Midpoint of a Line

$$\frac{-3+5}{2}, \frac{6+14}{2}$$

(1, 10)

B (5, 14)

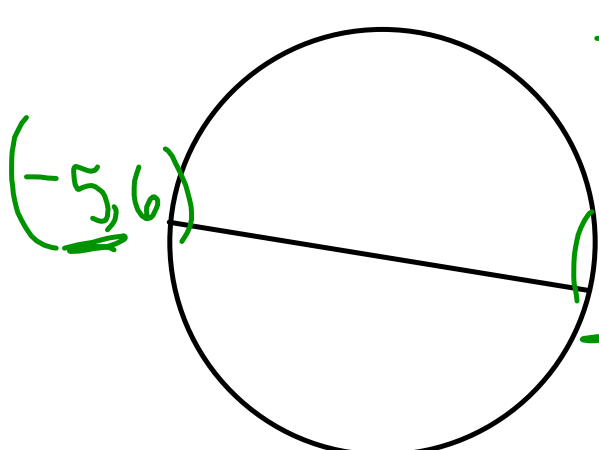$$\frac{2}{2} \qquad \frac{20}{2}$$

A(-3, 6)

$$midpoint\,(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

(1, 10)

Examples...

#1. The endpoints of a diameter of a circle are (-5,6) and (11,-12). Find the coordinates of the center of the circle.

(-5,6)

(11,-12)

$$\frac{-5+11}{2}, \frac{6-12}{2}$$

$$\frac{6}{2} \qquad \frac{-6}{2}$$
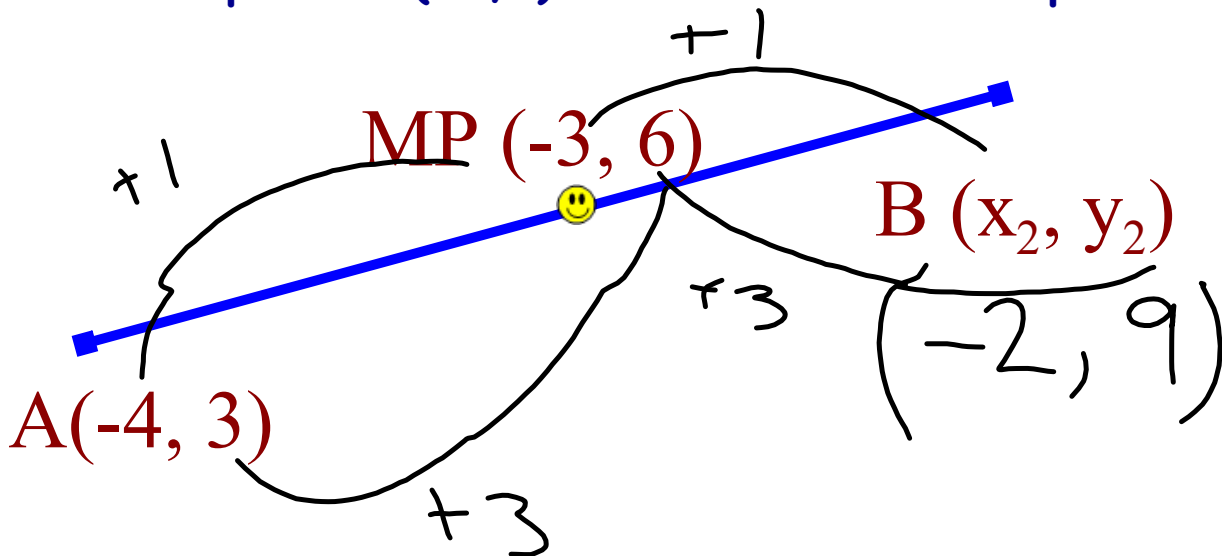
$$(3, -3)$$

