Notes - Optimization Problems.pdf

5.4 Optimization Problems I: Creating the Model

optimization problem
A problem where a quantity must be maximized or minimized following a set of guidelines or conditions.

constraint
A limiting condition of the optimization problem being modelled, represented by a linear inequality.

objective function
In an optimization problem, the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized.

feasible region
The solution region for a system of linear inequalities that is modelling an optimization problem.

Need to Know

- You can create a model for an optimization problem by following these steps:

  Step 1. Identify the quantity that must be optimized. Look for key words, such as *maximize* or *minimize*, *largest* or *smallest*, and *greatest* or *least*.

  Step 2. Define the variables that affect the quantity to be optimized. Identify any restrictions on these variables.

  Step 3. Write a system of linear inequalities to describe all the constraints of the problem. Graph the system.

  Step 4. Write an objective function to represent the relationship between the variables and the quantity to be optimized.

HOMEWORK???

x -> # of cans of pop
y -> # of cans of juice
x ∈ W y ∈ W

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3. A vending machine sells juice and pop.

- The machine holds, at most, 240 cans of drinks.
- Sales from the vending machine show that at least 2 cans of juice are sold for each can of pop.
- Each can of juice sells for $1.00, and each can of pop sells for $1.25.

Create a model that could be used to determine the maximum revenue from the vending machine.

← objective $R = 1.25x + 1.00y$

-> Graph

$x + y \le 240$ $y \ge 2x$

$x + y = 240$

x-int: $x + 0 = 240$ $(240, 0)$

y-int: $0 + y = 240$ $(0, 240)$

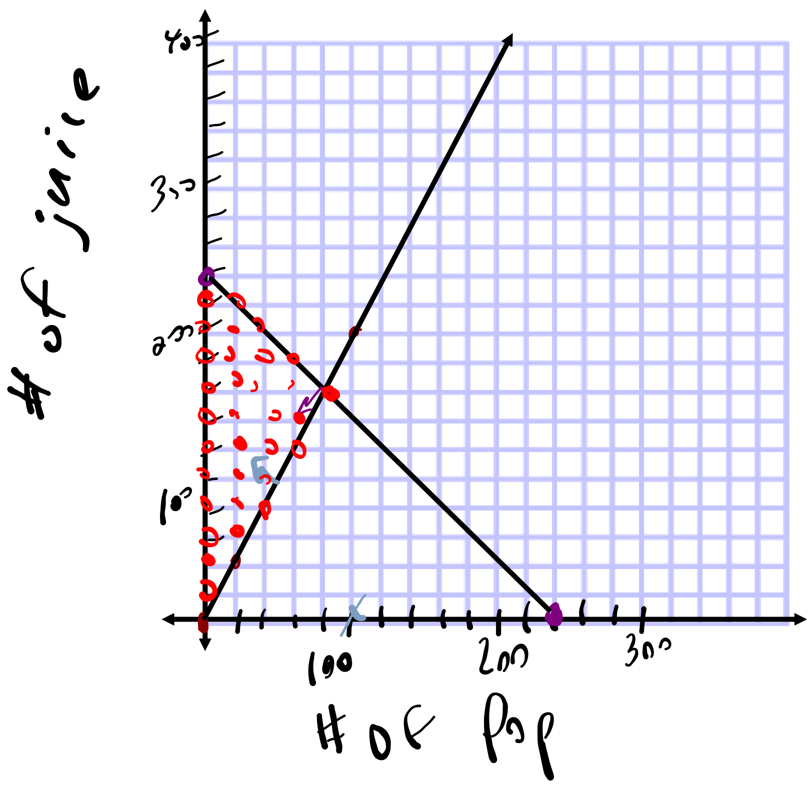
$y = 2x$

x	y
20	40
100	200

Test (100, 0)

$y \ge 2x$

LS	≥ RS
0	2(100)
	200
	NO



EXAMPLE 2 Creating a model for a maximization problem with positive real-number variables

A refinery produces oil and gas.

- At least 2 L of gasoline is produced for each litre of heating oil.
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- Gasoline is projected to sell for \$1.10 per litre. Heating oil is projected to sell for \$1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

Umberto's Solution

Let h represent the number of litres of heating oil.
Let g represent the number of litres of gasoline.

Restrictions:
$h \geq 0$ and $g \geq 0$, where $h \in \mathbf{R}$ and $g \in \mathbf{R}$

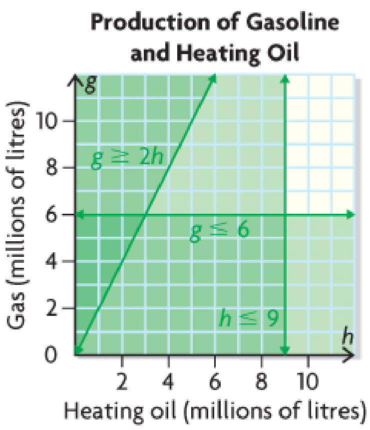
> I knew that this is an optimization problem because the total revenue has to be maximized.
>
> The two variables in the problem are the volume of heating oil and the volume of gasoline, both in litres. Litres are measured using positive real numbers.

Constraints:
Ratio of gasoline produced to oil produced:
$g \geq 2h$
Amount of gasoline that can be produced:
$g \leq 6\,000\,000$
Amount of oil that can be produced:
$h \leq 9\,000\,000$

> I created inequalities to represent the five constraints of the problem.
>
> I treated the restrictions on each variable as a constraint.

Let R represent total revenue from sales of gasoline and heating oil.
Objective function to maximize:
$R = 1.10g + 1.75h$

> I wrote an objective function to represent the relationship between the two variables (volume of heating oil and volume of gasoline) and the quantity to be maximized (total revenue).



