

**In Summary****Key Idea**

- Inductive reasoning involves looking at specific examples. By observing patterns and identifying properties in these examples, you may be able to make a general conclusion, which you can state as a conjecture.

**Need to Know**

- A conjecture is based on evidence you have gathered.
- More support for a conjecture strengthens the conjecture, but does not prove it.

**HW...**

*Questions 8, 10*

p. 12: #1 - 3; #6 - 11; 13; 15; 16

8. Dan noticed a pattern in the digits of the multiples of 3. He created the following table to show the pattern.

Multiples of 3	12	15	18	21	24	27	30
Sum of the Digits	3	6	9	3	6	9	3

- Make a conjecture based on the pattern in the table.
- Find a classmate who made a different conjecture. Discuss the reasonableness of both conjectures.
- Test one of the conjectures.

a) Sum of the digits is divisible by 3 when I have a multiple of 3.

b)

c)

5931

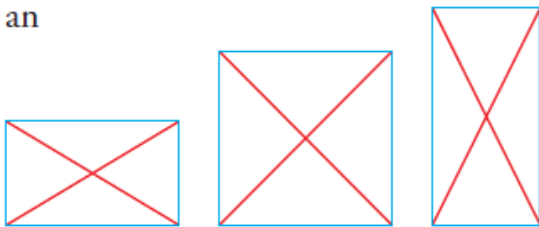
10. Make a conjecture about the temperature on November 1 in Hay River, Northwest Territories, based on the information in the chart below. Summarize the evidence that supports your conjecture.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Maximum Temperature (°C)	+3.1	-2.2	-1.1	-10.1	-1.6	-3.9	-3.2	+2.9	+1.8	-3.0

Range  $\rightarrow -11^{\circ}\text{C} \leftrightarrow 4^{\circ}\text{C}$

12. Ursula studied the diagonals of these rectangles to look for patterns. Make a conjecture about the diagonals of rectangles. What evidence supports your conjecture?

is an




*Intersection  
is in the  
center of  
the  
rectangle*

# REVIEW...

Lesson 2.1 - Inductive Reasoning.wmv

laporte306  54 videos

*Mr. Lien's Weekend!*



- Mr. Lien was shopping for food on Saturday, when he found a great deal!

0:12 / 3:54

## 1.3

## Using Reasoning to Find a Counterexample to a Conjecture

**GOAL**

Invalidate a conjecture by finding a contradiction.

To restate what you have read so far, a conjecture is a mathematical statement that has been proposed as a true statement, but not yet proven or disproved.

Once a conjecture is proven, it is a mathematical **fact**.

One method to test a conjecture is to attempt to **disprove** it by using a **counterexample**.

For example:

Conjecture: All prime numbers are odd.

Counterexample: But 2 is a prime number.

The counterexample disproves the conjecture, hence we can conclude that not all prime numbers are odd.

## EXAMPLE #2:

Conjecture:



For all real numbers  $x$ , the expressions  $x^2$  is greater than or equal to  $x$

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Here is a counterexample:

$$(0.5)^2 = 0.25, \text{ and } 0.25 \text{ is } \mathbf{not} \text{ greater than or equal to } 0.5$$

In fact, any number between 0 and 1 is a counterexample. The conjecture is false.

**COUNTEREXAMPLE???**

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EXAMPLE 3

Using reasoning to find a counterexample to a conjecture

Matt found an interesting numeric pattern:

$$1 \cdot 8 + 1 = 9$$

$$12 \cdot 8 + 2 = 98$$

$$123 \cdot 8 + 3 = 987$$

$$1234 \cdot 8 + 4 = 9876$$

Matt thinks that this pattern will continue.

Search for a counterexample to Matt's conjecture.

**Kublu's Solution**

$$1 \cdot 8 + 1 = 9$$

$$12 \cdot 8 + 2 = 98$$

$$123 \cdot 8 + 3 = 987$$

$$1234 \cdot 8 + 4 = 9876$$

The pattern seemed to be related to the first factor (the factor that wasn't 8), the number that was added, and the product.