Midpoint of a Line

$$midpoint\ (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

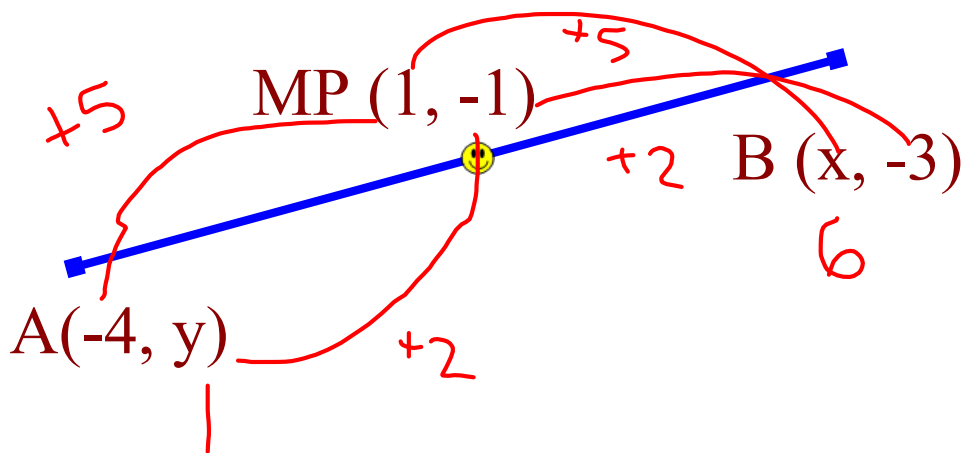**#2. One endpoint of a line segment is (-4,3). The midpoint is (-3,6). Find the other endpoint.**

$+1$

MP (-3, 6)

$+1$

B (x_2, y_2)

$+3$

$\left(-2, 9 \right)$

A(-4, 3)

$+3$

Midpoint of a Line

$$midpoint\,(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**#3. If the line segment joining (-4,y) to (x, -3) has a
midpoint of (1,-1), find the value of x and y.**

MP (1, -1)

A(-4, y)   B (x, -3)

$$1(1,-1) = \frac{-4 + X}{2} \;,\; \frac{y + -3}{2}$$

$$1 = \frac{-4 + X}{2} \qquad -1 = \frac{y + -3}{2}$$

$$2 = (-4) + X \qquad -2 = y + -3$$

$$6 = X \qquad -2 + 3 = y$$

$$1 = y$$

Homework...

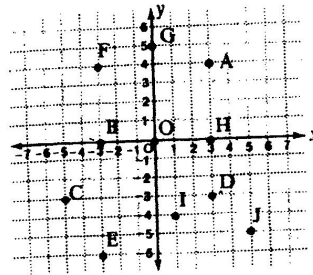Worksheet - Distance and Midpoint.doc

Sec. 6.6

#3,4,5a

Sec. 6.7

1(acdfgi)4,5

6.6   Exercise

A   Throughout this exercise, unless indicated otherwise, you may leave your final answer in simplified radical form.

1   Refer to the diagram. Find the length of each line segment.

(a) AH   (b) CD   (c) OF
(d) OA   (e) DJ   (f) EI
(g) DF   (h) AC   (i) FB

Distance Between Two Points

$P_1(x_1, y_1)$, $P_2(x_2, y_2)$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2   Find the distance from each point to the origin.

(a) $(6, 8)$ (b) $(-1, 8)$ (c) $\left(\frac{3}{4}, 1\right)$ (d) $(\sqrt{3}, 1)$ (e) $\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

3   Find the length of the line segment joining each pair of points.

(a) $(0, 0), (8, 6)$ (b) $(0, 2), (3, 3)$ (c) $(-3, 0), (8, -5)$
(d) $(-2, -2), (8, 0)$ (e) $(8, -8), (4, -1)$ (f) $(6, 8), (-6, -8)$

4   Find the distance from each point to $(1, 4)$.

(a) $(-1, 7)$ (b) $(-2, 6)$ (c) $(4, 6)$

(d) What do you notice about your answers?

B   Remember: Absolute value symbols are used to show distance because distance is always positive. ▶**PSP**

5   Find the perimeter of each triangle.

(a) $\triangle ABC$ $A(1, 5)$ $B(1, 2)$ $C(5, 2)$
(b) $\triangle PQR$ $P(4, 2)$ $Q(-5, -10)$ $R(4, -10)$
(c) $\triangle DEF$ $D(8, 10)$ $E(-7, -10)$ $F(-7, 10)$

6   A triangle has vertices $P(-1, 2)$, $Q(2, 6)$, $R(-4, 4)$.

(a) Find the perimeter.
(b) Classify the triangle as scalene, isosceles, or equilateral.

7   Three vertices of rectangle ABCD are $A(-8, 0)$, $B(4, 4)$ and $C(6, -2)$.

(a) Find the lengths of the sides.   (b) Find the length of the diagonal.

8   Calculate the area of the rectangle with vertices at $P(-3, 2)$, $Q(2, 4)$, $R(4, -1)$ and $S(-1, -3)$.

9   Decide whether the points $P(-2, -1)$ and $Q(5, -8)$ lie on the same circle with centre $C(1, -5)$. Give reasons for your answer.

