Name: Key

/40

Compound Interest

1. Complete the following chart: [15]

Principal	Rate/a	Time	Compounded	Formula	Amount (1)	Interest (1)
$1200	12%	5 a	Semi-annually	$A = 1200\left(1+\frac{0.12}{2}\right)^{10}$	$2149.02	$949.02
$480	6%	3 a	Quarterly	$A = 480\left(1+\frac{0.06}{4}\right)^{12}$	$573.90	$93.90
$10000	8%	12 a	Annually	$A = 10000\left(1+\frac{0.08}{1}\right)^{12}$	$25 181.70	$15 181.70
$5600	$7\frac{1}{4}\%$	10 a	Semi-annually	$A = 5600\left(1+\frac{0.0725}{2}\right)^{20}$	$11415.09	$5815.08
$80	$10\frac{1}{2}\%$	20 a	Monthly	$A = 80\left(1+\frac{0.105}{12}\right)^{240}$	$647.35	$567.35
$1 200 000	5%	7 a	Quarterly	$A = 1200000\left(1+\frac{0.05}{4}\right)^{28}$	1699190.76	$499 190.76

2. Examine how varying interest rates and compounding intervals affects the following investment. [18]

Principal	Rate/a	Time	Compounded	Formula	Amount (1)	Interest (1)
$12 000	8%	15 a	Annually	$A = 12000\left(1+\frac{0.08}{1}\right)^{15}$	$38 066.03	$26 066.03
$12 000	8%	15 a	Semi-Annually	$A = 12000\left(1+\frac{0.08}{2}\right)^{30}$	$38 920.77	$26 920.77
$12 000	8%	15 a	Quarterly	$A = 12000\left(1+\frac{0.08}{4}\right)^{60}$	$39 322.37	$27 322.37
$12 000	8%	15 a	Monthly	$A = 12000\left(1+\frac{0.08}{12}\right)^{180}$	$39 683.06	$27683.06
$12 000	8%	15 a	Daily	$A = 12000\left(1+\frac{0.08}{365}\right)^{5475}$	$39836.16	$27836.16
$12 000	8%	15 a	Simple Interest	$I = 12000(0.08)(15)$	$26 400	$14 400

+12000

3. Which of the following investments would be worth the most money after 20 years? [7]

$5000 at 8%/a compounded semi-annually	$7000 at 6%/a compounded daily	$17000 at 2%/a compounded monthly
$A = 5000\left(1+\frac{0.08}{2}\right)^{40}$	$A = 7000\left(1+\frac{0.06}{365}\right)^{7300}$	$A = 17000\left(1+\frac{0.02}{12}\right)^{240}$
$= \$24\,005.10$	$= \$23\,238.53$	$= \$25\,352.58$

rate of return
The ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent.

$$ROR = \frac{earn\ /\ lost}{invested}$$

EXAMPLE 3 p. 448 Determining the duration of a simple interest investment

Ingrid invested her summer earnings of $5000 at 8% simple interest, paid annually. She intends to use the money in a few years to take a holiday with a girlfriend.

a) How long will it take for the future value of the investment to grow to $8000?
b) What is Ingrid's **rate of return**?

Ingrid's Solution

a) $A = P + Prt$

P is $5000.
r is 8%, or 0.08.
A is $8000.

$$8000 = 5000 + (5000)(0.08)t$$
$$3000 = 400t$$
$$7.5 = t$$

I knew P, r, and A. I determined t by substituting these known values into the formula $A = P + Prt$ and solving for t.

Because I needed to isolate t, I knew that the $A = P + Prt$ form of the equation would have fewer solution steps than the $A = P(1 + rt)$ form would.

It will take 8 years for the future value of the investment to be at least $8000.

I knew 7.5 years would not work because the interest is paid annually. This meant that I had to round up to the next whole year. It also meant that, at 8 years, the future value would be more than $8000.

b) After 8 years:
$A = P + Prt$
$A = 5000 + (5000)(0.08)(8)$
$A = 8200$

At 8 years, the future value will be $8200.

Interest earned:
$8200 − $5000 = $3200

I determined the interest earned by subtracting the principal from the future value.

$$\text{Rate of return} = \frac{3200}{5000}$$
$$\text{Rate of return} = 0.64$$

I compared the interest earned with the principal to determine the rate of return.

The rate of return is 64% over 8 years.

EXAMPLE 3 p. 448 — Determining the duration of a simple interest investment

Ingrid invested her summer earnings of $5000 at 8% simple interest, paid annually. She intends to use the money in a few years to take a holiday with a girlfriend.

a) How long will it take for the future value of the investment to grow to $8000?
b) What is Ingrid's **rate of return**?

