(24, 0)$

y-int $(0, 24)$

sub

Vertices	Objective
	$B = \frac{x}{3} + \frac{y}{4}$
~~(0, 0)~~	
~~(14, 0)~~	
~~(0, 18)~~	
Max (14, 10)	7.2
(6, 18)	6.5

5.5 Optimization Problems II: Exploring Solutions

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Need to Know

- The solution to an optimization problem is usually found at one of the vertices of the feasible region.
- To determine the optimal solution to an optimization problem using linear programming, follow these steps:

Step 1. Create an algebraic model that includes:
- a defining statement of the variables used in your model
- the restrictions on the variables
- a system of linear inequalities that describes the constraints
- an objective function that shows how the variables are related to the quantity to be optimized

Step 2. Graph the system of inequalities to determine the coordinates of the vertices of its feasible region.

Step 3. Evaluate the objective function by substituting the values of the coordinates of each vertex.

Step 4. Compare the results and choose the desired solution.

Step 5. Verify that the solution(s) satisfies the constraints of the problem situation.

optimal solution
A point in the solution set that represents the maximum or minimum value of the objective function.

EXAMPLE #1...

The vertices of the feasible region of a graph of a system of linear inequalities are (-4, -8); (5, 0) and (1, -6). Which point would result in the minimum value of the objective function C = 0.50x + 0.60y?

	Vertices	$C = 0.50x + 0.60y$
Min	(-4, -8)	$0.5(-4) + 0.6(-8) = -6.8$ *
	(5, 0)	$0.50(5) + \cancel{0.60(0)} = 2.5$
	(1, -6)	$0.5(1) + 0.6(-6) = -3.1$

EXAMPLE #3...

Four MVHS teams are travelling to a basketball tournament in cars and minivans. 4×12

- Each team has no more than 2 coaches and 10 athletes TOTAL $= 48$
- Each car can take 4 team members. Each minivan can take 6 team members.
- No more than 6 cars are available, but more than 3 minivans are available.

Mr. Watters wants to know the combination of cars and minivans that will require the maximum number of vehicles... $V = x + y$

a) Create an algebraic model to represent this situation.

$x \rightarrow$ # of cars $x \in W$

$y \rightarrow$ # of vans $y \in W$

$x \leq 6$

$y \geq 3$

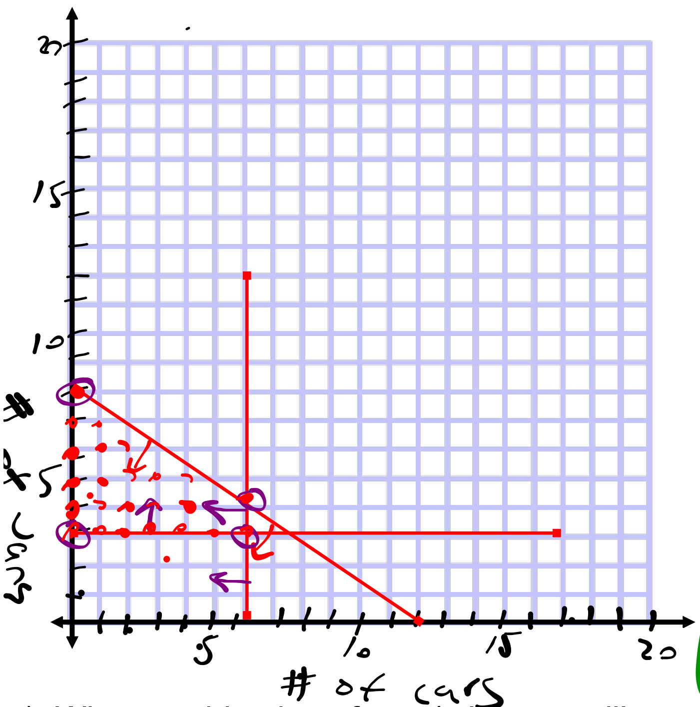
$\boxed{4x + 6y \leq 48}$

b) Graph the model.

$4x + 6y = 48$

y-int: $6y = 48$, $y = 8$ $(0, 8)$

x-int: $4x = 48$, $x = 12$ $(12, 0)$



Vertices	$V = x + y$
$(0, 3)$	3
$(0, 8)$	8
$(6, 3)$	9
$\boxed{\text{Max } (6, 4)}$	10

c) What combination of cars/minivans will result in the maximum number of vehicles?

6 cars 4 van

d) How many team members can travel in the maximum number of vehicles?

48

$4(6) + 6(4) = 48$

In Summary

Key Ideas

- The value of the objective function for a system of linear inequalities varies throughout the feasible region, but in a predictable way.
- The optimal solutions to the objective function are represented by points at the intersections of the boundaries of the feasible region. If one or more of the intersecting boundaries is not part of the solution set, the optimal solution will be nearby.

Need to Know

- You can verify each optimal solution to make sure it satisfies each constraint by substituting the values of its coordinates into each linear inequality in the system.
- The intersection points of the boundaries are called the vertices, or corners, of the feasible region.

HOMEWORK...

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