1.4 Proving Conjectures: Deductive Reasoning

GOAL

Prove mathematical statements using a logical argument.

Every day, you use **deductive thinking** to **deduce** new information.

In this course, you will use this method to deduce the properties of geometric figures and many geometric relationships. For example:

Step A General Statement	**And**	**Step B Particular Statement**	Thus	**Step C Conclusion**
During a game, five players are used on a basketball team.	And	UNB is playing basketball.	Thus	UNB uses five players on the basketball team.
All isosceles triangles have two equal sides.	And	Triangle ABC is isosceles.	Thus	Two sides of Triangle ABC are equal.

In Step A, based on your earlier knowledge (your experience, what you have learned in life) you accept certain general statements to be true. In Step B you are confronted with a particular case that is related to a general statement. Lastly, in Step C, you deduce a conclusion based upon Step A and B.

KEY TERMS...

deductive reasoning
Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

proof
A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

two-column proof
A presentation of a logical argument involving deductive reasoning in which the statements of the argument are written in one column and the justifications for the statements are written in the other column.

generalization
A principle, statement, or idea that has general application.

We can often use the **transitive property** in deductive reasoning. According to this property, **if two things are equal to the same thing, then they are equal to one another**. We can express this property mathematically:

If $a = b$ and $b = c$, then $a = c$.

Mathematical Proofs

A **proof** is a convincing argument that something is true.

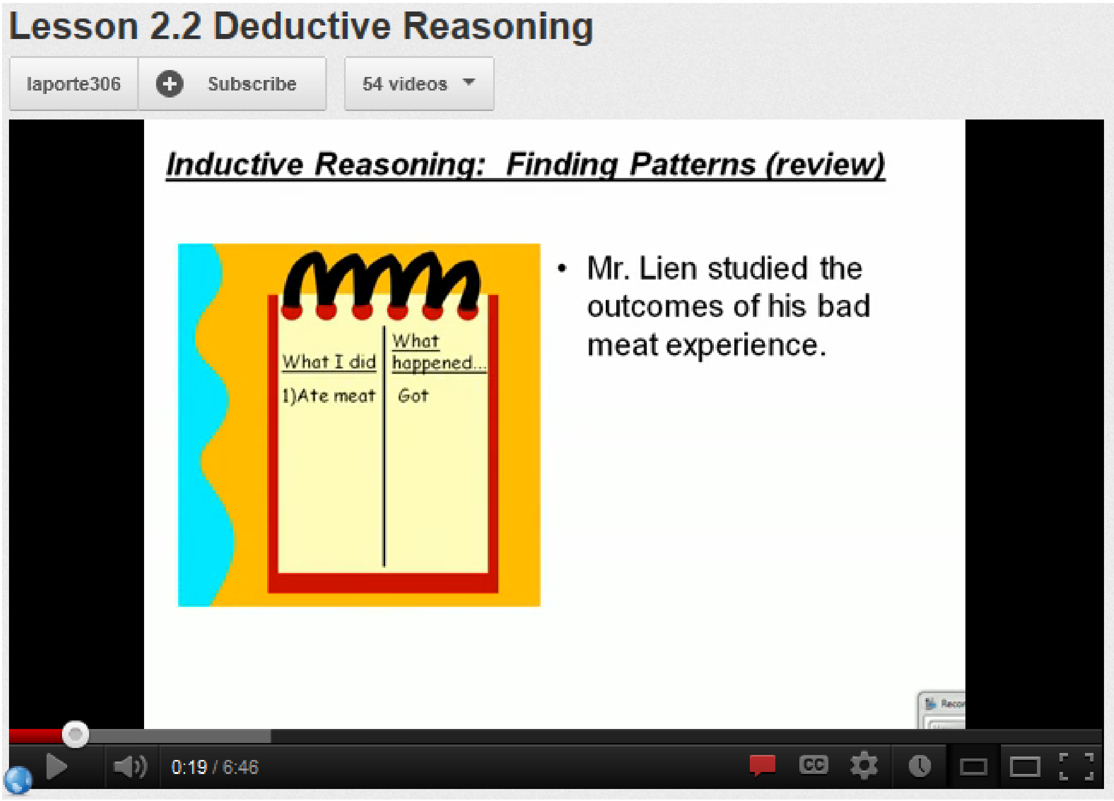
In mathematics, a proof starts with things that are agreed upon, called **postulates** or **axioms**, and then uses logic to reach a conclusion. Conclusions are often reached in geometry by observing data and looking for patterns. As you learned earlier, this type of reasoning is called **inductive reasoning** and the conclusion reached by inductive reasoning is called a **conjecture**.

A proof in geometry consists of a sequence of statements—each supported by a reason—that starts with a given set of premises and leads to a valid conclusion. This type of reasoning is called **deductive reasoning**. Each statement in a proof follows from one or more of the previous statements.

A **reason** (a fact) for a statement can come from the set of given premises or from one of the four types of other premises:

- Definitions
- Postulates
- Properties of algebra, equality or congruence
- Previously proven theorems

Once a conjecture is proved, it is called a **theorem**. As a theorem, it becomes a premise for geometric arguments you can use to prove other conjectures.


