5.1 Graphing Linear Inequalities in Two Variables

GOAL

Solve problems by modelling linear inequalities in two variables.

EXPLORE...

x y

• For which inequalities is (3, 1) a possible solution? How do you know?

a) $13 - 3x > 4y$
b) $2y - 5 \leq x$
c) $y + x < 10$
d) $y \geq 9$

VERIFY

LS	RS

Let's VERIFY...

a) LS > RS

LS	RS
13-3(3)	4(1)
4	4

Not a solution

b) LS ≤ RS

LS	RS
2(1) -5	3
-3	yes

c) LS < RS

LS	RS
1 + 3	10
4	yes

d) LS ≥ RS

LS	RS
1	9

No

EXAMPLE 2 Graphing linear inequalities with vertical or horizontal boundaries

Graph the solution set for each linear inequality on a Cartesian plane.

a) $\{(x, y) \mid x - 2 > 0, x \in \mathrm{R}, y \in \mathrm{R}\}$

b) $\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

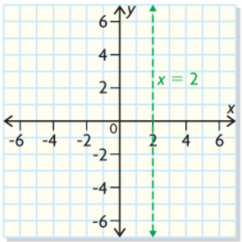
Wynn's Solution

a) $x - 2 > 0$
$x > 2$

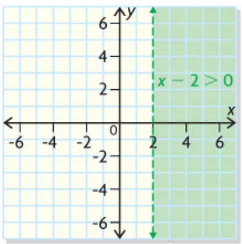
I isolated x so I could graph the inequality.

The variables represent numbers from the set of real numbers.
$x \in \mathrm{R}$ and $y \in \mathrm{R}$

The domain and range are stated as the set of real numbers. The solution set is continuous, so the solution region and its boundary will be green in my graph.



I drew the boundary of the linear inequality as a dashed green line because I knew that the linear inequality (>) does not include the possibility of x being equal to 2.



I needed to decide which half plane to shade. For x to be greater than 2, I knew that any point to the right of the boundary would work.

The solution region includes all the points in the shaded area because the solution set is continuous. The solution region does not include points on the boundary.

Communication Tip

If the solution set to a linear inequality is continuous and the sign does not include equality (< or >), a dashed green line is used for the boundary and the solution region is shaded green.

b) $-3y + 6 \geq -6 + y$
$-4y \geq -12$
$\frac{-4y}{-4} \leq \frac{-12}{-4}$
$y \leq 3$

Since the linear inequality has only one variable, y, I isolated the y.

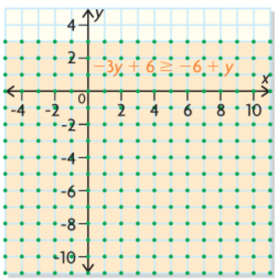
As I rearranged the linear inequality, I divided both sides by −4. That's why I reversed the sign from ≥ to ≤.

The variables represent integers.
$x \in \mathrm{I}$ and $y \in \mathrm{I}$

The domain and range are stated as being in the set of integers. I knew this means that the solution set is **discrete**.

discrete

Consisting of separate or distinct parts. Discrete variables represent things that can be counted, such as people in a room.



$\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

I knew that points with integer coordinates below the line
$y = 3$
were solutions, so I shaded the half plane below it orange.

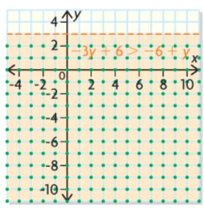
I knew the linear inequality includes 3, so points on the boundary with integer coordinates are also solutions to the linear inequality.

I stippled the boundary and the orange half plane with green points to show that the solution set is discrete.

The solution region includes only the points with integer coordinates in the shaded region and along the boundary.

Communication Tip

If the solution set to a linear inequality is discrete, the solution region is shaded orange and stippled with green points. If the sign includes equality (≥ or ≤), the boundary is also stippled. An example of this is shown to the left. If equality is not possible (< or >), the boundary is a dashed orange line. An example of this is shown below.



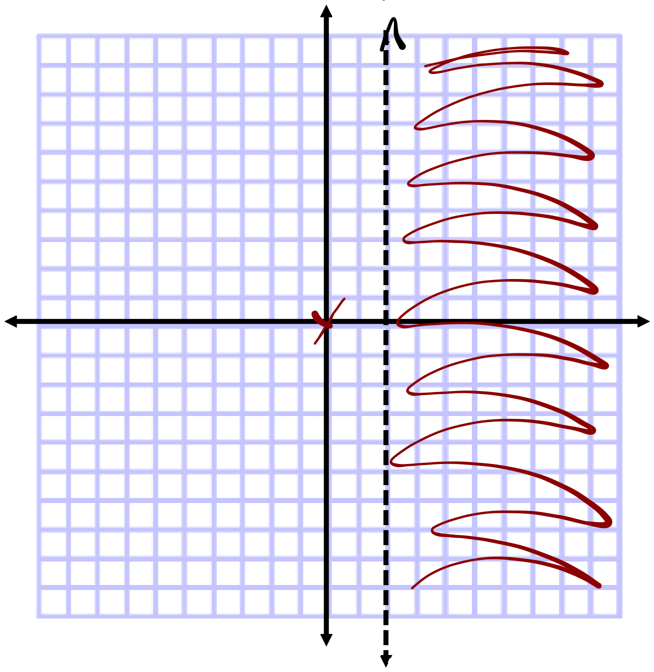
a) $\{(x, y) \mid x - 2 > 0, x \in \mathrm{R}, y \in \mathrm{R}\}$ ← TEST (0,0)

LS > RS

LS	RS
0 - 2	0
-2	NO

$x - 2 = 0$

$x = 2$ ← vertical



b) $\{(x, y) \mid -3y + 6 \geq -6 + y, x \in I, y \in I\}$

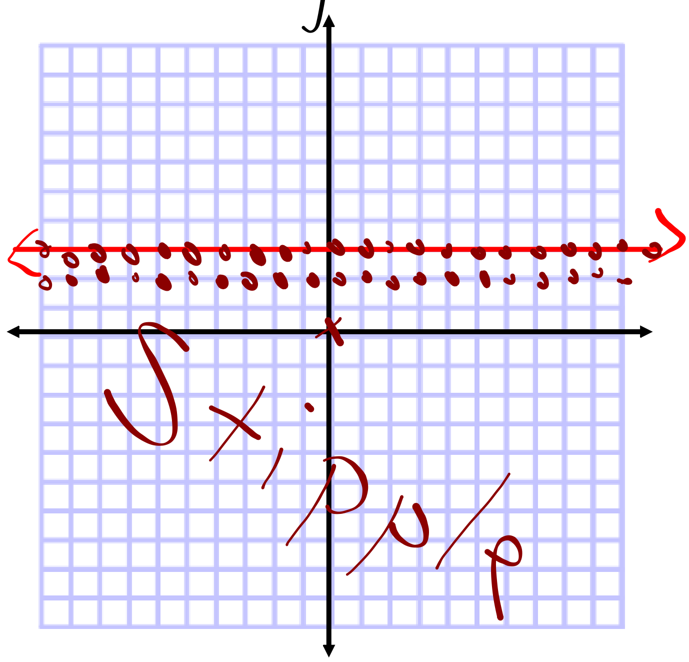
LS	$\geq$ RS
$-3(0) + 6$	$-6 + 0$
6	-6
	yes

$$-3y + 6 = -6 + y$$

$$-3y - y = -6 - 6$$

$$\frac{-4y}{-4} = \frac{-12}{-4}$$

$$y = 3$$ ← horizontal



HOMEWORK...

p. 221: #1, #2, #4 and #6