LEARN ABOUT the Math

*** Can be found on p. 226

A company makes two types of boats on different assembly lines: aluminum fishing boats and fibreglass bow riders.

- When both assembly lines are running at full capacity, a maximum of 20 boats can be made in a day.
- The demand for fibreglass boats is greater than the demand for aluminum boats, so the company makes at least 5 more fibreglass boats than aluminum boats each day.

What combinations of boats should the company make each day?

$x \to$ # of aluminum boats

$y \to$ # of fibreglass boats

$x \in W \quad y \in W$

"fiberglass depends on aluminum"

y x

$\boxed{x + y \le 20}$

$x + y = 20$

x-int $\quad x + 0 = 20$
$(20, 0)$

y-int $\quad 0 + y = 20$
$(0, 20)$

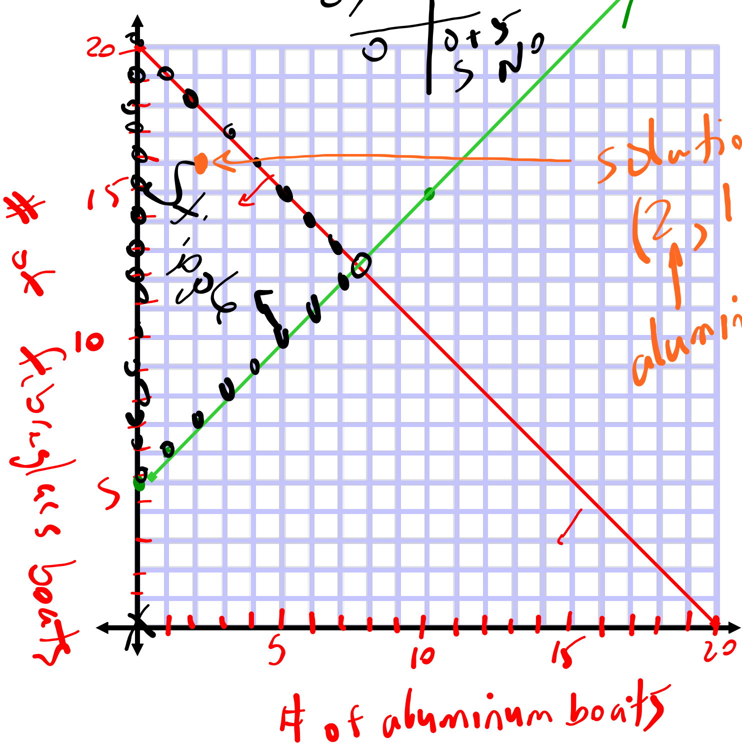
Test $(0,0)$

$\boxed{y \ge x + 5}$

LS	≥	RS
0		0+5
		5 No

$y = x + 5$

x	y
0	5
10	15



solution
(2, 16)
aluminum
fibreglass

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A company makes two types of boats on different assembly lines: aluminum fishing boats and fibreglass bow riders.

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EXAMPLE 1 Solving a problem with discrete whole-number variables using a system of inequalities

Mary's Solution: Using graph paper

Let a represent the number of aluminum fishing boats.
Let f represent the number of fibreglass bow riders.

$a \in W$ and $f \in W$

I knew I could solve this problem by representing the situation algebraically with a system of two linear inequalities and graphing the system.

Since only complete boats are sold, I knew that a and f are whole numbers and the graph would consist of discrete points in the first quadrant.

The relationship between the two types of boats can be represented by this system of inequalities:
$a + f \le 20$
$a + 5 \le f$

The two inequalities describe
- a combination of boats to a maximum of 20.
- at least 5 more fibreglass boats than aluminum boats.

$a + f = 20$

f-intercept:	a-intercept:
$0 + f = 20$	$a + 0 = 20$
$f = 20$	$a = 20$
(0, 20)	(20, 0)

$a + 5 = f$

f-intercept:	a-intercept:
$0 + 5 = f$	$a + 5 = 0$
$f = 5$	$a = -5$
(0, 5)	(−5, 0)

To graph each linear inequality, I knew I had to graph its boundary as a stippled line, and then shade and stipple the correct half plane.

To graph each boundary, I wrote each linear equation and then determined the a- and f-intercepts so I could plot and join them.

