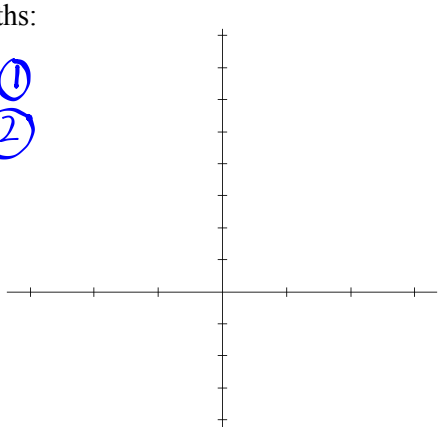
Warm Up

In Canada's Arctic, a coordinate system is one way of locating a position. Two surveying crews trek along the following paths:

A: Survey crew Hollander: $6x + y = 0$ ①
B: Survey crew Williams: $2x - y = -8$ ②



At which point will the two crews meet?

① + ② $8x + 0 = -8$

$\frac{8x}{8} = \frac{-8}{8}$

$x = -1$ ... ③

Sub ③ into ①

$6x + y = 0$

$6(-1) + y = 0$

$y = 6$

The two crews meet at $(-1, 6)$

Classifying Systems of Equations:

If a system of linear equations has one or more solutions, the system is said to be a **consistent system**. If a linear equation has no solution, it is said to be an **inconsistent system**.

If two equations represent the same line, then all points along the line are solutions to the system of equations. In such a case, the system is characterized as a **dependent system**. An **independent system** is one in which the two equations represent different lines.

Three possibilities when solving systems of equations in two variables...

Solutions to Systems of Linear Equations in Two Variables

One unique solution	No solution	Infinitely many solutions
		
One point of intersection	Parallel lines	Coinciding lines
System is consistent.	System is inconsistent.	System is consistent.
System is independent.	System is independent.	System is dependent.

True or False??

A consistent system is a system that always has a unique solution.

A dependent system is a system that has no solution.

If two lines coincide, the system is dependent.

If two lines are parallel, the system is independent.

Inconsistent System

- sometimes there may be no solutions when the lines are parallel.
- indicator is getting 0 = # in your solution.

Example: Solve... $y = 3x - 5$ & $y = 3x + 2$

$y = 3x - 5$ ①

$y = 3x + 2$ ②

① - ② $0 = 0 - 7$

$0 = -7$

not possible

no solution

Dependent System

- sometimes there may be infinitely many solutions when the lines are the same
- indicator is getting 0 = 0 in your solution.
- will have an infinite number of solutions.

Example: Solve... $3x - y = 5$ & $-15 + 9x = 3y$

$3x - y = 5$ ... ①

$3x - y = 5$ ... ②

① - ② $0 + 0 = 0$

$0 = 0$

infinite # of solutions

$\div 3$ $-5 + 3x = y$

$3x - y = 5$

Dependent Systems:

How many solutions?....

$3x + 5y = 9$ ①

$6x = 18 - 10y$

$\div 2$ $3x = 9 - 5y$

$3x + 5y = 9$ ②

$2x + 3y - 4 = 0$ ①

$6y - 8 = -4x$

$\div 2$ $3y - 4 = -2x$

$2x + 3y - 4 = 0$ ②

Same line

infinite # of solutions

Applications

In January of the year 2000, John was eleven times as old as his son William. In January of 2012, he will be three times as old as his son. How old was William in January of 2000?

Step 1: Read the problem (multiple times!)

Step 2: Define the two variables in the problem

Step 3: Set up the equations from the problem
(# equations = # unknowns)

Step 4: Solve the system of equations

Step 5: State your conclusion

Let x = William's age in 2000

Let y = John's age in 2000

$y = 11x$ ①

$y + 12 = 3(x + 12)$ ②

Sub ① into ②

$11x + 12 = 3(x + 12)$

$11x + 12 = 3x + 36$

$11x - 3x = 36 - 12$

$8x = 24$

$x = 3$

William is 3 years old in the year 2000.

In January of the year 2000, John was eleven times as old as his son William. In January of 2012, he will be three times as old as his son. How old was William in January of 2000?

Word Problems

In a student election, 584 students voted for one or the other of two candidates for president. If the winner received 122 more votes than the loser, how many votes were cast for each candidate?

Step 1: Develop two equations

Step 2: Solve the system of equations

x = votes for winner

y = votes for loser

$x + y = 584$ ①

$x = y + 122$

$x - y = 122$ ..... ②

① − ② $\frac{2y}{2} = \frac{462}{2}$

$y = 231$ ..... ③

Sub ③ into ②

$x = y + 122$

$= 231 + 122$

$= 353$

The votes were 353 and 231.

Money Problems

I have $11.60, all dimes and quarters, in my pocket. I have 32 more dimes than quarters. How many dimes, and how many quarters, do I have?

x = # of dimes
y = # of quarters

$x = y + 32$ ... ①

$0.10x + .25y = 11.60$ ... ②

② × 100 $10x + 25y = 1160$

Sub ① into ② $10x + 25y = 1160$
$10(y+32) + 25y = 1160$
$10y + 320 + 25y = 1160$
$35y = 1160 - 320$
$35y = 840$
$y = 24$ ..... ③

Sub ③ into ① $x = y + 32$
$= 24 + 32$
$= 56$

He/she has 24 quarters & 56 dimes.

Check $0.10 \times 56 = 5.60$
$0.25 \times 24 = 6.00$
$\$11.60$

Interest Problems

A total of $12,000 is invested in two funds paying 9% and 11% simple interest. If the yearly interest is $1,180, how much of the $12,000 is invested at each rate?

Example #2:

Sales personnel at a sporting goods store are given a choice of 2 payment methods...

PLAN A: A weekly salary of $200 plus a 2 % commission on all sales.

PLAN B: No weekly salary but a 5 % commission on all sales.

Which is the better plan for the employee?

Page 441: #1 & #4

Page 448: #4, #5, #7

PRACTICE PROBLEMS...

p. 410: #11, 12, 13

p. 425: #11, 16, 17, 18

p. 437: #8, 10,