Chapter 7:

Similarity and Transformations

Overview

To Find Scale Factor = $\frac{\text{Scale Length}}{\text{Original Length}}$

Given scale factor As a decimal or fraction

Find the scale dimensions
original x scale factor

Find the original dimensions
scale ÷ scale factor

Scale Diagrams

For an enlargement or reduction, the scale factor is: $\frac{\text{Length on scale diagram}}{\text{Length on original diagram}}$

An enlargement has a scale factor > 1. A reduction has a scale factor < 1.

Similar Triangles

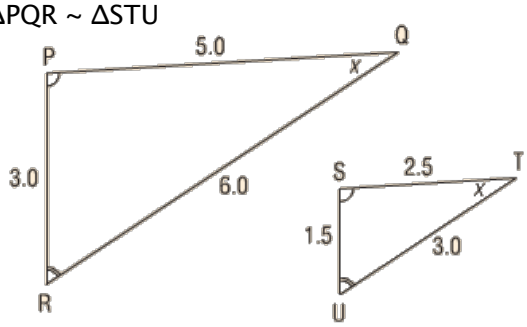
When we check whether two triangles are similar:

- their corresponding angles must be equal:
  $\angle P = \angle S$ and $\angle Q = \angle T$ and $\angle R = \angle U$
  or
- their corresponding sides must be proportional:
  $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$

Any of the ratios $\frac{PQ}{ST}$, $\frac{QR}{TU}$, and $\frac{PR}{SU}$ is the scale factor.

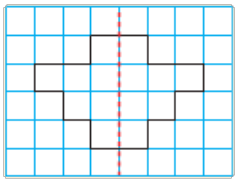
Similarity Statements

$\Delta PQR \sim \Delta STU$



Line Symmetry

A shape has line symmetry when a line divides the shape into two congruent parts so that one part is the image of the other part after a reflection in the line of symmetry.



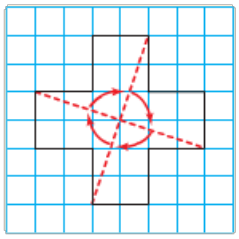
HORIZONTAL ——— y =

VERTICAL | x=

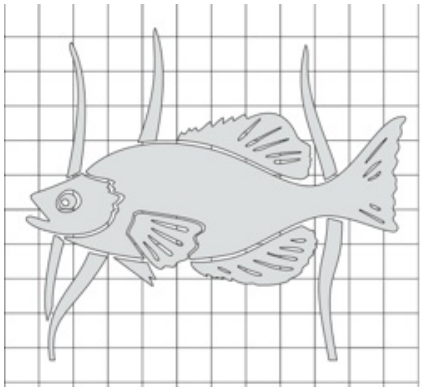
Rotational Symmetry

A shape has rotational symmetry when it coincides with itself after a rotation of less than 360° about its centre. The number of times the shape coincides with itself is the order of rotation.

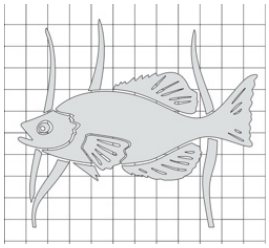
The angle of rotation symmetry = $\frac{360°}{\text{the order of rotation}}$

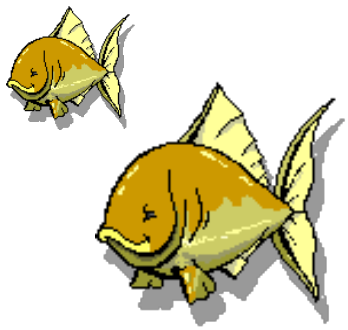


Counterclockwise
Clockwise
Point of Rotation
Degree



Scale Diagrams:





A diagram that is an enlargement or reduction of another diagram.

The measurements in each diagram are compared.

Scale Factor = $\frac{\textit{Length of Scale Diagram}}{\textit{Length of Original Diagram}}$

The scale factor can be written as a fraction or decimal.

If the scale factor is less than one, the diagram is a reduction, larger than one indicates the diagram is an enlargement.

This photo of longhouses has dimensions 9 cm by 6 cm.

The photo is to be enlarged by a scale factor of $\frac{7}{2}$.

Calculate the dimensions of the enlargement.

original

6 cm

9cm

scale

Sometimes you are only given the scale diagram....