10   Given that $P(2, -1)$, $Q(-4, 7)$ and $R(3, 6)$ lie on a circle, show that the point $C(-1, 3)$ is the centre of that circle.

▶**PSP** 11   If 3 pots boil 3 cabbages in 33 min, then how long will it take 10 pots to boil 10 cabbages?

C 12   If (x_1, y_1) and (x_2, y_2) are two points on the line $y = mx + b$, then show that the distance between the two points is given by the expression $|x_2 - x_1|\sqrt{1 + m^2}$.

6.6 Exercise, page 229
1.a)4 units b)8 units c)5 units d)5 units
e)2√2 units f)2√5 units g)√85 units
h)√113 units i)4 units 2.a)10 units b)√65 units
c)$\frac{5}{4}$ units d)2 units e)1 unit 3.a)10 units
b)√10 units c)√146 units d)2√26 units
e)√65 units f)20 units 4.a)√13 units
b)√13 units c)√13 units d)equal 5.a)12 units
b)36 units c)60 units 6.a)√5 + 2√10 + √13) units
b)scalene 7.a)AB = CD = 4√10 units,
BC = AD = 2√10 units b)10√2 units
8. 29 square units 9. yes 11. 33 min

6.7   *Midpoint of a Line Segment*

You can use your skills with average to find the midpoint of a line segment. ►PSP

> If $P(x, y)$ is the midpoint of a line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$, then the coordinates of P are given by
> $$P(x, y) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right).$$
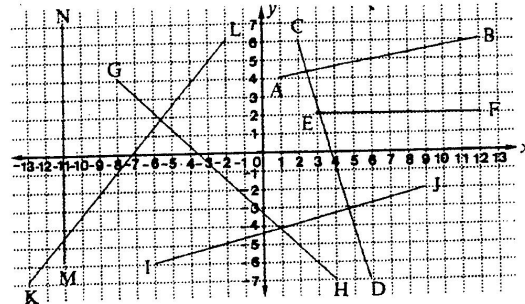
6.7   Exercise

A 1 Find the midpoint of the line joining each pair of points.

(a) $(2, 4), (6, 8)$ (b) $(-6, 4), (-8, 8)$ (c) $(5, 4), (7, 1)$

(d) $(-6, -2), (4, -6)$ (e) $(-3, 5), (5, -3)$ (f) $(-3.5, 7), (5, -4)$

(g) $(-3, -6), (3, 6)$ (h) $(a, b), (0, 0)$ (i) $(2a, 0), (0, 4b)$

2 Find the coordinates of the midpoint of each line segment.

3 The points where the diameter meets a circle are $(-11, -4)$ and $(5, 6)$. What are the coordinates of the centre of the circle?

B You may find it helpful to use a calculator. ►PSP

4 The midpoint of AB is given by $M(-1, 7)$ for the points $A(-5, 8)$ and $B(x, y)$. Find x and y.

5 Two points are given by $P(-4, y)$ and $Q(x, -2)$. If the midpoint is given by $M(2, 3)$, find x and y.

6 What are the coordinates of the points which divide a line with endpoints $(-10, -16)$ and $(2, 24)$ into 4 equal parts?

7 $\triangle ABC$ is given by $A(-8, \frac{1}{2})$, $B(-2, 7)$ and $C(6, -1)$. Median AP is to be drawn. What are the coordinates of the endpoints of median AP?

8 The vertices of an isosceles triangle are given by $A(1, 7)$, $B(-5, 1)$ and $C(7, 1)$. Determine whether the triangle formed by joining the midpoints of the sides of $\triangle ABC$ is also isosceles. ►PSP

C 9 Prove that the line joining the midpoints of any two sides of $\triangle ABC$ where $A(4, 7)$, $B(-2, 5)$, and $C(-10, -1)$, is parallel to the third side.

6.7 Exercise, page 231
1.a)(4, 6) b)(−7, 6) c)(6, 2.5) d)(−1, −4) e)(1, 1)
f)(0.75, 1.5) g)(0, 0) h)$\left(\frac{a}{2}, \frac{b}{2}\right)$ i)(a, 2b)
2. AB: (6.5, 5), CD: $\left(4, \frac{-1}{2}\right)$, EF: (7.5, 2),
GH: (−2, −1.5), IJ: (1.5, −4), KL: (−7.5, −0.5),
MN: (−11, 0.5) 3. (−3, 1) 4. x = 3, y = 6
5. x = 8, y = 8 6.(−7, −6), (−4, 4), (−1, 14)
7. $\left(-8, \frac{1}{2}\right), (2, 3)$ 8. yes

Chapter 6 Review

Part 1:

Find the slope of the line through each of the points.

    a) (6,6) (6,-11)   b) (10, -14), (-2, 2)

    c) (-7,-6), (-20,-1)   d) (-20,14), (11, -18)

Part 2:

Write the following equations in slope-intercept form, and then state the slope, y-intercept and x-intercept.