> I graphed the system of inequalities in the 1st quadrant because of the restrictions on the variables. The feasible region is a right triangle and includes all points on its boundaries.
>
> I think I can use the objective function to determine which point in the feasible region represents the combination of oil and gas that will result in the maximum revenue, but I am not sure how yet.

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The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

x → # of liters of oil $x \in R$

y → # of liters of gas $y \in R$

$x \leq 9\,000\,000$ $y \leq 6\,000\,000$

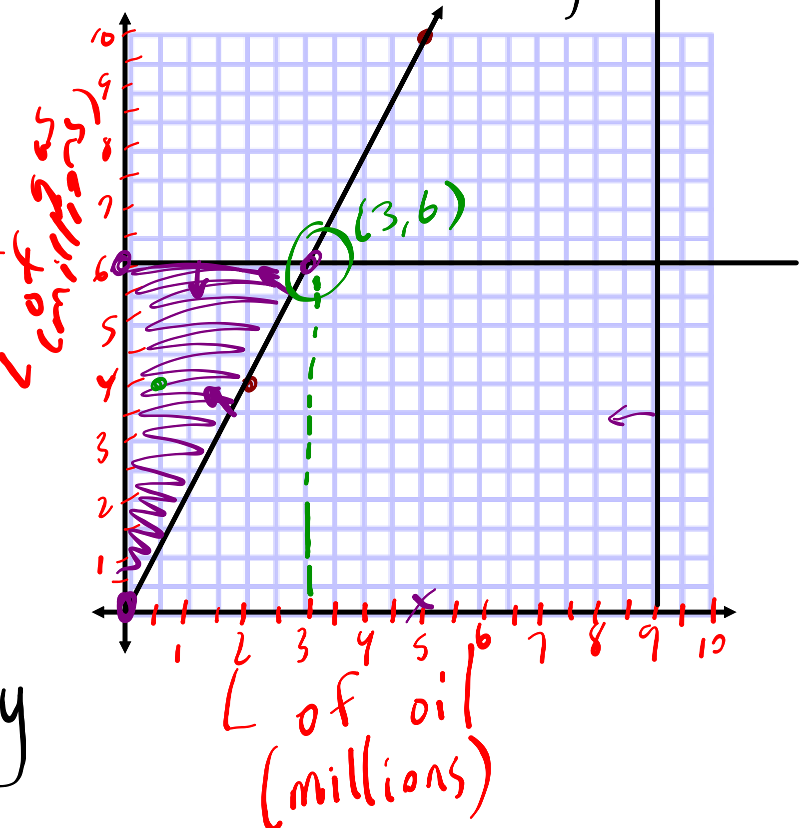
$y \geq 2x$

LS ≥ RS

Test	(5,0)
0	2(5)
No	10

$y = 2x$

x	y
~~10~~	~~20~~
5	10
2	4



Objective

$R = 1.75x + 1.10y$

```
1.75(500000)+1.1
(4000000)
         5275000
1.75(3000000)+1.
1(6000000)
        11850000
```

EXAMPLE of an OPTIMIZATION Problem...

Mick and Keith make MP3 covers to sell, using beads and stickers.

- ✓ At most, 45 covers with stickers and 55 bead covers can be made per day.
- ✓ Mick and Keith can make 45 or more covers, in total, each day.
- It costs \$0.75 to make a cover with stickers, \$1.00 to make one with beads.

Let x represent the number of covers with stickers and let y represent the number of bead covers.

Let C represent the cost of making the covers.

RESTRICTIONS: $x \in W$ $y \in W$

CONSTRAINTS: $x \le 45$ $y \le 55$ $x + y \ge 45$

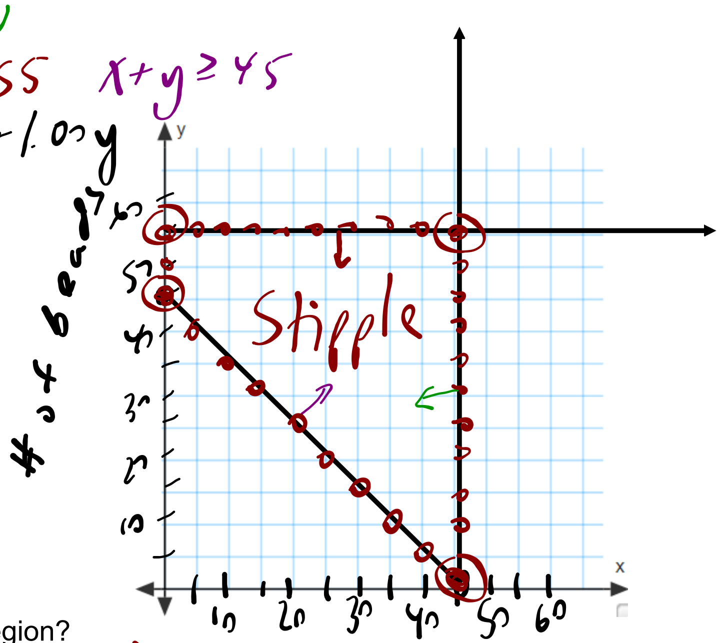
OBJECTIVE FUNCTION: $C = 0.75x + 1.00y$

a) Graph the solution set.

$x + y = 45$

x-int $(45, 0)$

y-int $(0, 45)$



b) What are the vertices of the feasible region?

① $(0, 45)$ ② $(0, 55)$ ③ $(45, 0)$ ④ $(45, 55)$

c) Which point would result in the maximum value of the objective function?

d) Which point would result in the minimum value of the objective function?

	$C = 0.75x + 1.00y$
$(0, 45)$	45
$(0, 55)$	55
Min $(45, 0)$	33.75
Max $(45, 55)$	0.75(45)+1(55) 88.75

HOMEWORK...

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Attachments

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