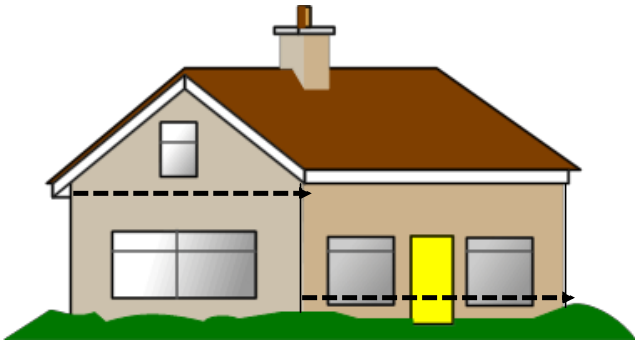
A scale may be given as a ratio.

The scale on this scale diagram of a house is 1:150.

This means that 1cm on the diagram represents 150 cm or 1.5m on the house.

In other words... the scale factor is $\frac{1}{150}$

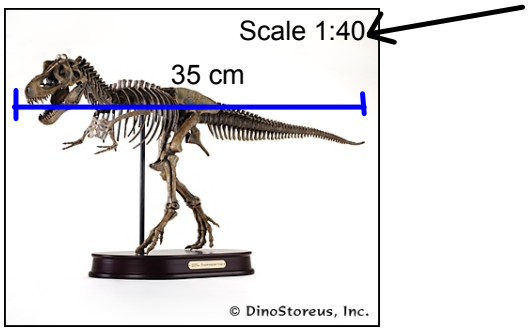
How wide is the actual house??



Scale Diagrams:

Day 2

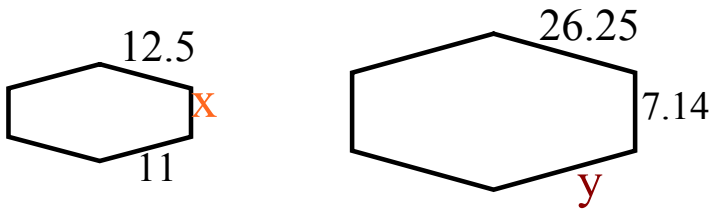
2) The following is a scale diagram of "Sue" the T-Rex. Using the ratio determine the true length of Sue



Warm Up

Solution

1) Find the length of the missing sides of the similar polygons



full ratio

$$\frac{12.5}{26.25} = \frac{x}{7.14} = \frac{11}{y}$$

$$\frac{12.5}{26.25} = \frac{x}{7.14}$$

cross multiply

$$26.25\,x = (12.5)(7.14)$$

$$26.25\,x = 89.25$$

solve for x

$$\frac{26.5\,x}{26.25} = \frac{89.25}{26.25}$$

$$x = 3.4$$

$$\frac{12.5}{26.25} = \frac{11}{y}$$

$$12.5\,y = (11)(26.25)$$

$$12.5\,y = 288.75$$

solve for y

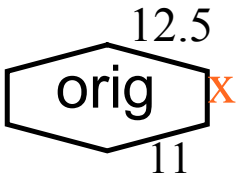
$$\frac{12.5\,y}{12.5} = \frac{288.75}{12.5}$$

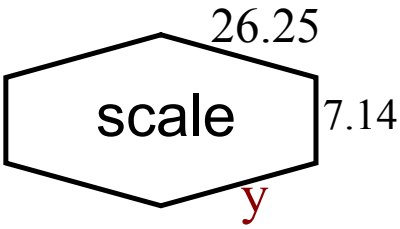
$$y = 23.1$$

Warm Up

1) Find the length of the missing sides of the similar polygons

Show work

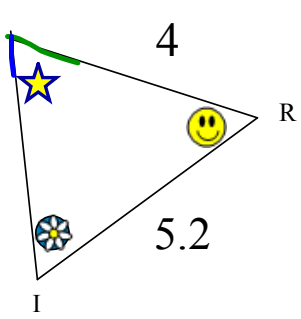




Are these triangles similar?