	A	B
1	$1 \cdot 8 + 1$	9
2	$12 \cdot 8 + 2$	98
3	$123 \cdot 8 + 3$	987
4	$1234 \cdot 8 + 4$	9876
5	$12345 \cdot 8 + 5$	98765
6	$123456 \cdot 8 + 6$	987654
7	$1234567 \cdot 8 + 7$	9876543
8	$12345678 \cdot 8 + 8$	98765432
9	$123456789 \cdot 8 + 9$	987654321

I used a spreadsheet to see if the pattern continued. The spreadsheet showed that it did.

$$12345678910 \cdot 8 + 10 = 98765431290$$

$$1234567890 \cdot 8 + 10 = 9876543130$$

$$12345678910 \cdot 8 + 0 = 98765431280$$

$$1234567890 \cdot 8 + 0 = 9876543120$$

When I came to the tenth step in the sequence, I had to decide whether to use 10 or 0 in the first factor and as the number to add. I decided to check each way that 10 and 0 could be represented.

The pattern holds true until 9 of the 10 digits are included. At the tenth step in the sequence, a counterexample is found.

Since the pattern did not continue, Matt's conjecture is invalid.

Revised conjecture: When the value of the addend is 1 to 9, the pattern will continue.

I decided to revise Matt's conjecture by limiting it.



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### *Your Turn*

If Kublu had not found a counterexample at the tenth step, should she have continued looking? When would it be reasonable to stop gathering evidence if all the examples supported the conjecture? Justify your decision.

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### *Answer*

If Kublu had not found a counterexample at the 10th step, she could have still stopped there. With the quantity of evidence found to support the conjecture, and a two-digit number further validating the conjecture, the conjecture could be considered strongly supported. If she had wanted to do one more example, then it might have been logical to try a three-digit number to see if the conjecture was valid in that case.

# Mathematical HISTORY???

## Goldbach's Conjecture

One famous example of an unproven conjecture has remained undecided for nearly 300 years.

In the early 1700's, Christian Goldbach, a Prussian mathematician, noticed that many even numbers greater than 2 can be written as the sum of two primes. Expanding on examples like these, Goldbach wrote the following conjecture:

$4 = 2 + 2$	$10 = 3 + 7$	$16 = 3 + 13$
$6 = 3 + 3$	$12 = 5 + 7$	$18 = 5 + 13$
$8 = 3 + 5$	$14 = 3 + 11$	$20 = 3 + 17$

**Conjecture:** Every even number greater than 2 can be written as the sum of two primes.

To this day, no one has proven **Goldbach's Conjecture** or found a counterexample to show that it is false. It is still unknown whether this conjecture is true or false. It is known, however, that all even numbers up to  $4 \times 10^{18}$  confirm Goldbach's Conjecture.



Christian Goldbach (1690 – 1764) was a German mathematician famous for his eponymous Conjecture. Goldbach's Conjecture is one of the most infamous problems in mathematics, and states that every even integer number greater than 2 can be expressed as the sum of two prime numbers. For example,  $4=2+2$ ,  $6=3+3$ , and  $8=3+5$ . While there have not been any counter-examples found up through  $4 \times 10^{18}$  (as of 2012), the conjecture has not yet been formally proven.

## 1.2

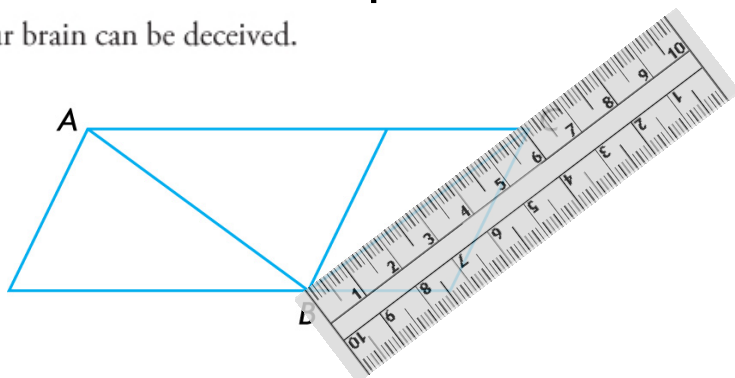
Exploring the Validity  
of Conjectures

## GOAL

Determine whether a conjecture is valid.