P → $ - invest
I → $ - earn
A → $ - end

$I = A - P$

$I = 8000 - 5000$

$I = 3000$

a)

$t = \frac{I}{Pr}$

$I = 5000(0.08)(8)$

$I = 3200$

$t = \frac{3000}{5000 \times 0.08}$

$t = 7.5$ years

↓ Paid annually

$t = 8$ years

b) $ROR = \frac{\$ \text{ earn}}{\$ \text{ invest}}$

$= \frac{3200}{5000} \times 100\%$

$= 64\%$

rate of return

The ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent.

$$ROR = \frac{earn\ /\ lost}{invested}$$

Interest

Principal

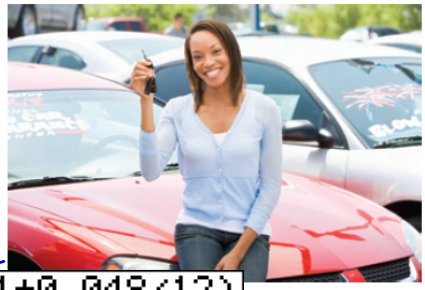
EXAMPLE 4 p. 463

Comparing interest on investments with different compounding periods

Céline wants to invest $3000 so that she can buy a new car in the next 5 years. Céline has the following investment options:

A. 4.8% compounded annually
B. 4.8% compounded semi-annually
C. 4.8% compounded monthly
D. 4.8% compounded weekly
E. 4.8% compounded daily

Popular



A

```
3000(1+0.048/1)^
5
     3792.518151
■
```

B

```
3000(1+0.048/2)^
10
     3802.951801
■
```

C

```
3000(1+0.048/12)
^60
     3811.922156
■
```

D

```
3000(1+0.048/52)
^(52*5)
     3813.325288
```

E

```
3000(1+0.048/365
)^(365*5)
     3813.687273
```

p. 465

Rule of 72

A simple formula for estimating the doubling time of an investment; 72 is divided by the annual interest rate as a percent to estimate the doubling time of an investment in years.

The Rule of 72 is most accurate when the interest is compounded annually.

EXAMPLE 5 Estimating doubling times for investments

Both Berta and Kris invested $5000 by purchasing Canada Savings Bonds. Berta's CSB earns 8%, compounded annually, while Kris's CSB earns 9%, compounded annually.

a) Estimate the doubling time for each CSB.

Time Needed to Double $

$$\text{Rule of } 72 = \frac{72}{\text{Rate}}$$

Berta (8%) vs Kris (9%)

$t = \frac{72}{8}$ | $t = \frac{72}{9}$

$t = 9$ yrs | $t = 8$ years

b) $t = ?$ for $20 000

$5000 \xrightarrow{\times 2} 10\,000 \xrightarrow{\times 2} 20\,000$

Berta 9 yrs + 9 yrs = 18 yrs

Kris 8 yrs + 8 yrs = 16 yrs

Present Value...

Principal

$ needed to invest NOW to get a fixed amount later

$$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$$

$A = P(1+\frac{r}{n})^{nt}$

$\frac{A}{(1+\frac{r}{n})^{nt}} = \frac{P(1+\frac{r}{n})^{nt}}{(1+\frac{r}{n})^{nt}}$

$\frac{A}{(1+\frac{r}{n})^{nt}} = P$

8.4 Compound Interest: Present Value

GOAL

Determine the principal or present value of an investment, given its future value and compound interest rate.

EXAMPLE 2 p. 475 — Determining the present value of an investment that is compounded quarterly

Agnes and Bill are musicians. They have researched the costs to set up a small recording studio. They estimate that $40 000 will pay for the soundproofing, recording equipment, and computer hardware and software that they need. They plan to set up the studio in 3 years and have invested money at 9.6%, compounded quarterly, to save for it.

a) How much money should they have invested?
b) How much interest will they earn over the term of their investment?

A

P = ?

a) $P = \frac{A}{(1+\frac{r}{n})^{nt}}$

$P = \frac{40000}{(1+\frac{0.096}{4})^{12}}$

40000/(1+0.096/4)^12
30092.65538

P = $30092.66

b) $I = A - P$

$I = 40000 - 30092.66$

$9907.34

c) ROR = 9907.34/30092.66*100
32.92277918

= 33%

HOMEWORK...

p. 468: **Rule of 72...**

#3 (only estimate the doubling time)

#5a & #8

Compound Interest (Future Value)

#10 & #12

p. 478: **Compound Interest (Present Value)**

#4, #6, #7, & #9