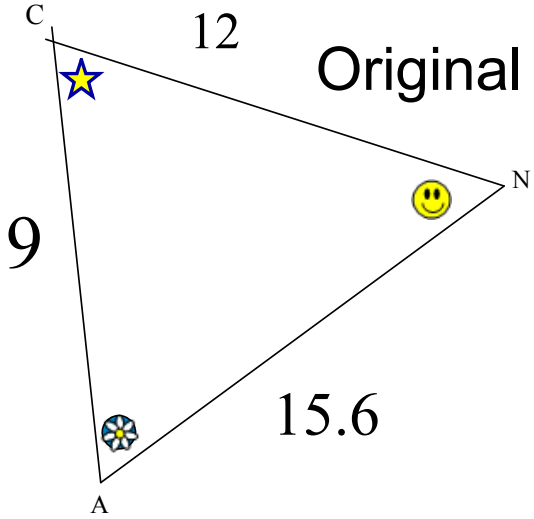
Triangles are just polygons

Scale



3

Original



Step 1) Match up Angles

TRI CNA

erase

Let's Compare sides

Step 2) Set up ratios

$$\frac{TR}{CN} = \frac{RI}{NA} = \frac{TI}{CA}$$

$$\frac{4}{12} = \frac{5.2}{15.6} = \frac{3}{9}$$

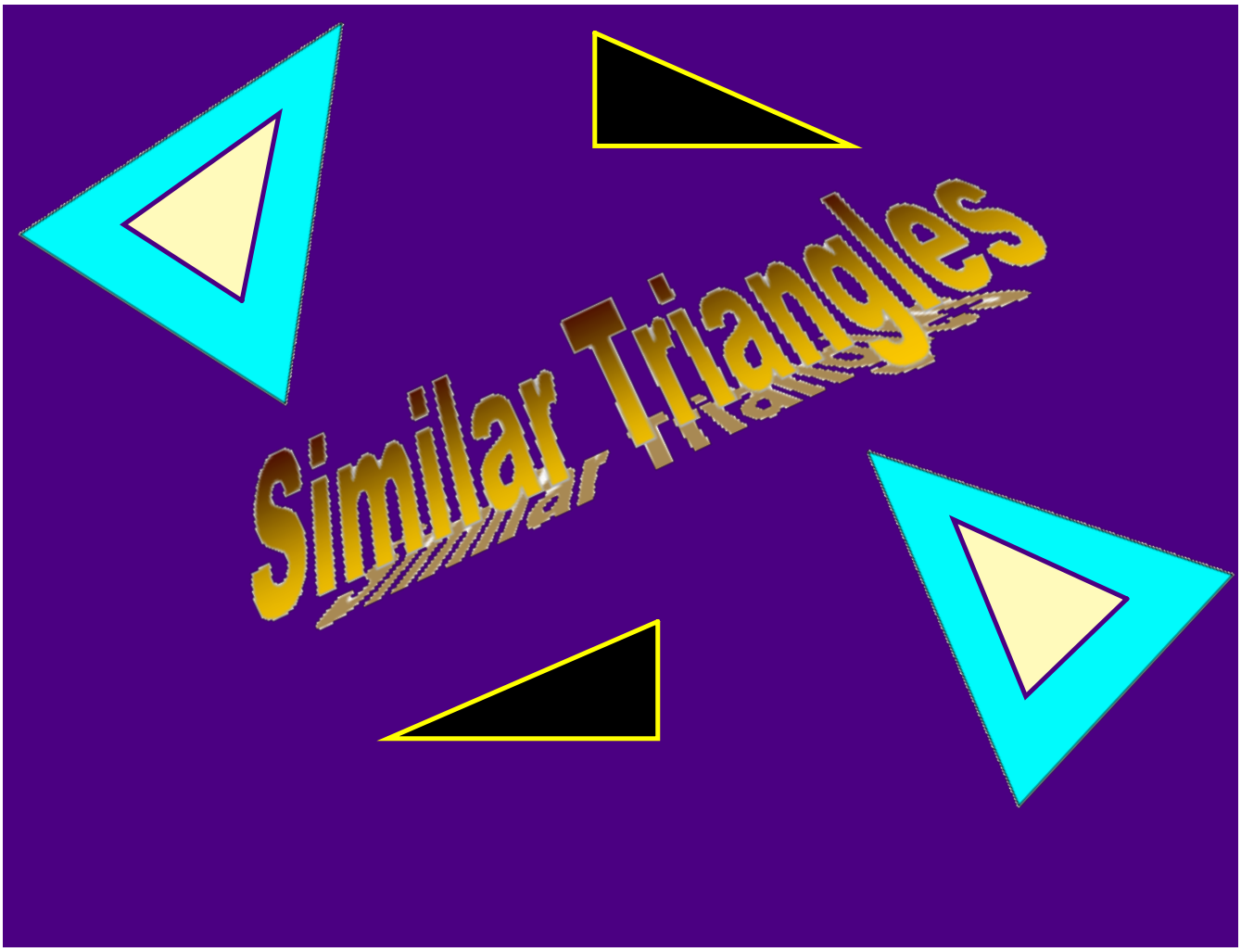
$$0.3\overline{3} = 0.3\overline{3} = 0.3\overline{3}$$