$a + 5 = f$
$(5) + 5 = f$
$10 = f$
(5, 10) is a point on this boundary.

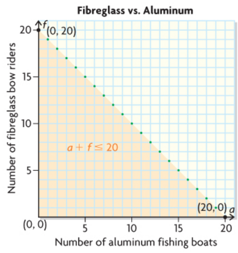
For $a + 5 = f$, I knew (−5, 0) wasn't going to be a point on the boundary, because it's not in the first quadrant, so I chose another point by solving the equation for $a = 5$.

Test (0, 0) in $a + f \le 20$.

LS	RS
$a + f$	20
$0 + 0$	
0	

Since $0 \le 20$, (0, 0) is in the solution region.

I tested point (0, 0) to determine which half plane to shade for $a + f \le 20$.



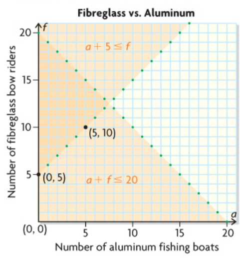
I drew a green stippled boundary connecting (0, 20) and (20, 0) and shaded the half plane below it orange, because the solution region is discrete.

Test (0, 0) in $a + 5 \le f$.

LS	RS
$a + 5$	f
$0 + 5$	0
5	

Since 5 is not less than or equal to 0, (0, 0) is not in the solution region.

I tested (0, 0) to determine which half plane to shade for $a + 5 \le f$.

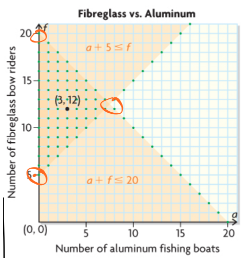


I plotted the points (0, 5) and (5, 10) on the same coordinate plane. I used these points to draw a green stippled boundary for $a + 5 \le f$.

I shaded the half plane above the boundary orange, since the test point (0, 0) is not a solution to the linear inequality and the solution region is discrete.

I knew that the solution set for the system of linear inequalities is represented by the intersection or overlap of the solution regions of the two inequalities. This made sense since points in this region satisfy both inequalities.

I knew that the triangular solution region included discrete points along its three boundaries, including the y-axis from $y = 5$ to $y = 20$.



Since the solution set for the system contains only discrete points with whole-number coordinates, I stippled its solution region.

I knew that any whole-number point in the triangular solution region is a possible solution. For example, (3, 12) is a possible solution.

$\{(a, f) \mid a + f \le 20, a \in W, f \in W\}$
$\{(a, f) \mid a + 5 \le f, a \in W, f \in W\}$

Any point with whole-number coordinates in the intersecting or overlapping region is an acceptable combination. For example, 3 aluminum boats and 12 fibreglass boats is an acceptable combination.

I knew that (3, 12) worked because this gives a total of 15 boats with 9 more fibreglass boats than aluminum boats.

Reflecting

A. Is every point on the boundaries of the solution region a possible solution? Explain.

B. Are the three points where the boundaries intersect part of the solution region? Explain.

C. How would the graph change if fewer than 25 boats were made each day?

D. All points with whole-number coordinates in the solution region are valid, but are they all reasonable? Explain.

Answers

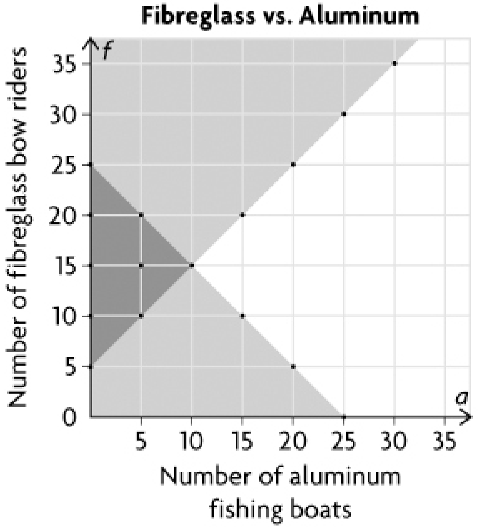
A. No. Only whole-number coordinate points on the boundaries are part of the solution region, because the variables represent numbers of boats and only whole numbers of boats make sense.

B. Yes. Equality is possible for both inequalities, and all of these points have whole-number coordinates: (0, 5), and (0, 20).) Intersection not whole numbers.