**EXPLORE** *the Math* p. 16

Your brain can be deceived.

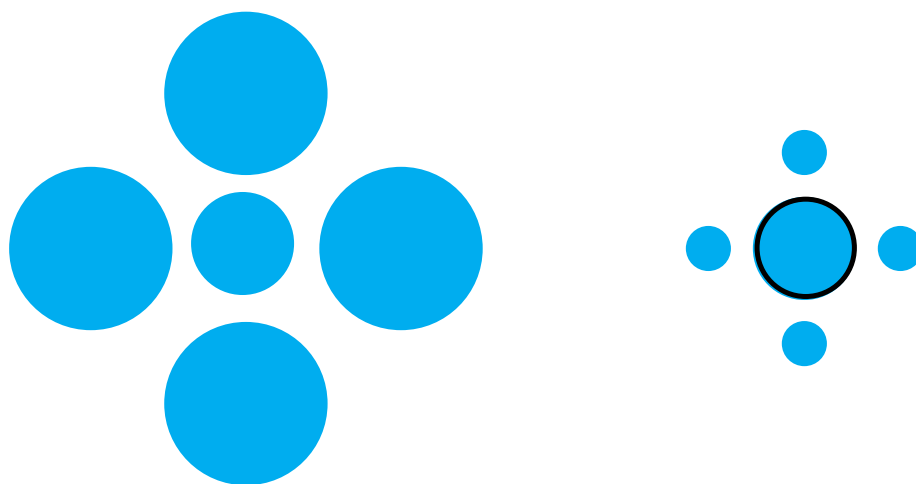


Make a conjecture about diagonal  $AB$  and diagonal  $BC$ .

**?** How can you check the validity of your conjecture?

## *EXPLORE the Math*

Your brain can be deceived.

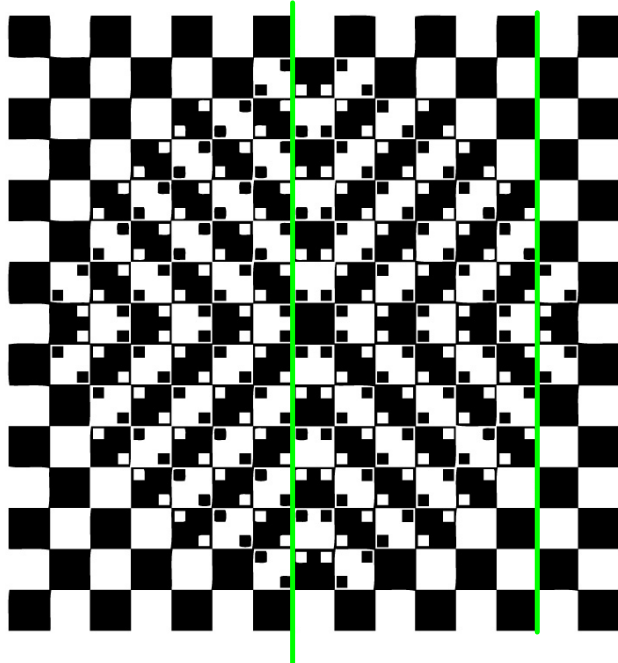


Make a conjecture about the circles in the centre.

**?** How can you check the validity of your conjecture?

## EXPLORE the Math

Your brain can be deceived.