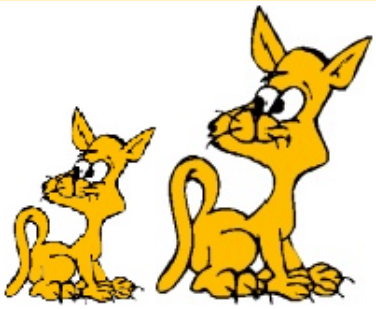
Since corresponding sides are proportionate and angles are equal then

$$\triangle TRI \sim \triangle CNA$$

therefore

similar


Similar Triangles



The cat on the right is an enlargement of the cat on the left. They are exactly the same shape, but they are NOT the same size.

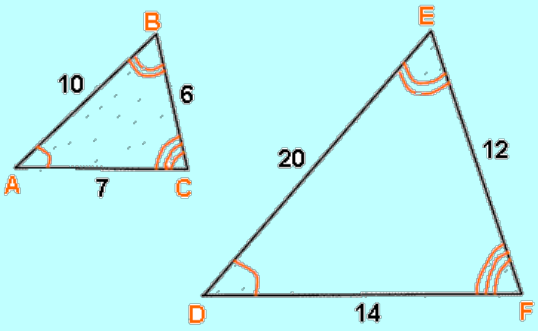
These cats are similar figures.

Objects, such as these two cats, that have the same shape, but do not have the same size, are said to be "similar".

The mathematical symbol used to denote similar is ~ .

Similar Symbol

~



Facts about similar triangles:

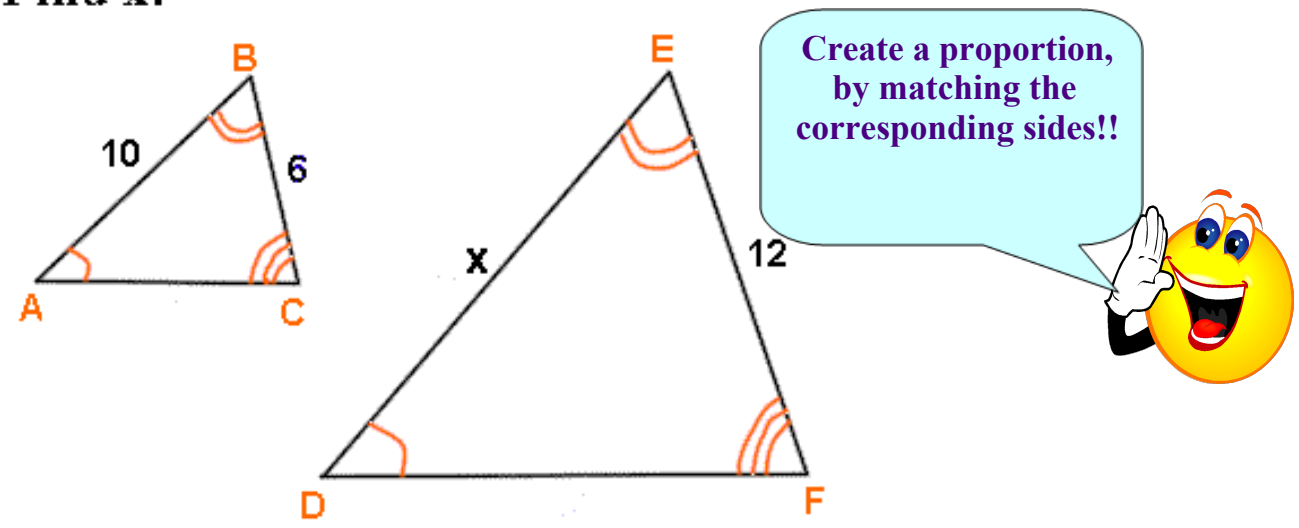
$\measuredangle A = \measuredangle D$	$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$
$\measuredangle B = \measuredangle E$	
$\measuredangle C = \measuredangle F$	

