

HOMEWORK???

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2. A fast-food concession stand sells hotdogs and hamburgers.
- ✓ Daily sales can be as high as 300 hamburgers and hot dogs combined.
 - ✓ The stand has room to stock no more than 200 hot dogs and no more than 150 hamburgers.
 - Hot dogs are sold for \$3.25, and hamburgers are sold for \$4.75.
- Create a model that could be used to determine the combination of hamburgers and hot dogs that will result in maximum sales.

Objective

$$S = 3.25x + 4.75y$$

↳ Graph

$x \rightarrow$ # of hotdogs sold
 $y \rightarrow$ # of hamburgers sold
 $x \in \mathbb{W}$ $y \in \mathbb{W}$

$$x + y \leq 300$$

$$x \leq 200$$

$$y \leq 150$$

$$x + y = 300$$

x-int $x + 0 = 300$
 $(300, 0)$

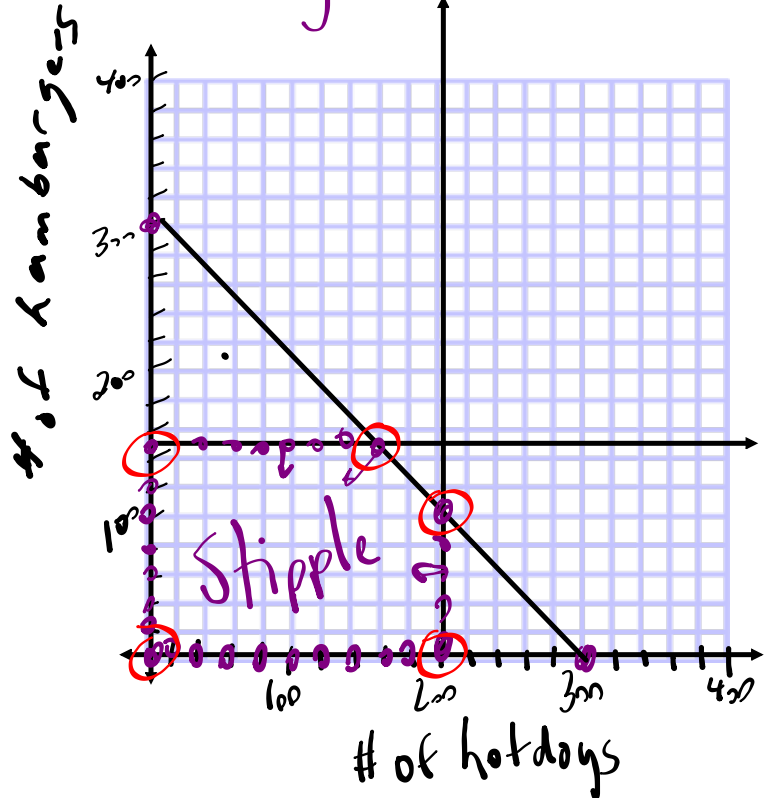
y-int $0 + y = 300$
 $(0, 300)$

~~$(200, 0)$~~

~~$(200, 100)$~~

* $(150, 150)$

~~$(0, 150)$~~



EXAMPLE 2 Creating a model for a maximization problem with positive real-number variables

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A refinery produces oil and gas.

- At least 2 L of gasoline is produced for each litre of heating oil.
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- Gasoline is projected to sell for \$1.10 per litre. Heating oil is projected to sell for \$1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.



Umberto's Solution



Let h represent the number of litres of heating oil.
Let g represent the number of litres of gasoline.

Restrictions:

$$h \geq 0 \text{ and } g \geq 0, \text{ where } h \in \mathbb{R} \text{ and } g \in \mathbb{R}$$

Constraints:

Ratio of gasoline produced to oil produced:

$$g \geq 2h$$

Amount of gasoline that can be produced:

$$g \leq 6\,000\,000$$

Amount of oil that can be produced:

$$h \leq 9\,000\,000$$

Let R represent total revenue from sales of gasoline and heating oil.

Objective function to maximize:

$$R = 1.10g + 1.75h$$

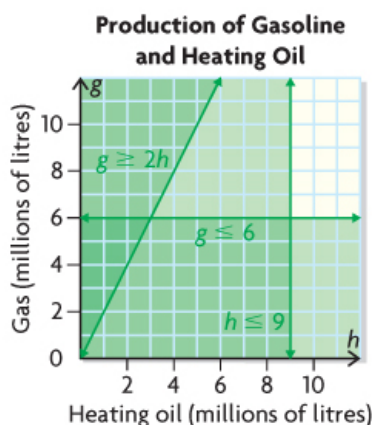
I knew that this is an optimization problem because the total revenue has to be maximized.

The two variables in the problem are the volume of heating oil and the volume of gasoline, both in litres. Litres are measured using positive real numbers.

I created inequalities to represent the five constraints of the problem.

I treated the restrictions on each variable as a constraint.

I wrote an objective function to represent the relationship between the two variables (volume of heating oil and volume of gasoline) and the quantity to be maximized (total revenue).



I graphed the system of inequalities in the 1st quadrant because of the restrictions on the variables. The feasible region is a right triangle and includes all points on its boundaries.

I think I can use the objective function to determine which point in the feasible region represents the combination of oil and gas that will result in the maximum revenue, but I am not sure how yet.

