4.4 Fractional Exponents and Radicals

LESSON FOCUS Relate rational exponents and radicals.
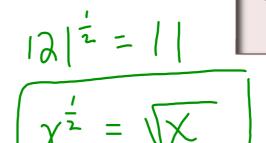
Make Connections

Coffee, tea, and hot chocolate contain caffeine. The expression $100(0.87)^{\frac{1}{2}}$ represents the percent of caffeine left in your body $\frac{1}{2}$ h after you drink a caffeine beverage.

Given that $0.87^1 = 0.87$ and $0.87^0 = 1$, how can you estimate a value for $0.87^{\frac{1}{2}}$?

Use a calculator to complete the table.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} =$ 1
4	$4^{\frac{1}{2}} =$ 2
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$

$121^{\frac{1}{2}} = 11$

$$x^{\frac{1}{2}} = \sqrt{x}$$

Use a calculator to complete the table.

x	$x^{\frac{1}{3}}$
1	$1^{\frac{1}{3}} = 1$
8	$8^{\frac{1}{3}} = 2$
27	$27^{\frac{1}{3}} = 3$
64	
125	

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$x^{\frac{1}{10}} = \sqrt[10]{x}$$

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$$x^{\frac{2}{3}} = \left(x^{\frac{1}{3}}\right)^2$$

$$\left(\sqrt[3]{x}\right)^2$$

Example 1   Evaluating Powers of the Form $a^{\frac{1}{n}}$

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ b) $0.49^{\frac{1}{2}}$ c) $(-64)^{\frac{1}{3}}$ d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

✓ **SOLUTION**

$\sqrt[3]{27}$ $\sqrt{0.49}$ $\sqrt[3]{-64}$ $\sqrt{\dfrac{4}{9}}$

3 0.7 -4 $\dfrac{\sqrt{4}}{\sqrt{9}} = \dfrac{2}{3}$

CHECK YOUR UNDERSTANDING

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Examples: Express each exponential in radical form, then evaluate.

1. $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2$

2^2

4

2. $125^{-\frac{1}{3}} = \dfrac{1}{125^{\frac{1}{3}}} = \dfrac{1}{\sqrt[3]{125}}$

$\qquad = \dfrac{1}{5}$

3. $32^{-\frac{7}{5}} =$

4. $\dfrac{3}{9^{-\frac{3}{2}}} =$

Express as a ~~exponent~~: Single power

$\sqrt{4} = 4^{\frac{1}{2}}$

a) $\sqrt[5]{32}$ b) $\sqrt[3]{-64}$ c) $\left(\sqrt[2]{144}\right)^3$

$32^{\frac{1}{5}}$ $(-64)^{\frac{1}{3}}$ $144^{\frac{3}{2}}$

Express as a Radical:

a) $8^{\frac{5}{3}}$

$\left(\sqrt[3]{8}\right)^5$

$(2)^5$

32

b) $49^{\frac{3}{2}}$

$\left(\sqrt{49}\right)^3$

147

c) $(-125)^{\frac{2}{3}}$

$\left(\sqrt[3]{-125}\right)^2$

25

Example 3 — Evaluating Powers with Rational Exponents and Rational Bases

$0.4 = \frac{4}{10} = \frac{2}{5}$

Evaluate.

a) $0.04^{\frac{3}{2}}$ b) $27^{\frac{4}{3}}$ c) $(-32)^{0.4}$ d) $1.8^{\frac{2}{5}}$

✓ **SOLUTION**

$\left(\sqrt{0.04}\right)^3$

$(0.2)^3$

0.008

$\left(\sqrt[3]{27}\right)^4$

3^4

81

$\left(\sqrt[5]{32}\right)^2$

$(-2)^2$

4

$\left(\sqrt[5]{1.8}\right)^2$

CHECK YOUR UNDERSTANDING

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Switch to power

$$\left(\sqrt[4]{81}\right)^3 \qquad \left(\sqrt[3]{27}\right)^2 \qquad \left(\sqrt[2]{64}\right)^3$$

$$81^{\frac{3}{4}} \qquad 27^{\frac{2}{3}} \qquad 64^{\frac{3}{2}}$$

Switch to radical.

$$27^{\frac{5}{4}} \qquad 49^{\frac{3}{2}} \qquad 121^{2.5}$$

$$\left(\sqrt[4]{27}\right)^5 \qquad \left(\sqrt{49}\right)^3 \qquad 121^{\frac{5}{2}}$$

$$\left(\sqrt{121}\right)^5$$

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