Angles equal and sides are proportionate

$\therefore \quad \Delta ABC \sim \Delta DEF$

WHAT YOU HAVE TO INCLUDE ON A TEST

Find x:



Write the Similarity Statement:

Write the proper ratios:

Fill in the ratios:

Solve:

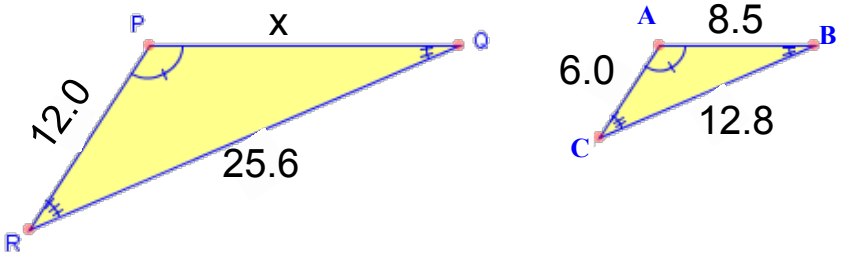
Try This !!

Solve for x .

2 ratios needed

You only need a full ratio and a ratio with the missing side

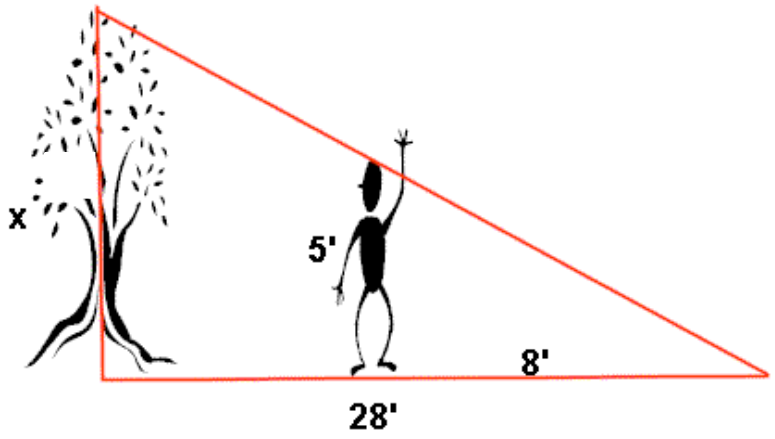
a)



Similarity statement

$\frac{\quad}{\quad} = \frac{\quad}{\quad}$

7.



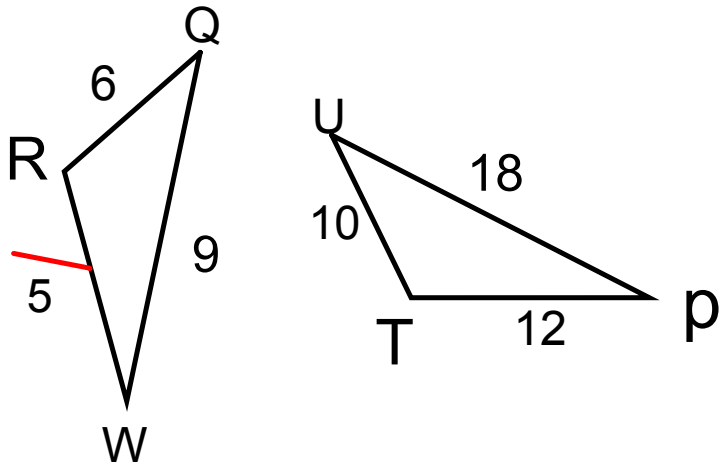
At a certain time of the day, the shadow of a 5' boy is 8' long. The shadow of a tree at this same time is 28' long. How tall is the tree?