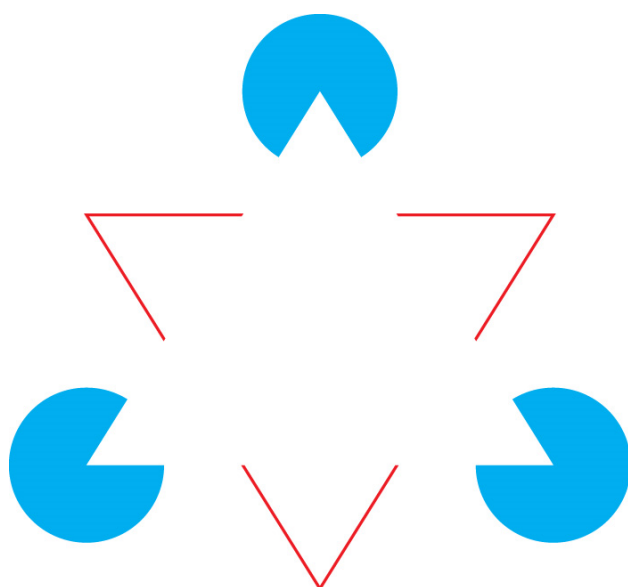


Make a conjecture about the lines.

**?** How can you check the validity of your conjecture?

## **EXPLORE** the Math

Your brain can be deceived.



Make a conjecture about the number of triangles.

**?** How can you check the validity of your conjecture?

## Reflecting

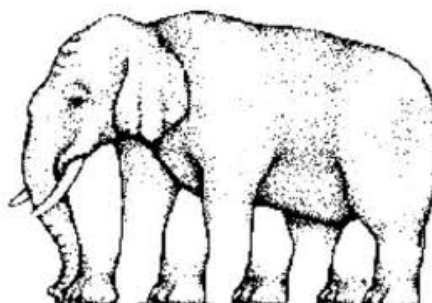
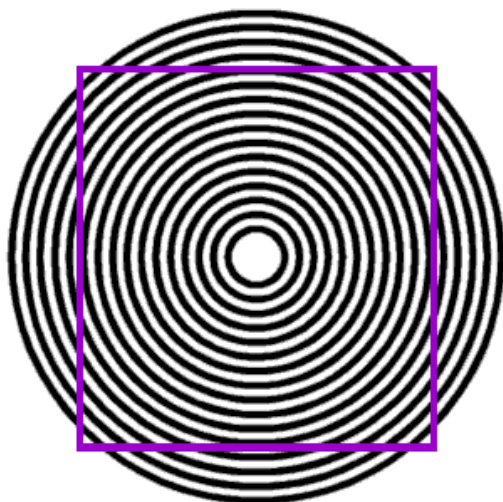
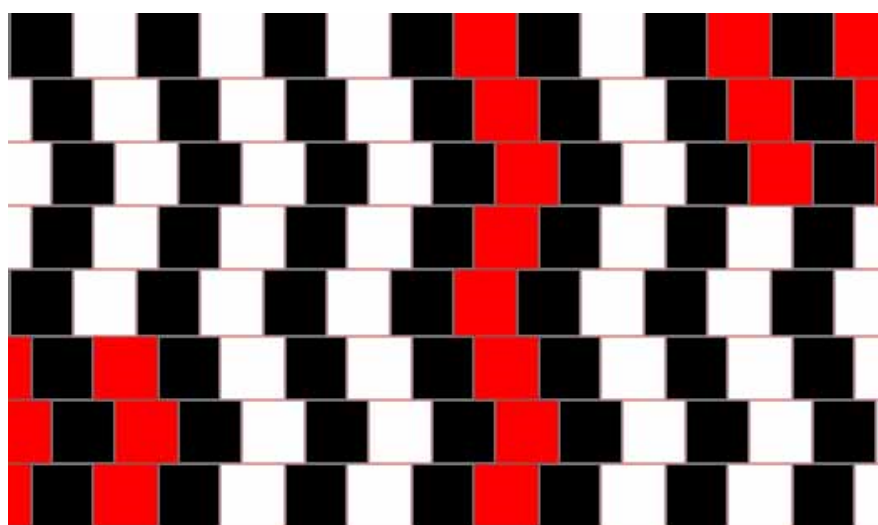
- A. Describe the steps you took to verify your conjectures.
- B. After collecting evidence, did you decide to revise either of your conjectures? Explain.
- C. Can you be certain that the evidence you collect leads to a correct conjecture? Explain.