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$x \rightarrow$ # of liters of oil

$y \rightarrow$ # of liters of gas

$x \in \mathbb{R}$

$y \in \mathbb{R}$

Test
(2, 0)
 $0 \geq 25$

$y \geq 2x$

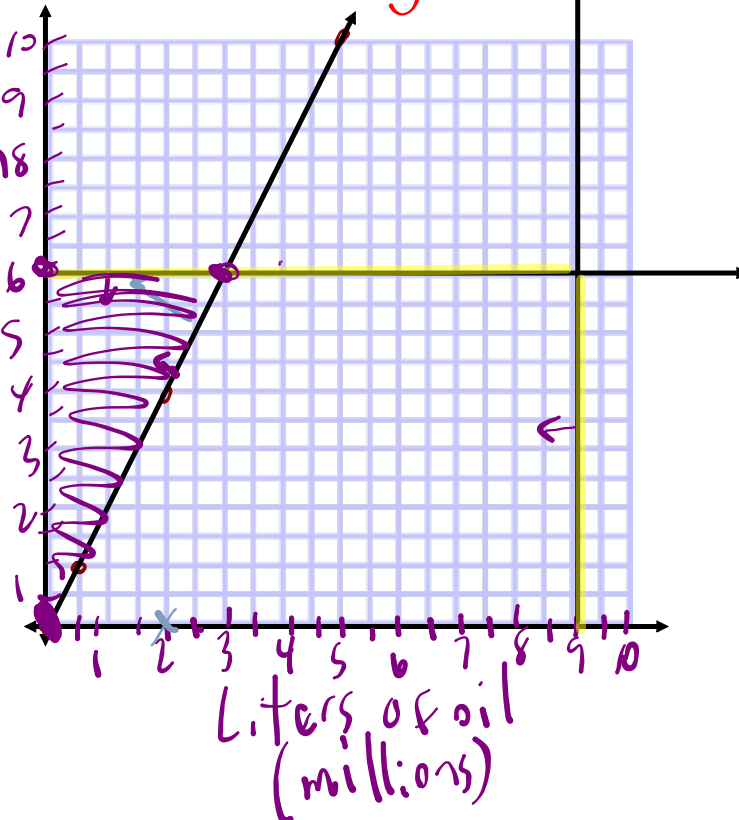
$x \leq 9000000$

$y \leq 6000000$

$0 \geq 25$
 $0 \geq 25$
 $0 \geq 25$

x	y
2	4
5	10

↳ of gas
↳ of oil



EXAMPLE of an OPTIMIZATION Problem...

Mick and Keith make MP3 covers to sell, using beads and stickers.



- At most, 45 covers with stickers and 55 bead covers can be made per day.
- Mick and Keith can make 45 or more covers, in total, each day.
- It costs \$0.75 to make a cover with stickers, \$1.00 to make one with beads.

Let x represent the number of covers with stickers and let y represent the number of bead covers.

Let C represent the cost of making the covers.

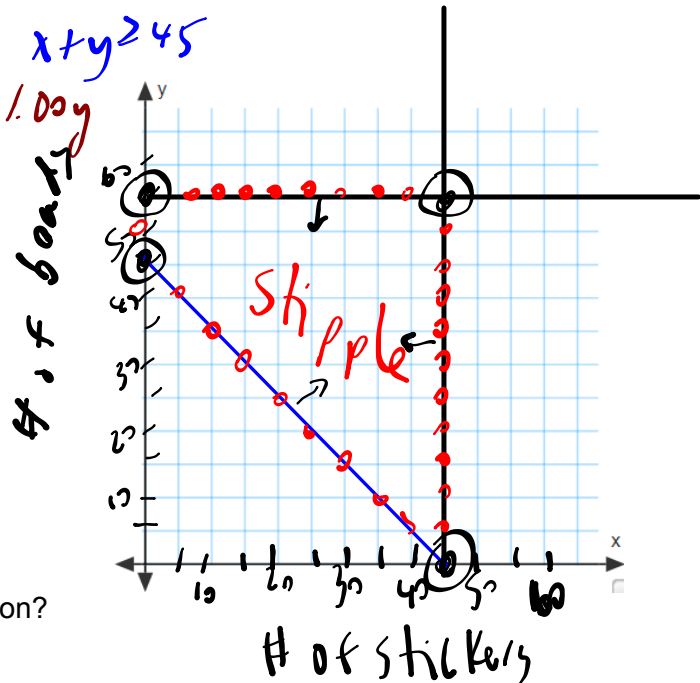
RESTRICTIONS: $x \leq 45$ $y \leq 55$

CONSTRAINTS: $x \leq 45$ $y \leq 55$ $x + y \geq 45$

OBJECTIVE FUNCTION: $C = 0.75x + 1.00y$

a) Graph the solution set.

$x + y = 45$
 $x \text{ int } - (45, 0)$
 $y \text{ int } (0, 45)$



b) What are the vertices of the feasible region?

- ① (0, 45) ③ (45, 0)
 ② (0, 55) ④ (45, 55)

c) Which point would result in the maximum value of the objective function?

d) Which point would result in the minimum value of the objective function?

	$C = 0.75x + 1.00y$
min	(0, 45) 45
	(0, 55) 55
	(45, 0) 33.75
Max	(45, 55) $.75 \cdot 45 + 1 \cdot 55 = 88.75$

HOMEWORK...

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