Explanation

Choose:

- 8.5'
- 16'
- 17.5'
- 20'

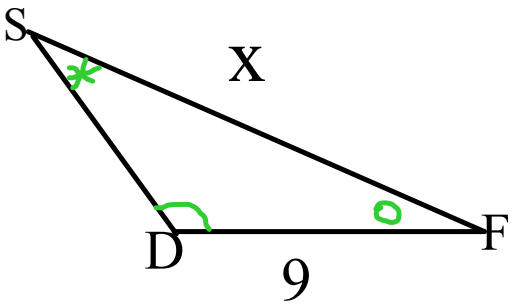
Show your work

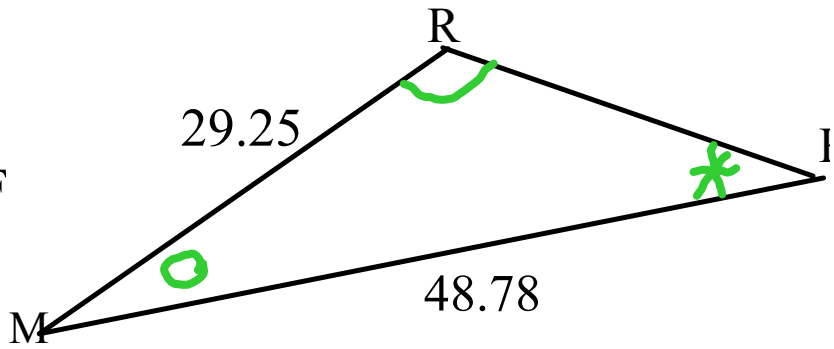

Q
6
R
9
5
W
U
18
10
T
12
p

Similar Triangles

Day 2

i)Determine if the triangles are similar
ii) Write the Ratios
iii) Fill in ratios
iv) solve for "x"





Similar Triangles

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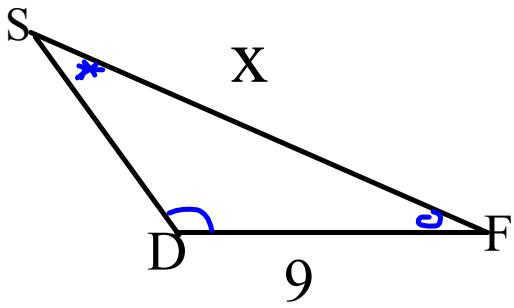
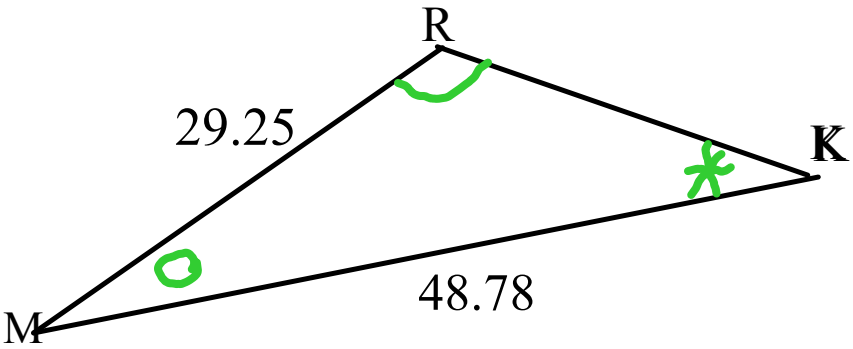
i)Determine if the triangles are similar



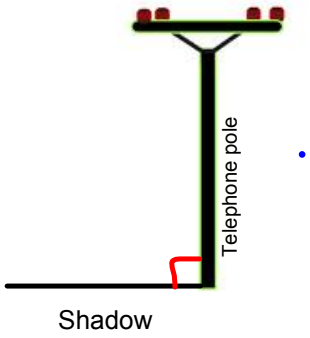
ii) Write the Ratios

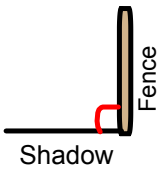
iii) Fill in ratios



iv) solve for "x"

A telephone pole that is 62 ft tall cast a shadow that is 40 ft long. Find the height of a fence pole that cast a 4 ft shadow.