## Answers

- A. Both measurement and visual inspection helped to verify or discredit the conjectures.
- B. My conjectures changed as follows after collecting more evidence:
  - First image: Both diagonals are the same length.
  - Second image: The centre circles of the figures are the same size.
  - Third image: The rows and columns of white and black shapes are placed in straight lines.
  - Fourth image: There are no triangles in the figure.
- C. For these images, the revised conjectures hold true for the accuracy of the tools I used. I cannot be absolutely sure that my new conjectures are valid until the precision of the tools is considered.

Some other optical illusions...






*M. C. Escher...*

<http://www.mcescher.com/>



Three Dragons

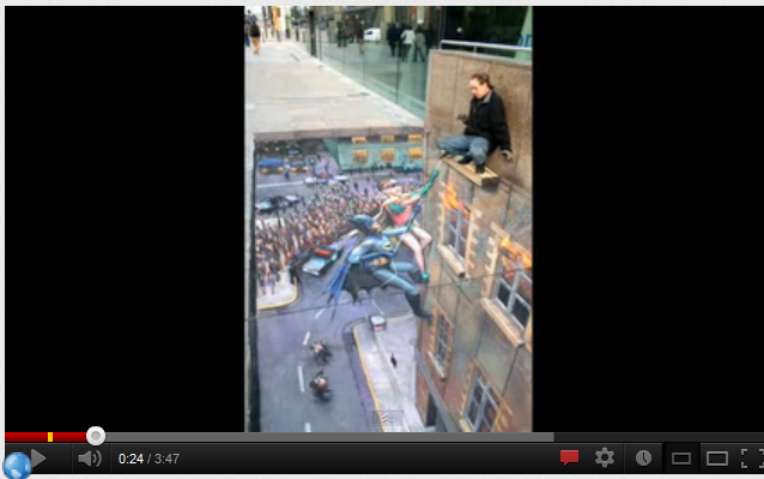


 three\_dragons.wmv

## 3D Chalk Art.. Julian Beever

extreme 3D sidewalk chalk drawings.

moviemagician101  22 videos



*"Julian Beever is an English, Belgium-based chalk artist who has been creating trompe-l'œil chalk drawings on pavement surfaces since the mid-1990s. His works are created using a projection called **anamorphosis**, and create the illusion of three dimensions when viewed from the correct angle."*

**44 Amazing Julian Beever's 3D Pavement Drawings**

## 1.2 - Validity of Conjectures?

In Summary page 17

### Key Idea

- Some conjectures initially seem to be valid, but are shown not to be valid after more evidence is gathered.

### Need to Know

- The best we can say about a conjecture reached through inductive reasoning is that there is evidence either to support or deny it.
- A conjecture may be revised, based on new evidence.

## 1.3 - Counterexamples

In Summary page 22

### Key Ideas

- Once you have found a counterexample to a conjecture, you have disproved the conjecture. This means that the conjecture is invalid.
- You may be able to use a counterexample to help you revise a conjecture.

### Need to Know

- A single counterexample is enough to disprove a conjecture.
- Even if you cannot find a counterexample, you cannot be certain that there is not one. Any supporting evidence you develop while searching for a counterexample, however, does increase the likelihood that the conjecture is true.

## HOMework...

p. 17: #1 & 2

p. 22: #1, 3, 5, 8, 12, 17

## Attachments

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1s3e3 final.mp4

three\_dragons.wmv