C. $a + f \leq 20 \rightarrow a + f \leq 25$

The solution region would be larger, because its boundary would move up.

$x + y \leq 25$
$y \geq x + 5$



D. This would depend on the market. For example, if there was a high demand for boats, then points in the solution region with high coordinates, such as (7, 13), would probably make more sense. If there was a low demand for fishing boats, then points with low x-coordinates, such as (0, 20), would make more sense.

*** Can be found on p. 233

EXAMPLE 3 Solving graphically a problem with continuous positive variables

A sloop is a sailboat with two sails: a mainsail and a jib. When a sail is fully out or up, it is said to be "out 100%." When the winds are high, sailors often reef, or pull in, the sails to be less than their full capability.

- Jim is sailing in winds of 22 knots, so he wants no more than 80% of the mainsail out. $y \le 80$
- Jim also wants more mainsail out than jib. $y > x$

What possible combinations of mainsail and jib can Jim have out?

y x

$x \to$ % of jib

$y \to$ % of main

$\left.\begin{matrix} x \in R \\ y \in R \end{matrix}\right\}$ Quad I

Louise's Solution: Using graph paper

Let m represent the percent of mainsail out.
Let j represent the percent of jib out.

$m \ge 0$ and $j \ge 0$, where $m \in \mathrm{R}$, $j \in \mathrm{R}$

Quad I

> I knew that I could solve the problem by representing it algebraically with a system of two linear inequalities and then graphing it.
>
> I knew that the graph would be in the first quadrant since there can't be negative percents of sails out.
>
> I also knew that the solution region would be continuous since decimal percents are possible.

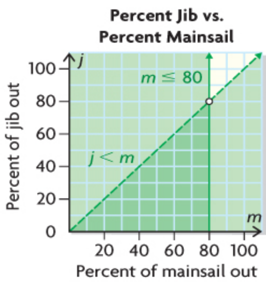
The relationship between the two types of sails can be represented by the following system of two linear inequalities:
$m \le 80$
$j < m$

> The inequalities describe the following information:
> - No more than 80% of the mainsail can be out.
> - Less jib than mainsail must be out.

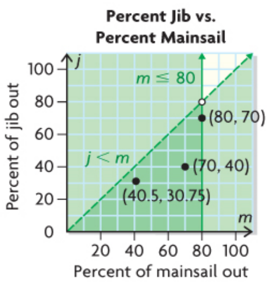
$m \le 80$
Boundary: $m = 80$
Boundary is a vertical line with an m-intercept of 80.

$j < m$
Boundary: $j = m$
Boundary line has a slope of 1 and a j-intercept of 0.

> I decided to use m as the independent variable.
>
> I examined each inequality to determine its boundary:
> - Since m is the independent variable, I knew the boundary for $m \le 80$ would be a vertical line through $m = 80$.
> - I knew the boundary of $j < m$ has a slope of 1 and passes through the point (0, 0).



> For $m \le 80$, I drew a solid green vertical line through $m = 80$ and then shaded the half plane to its left green, since the inequality sign is $\le$.
>
> For $j < m$, I drew a green dashed line through (0, 0) with a slope of 1 and I shaded the half plane below green since the inequality is $<$.
>
> I drew an open dot where the dashed boundary intersects the solid boundary to show that point isn't part of the solution region.
>
> The solution region for the system is a right triangle and consists of all the points in the overlapping region, including the solid boundary and the m-axis from 0 to 80.



> I looked for several solutions in the solution region. I knew that I could choose points with decimal coordinates since the solution region is continuous.

$\{(m, j) \mid m \le 80, m \ge 0, j \ge 0, m \in \mathrm{R}, j \in \mathrm{R}\}$
$\{(m, j) \mid j < m, m \ge 0, j \ge 0, m \in \mathrm{R}, j \in \mathrm{R}\}$

Any point in the solution region represents an acceptable combination. For example,
- 80% of the mainsail and 70% of the jib can be out.
- 70% of the mainsail and 40% of the jib can be out.
- 40.5% of the mainsail and 30.75% of the jib can be out.

HOMEWORK...

#6

p. 236: #7 - 10

NOTE: Each question requires a graph to get possible solutions!