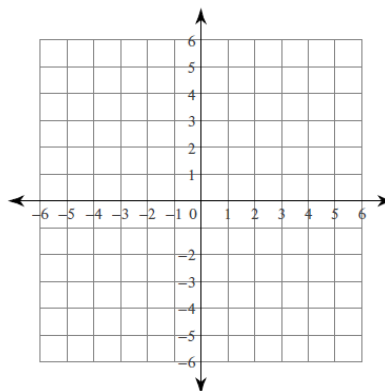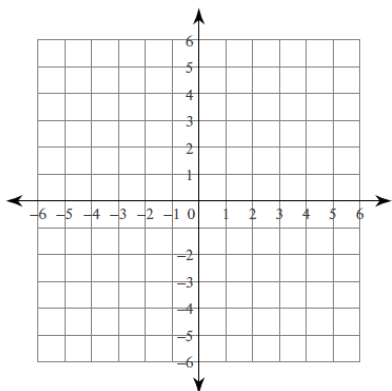 a) $y= \dfrac{2x}{5} -5$ b) y= -3x -5   c) y= 6x - 3   d) $y= \dfrac{9x}{2} -4$

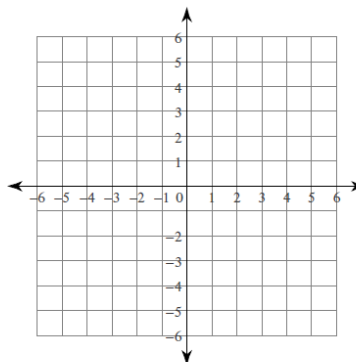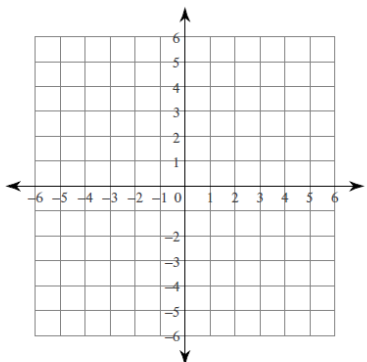Part 3: Graph the following

a) $y = \dfrac{6}{5}x - 2$

b) 2) $y = \dfrac{4}{3}x + 1$

c) $9x + y = 5$

d) $2x + y = 5$

Part 4:

Write the general form of the equation of each line given

a) $\text{Slope} = -\dfrac{3}{5}, \quad \text{y-intercept} = 5$

b) $\text{Slope} = 9, \quad \text{y-intercept} = 4$

c) slope= -2, x-intercept= -6

d) slope=$\dfrac{1}{2}$, x-intercept= -6

Part 6:
Write the equation of a line in point slope form and in then slope intercept form for each of the following:

a) slope = -5
   point= (-4,9)

b) slope = -2/3
   point= (5, -1)

c) point= (-6, -1)   Point= (2,5)

Part 7:
Which of the following are perpendicular or parallel?

a) y= 3x +6,   y = 3x -3

b) y= $\frac{-1}{2}$ x -5,   y = 2x + 5

c) (5,4) (11, -2) with (7,6) (3,2)

d) (7,6) (3,2) with (7,-3) (11,1)

Part 8:
Write the equation of a line, in point slope form for the following :

a)through: $(2, 0)$, parallel to $y = \frac{2}{3}x$

b)through: $(-2, 4)$, parallel to $y = -\frac{3}{2}x + 3$

c)through: $(2, 4)$, perp. to $y = -\frac{2}{7}x - 5$

d)through: $(5, 0)$, perp. to $y = -x + 5$

Part 9:
Write the equation of a line for the following:

a) Find the equation of a line that passes through (-2,4) and has ae slope perpendicular to $y = 2x + 3$.

b) Find the equation of a line that passes through the points (1,-3) and (-5,2)

c) Find the equation of a line that passes through the points (2,5) and (-11,-3)

d)Find the equation of a line that has the same x-intercept as this equation $6x + 12 = 3y$, and also passes through the point (3,-5).

Part 10:
Determine the distance and midpoint for the following lines

a) (-4,3) (5,6)   b) (0,-9) (-7, 2)