Lesson 2.2 Deductive Reasoning
laporte306
Subscribe
54 videos
Inductive Reasoning: Finding Patterns (review)
What I did
What happened...
1)Ate meat
Got
Mr. Lien studied the outcomes of his bad meat experience.
0:19 / 6:46

Let $x \to$ middle #

LEARN ABOUT the Math

Jon discovered a pattern when adding integers:

$$1 + 2 + 3 + 4 + 5 = 15$$
$$(-15) + (-14) + (-13) + (-12) + (-11) = -65$$
$$(-3) + (-2) + (-1) + 0 + 1 = -5$$

$5x$

$(x-2)+(x-1)+x+(x+1)+(x+2)$

$\boxed{5x}$

5 times middle #

He claims that whenever you add five consecutive integers, the sum is always 5 times the median of the numbers.

? How can you prove that Jon's conjecture is true for all integers?

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EXAMPLE 1 Connecting conjectures with reasoning

Prove that Jon's conjecture is true for all integers.

Pat's Solution

$$5(3) = 15$$
$$5(-13) = -65$$
$$5(-1) = -5$$

The median is the middle number in a set of integers when the integers are arranged in consecutive order. I observed that Jon's conjecture was true in each of his examples.

$$210 + 211 + 212 + 213 + 214 = 1060$$
$$5(212) = 1060$$

I tried a sample with greater integers, and the conjecture still worked.

Let x represent any integer.
Let S represent the sum of five consecutive integers.
$S = (x - 2) + (x - 1) + x + (x + 1) + (x + 2)$

I decided to start my **proof** by representing the sum of five consecutive integers. I chose x as the median and then wrote a **generalization** for the sum.

proof
A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

generalization
A principle, statement, or idea that has general application.

$$S = (x + x + x + x + x) + (-2 + (-1) + 0 + 1 + 2)$$
$$S = 5x + 0$$

I simplified by gathering like terms.

$$S = 5x$$
Jon's conjecture is true for all integers.

Since x represents the median of five consecutive integers, $5x$ will always represent the sum.

APPLY the Math p. 28

$4-1=3$
$9-4=5$
$64-49=15$
$100-81=19$

EXAMPLE 2 Using deductive reasoning to generalize a conjecture

Let $x \to 1^{st}$ #

$x^2 - (x-1)^2$
$x^2 - (x^2 - 2x + 1)$
$\cancel{x^2} - \cancel{x^2} + 2x - 1$
ODD

In Lesson 1.3, page 19, Luke found more support for Steffan's conjecture from Lesson 1.1, page 9—that the difference between consecutive perfect squares is always an odd number.

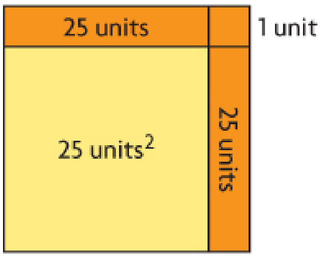
Determine the general case to prove Steffan's conjecture.

Back to previous lesson

Gord's Solution

The difference between consecutive perfect squares is always an odd number.

Steffan's conjecture has worked for consecutive perfect squares with sides of 1 to 7 units.



$26^2 - 25^2 = 2(25) + 1$
$26^2 - 25^2 = 51$

I tried a sample using even greater squares: 26^2 and 25^2.

The difference is the two sets of 25 unit tiles, plus a single unit tile.

Let x be any natural number.
Let D be the difference between consecutive perfect squares.
$D = (x + 1)^2 - x^2$

Since the conjecture has been supported with specific examples, I decided to express the conjecture as a general statement. I chose x to be the length of the smaller square's sides. The larger square's sides would then be $x + 1$.

$D = x^2 + x + x + 1 - x^2$
$D = x^2 + 2x + 1 - x^2$
$D = 2x + 1$

I expanded and simplified my expression. Since x represents any natural number, $2x$ is an even number, and $2x + 1$ is an odd number.

Steffan's conjecture, that the difference of consecutive perfect squares is always an odd number, has been proved for all natural numbers.

Review : Squaring a Binomial

$$(x-1)^2$$

$$(x-1)(x-1)$$

$$x^2 - x - x + 1$$

$$x^2 - 2x + 1$$

3 Step Rule

1. $(1^{st})^2$
2. $1^{st} \times 2^{nd} \times 2$
3. $(2^{nd})^2$

$$x^2 - 2x + 1$$

ex: $(4x+3)^2$

$$16x^2 + 24x + 9$$

Let's do one together...

Two Digit #

① $\boxed{10a + b}$

② $-(a + b)$

$\boxed{9a}$

Multiple of 9

In Summary p. 31

Key Idea

- Deductive reasoning involves starting with general assumptions that are known to be true and, through logical reasoning, arriving at a specific conclusion.

Need to Know

- A conjecture has been proved only when it has been shown to be true for every possible case or example. This is accomplished by creating a proof that involves general cases.
- When you apply the principles of deductive reasoning correctly, you can be sure that the conclusion you draw is valid.
- The transitive property is often useful in deductive reasoning. It can be stated as follows: Things that are equal to the same thing are equal to each other. If $a = b$ and $b = c$, then $a = c$.
- A demonstration using an example is *not* a proof.

HOMEWORK...

p. 31: #1, 2
#4, 5
#7, 8
#10, 11
#15, 17