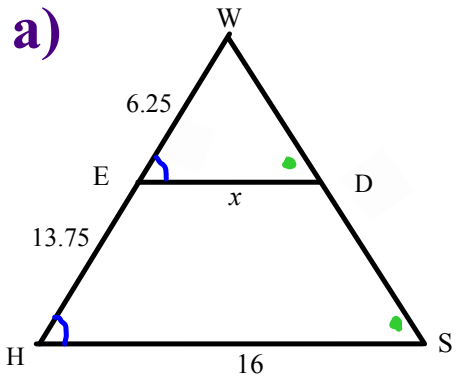


Try This !!

Solve for x .

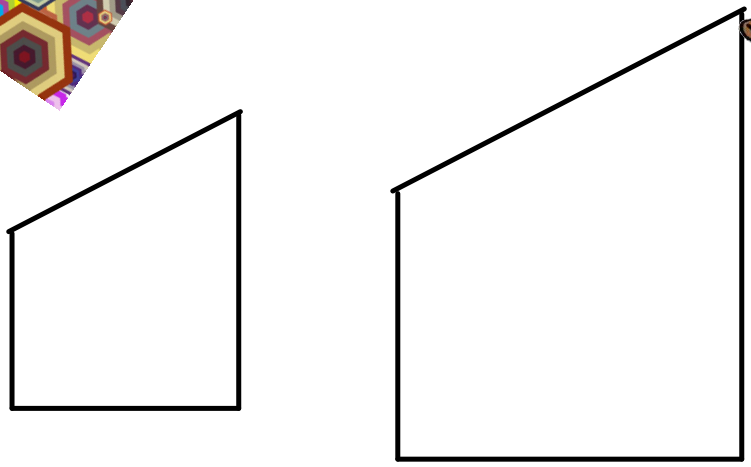
Remember to include a similarity statement

a)



SECTION 7.3

Similar Polygons



Polygons are 2-dimensional shapes. They are made of straight lines, and the shape is "closed" (all the lines connect up).

Polygon
(straight sides)

Not a Polygon
(has a curve)

Not a Polygon
(open, not closed)

Similar Polygons are enlargements or reductions of each other
: Same shape, but not necessarily the same size

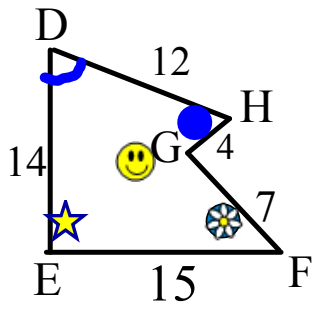
Corresponding: similar in position or purpose
: the same size; reduced or enlarged
- between same scaled sides

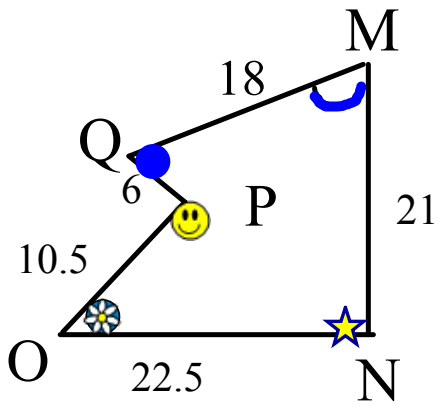
Properties of Similar Polygons
Their corresponding angles are equal Their corresponding sides are proportional

BOTH MUST BE TRUE

Symbol for similar is $\sim$

Are the following Similar Polygons?





Step1) Match up the Angles

Step 2) Match up sides and compare their ratio

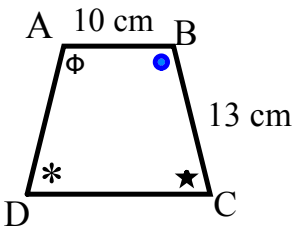
Big over Small

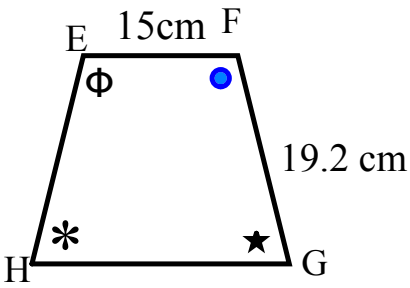
But doesn't matter just ratio must be the same in order to be similar

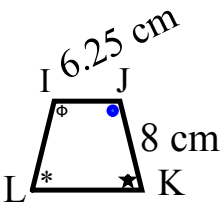
Put in the Values

Identifying Similar Polygons

Which two polygons are similar?







Compare Polygon ABCD and EFGH

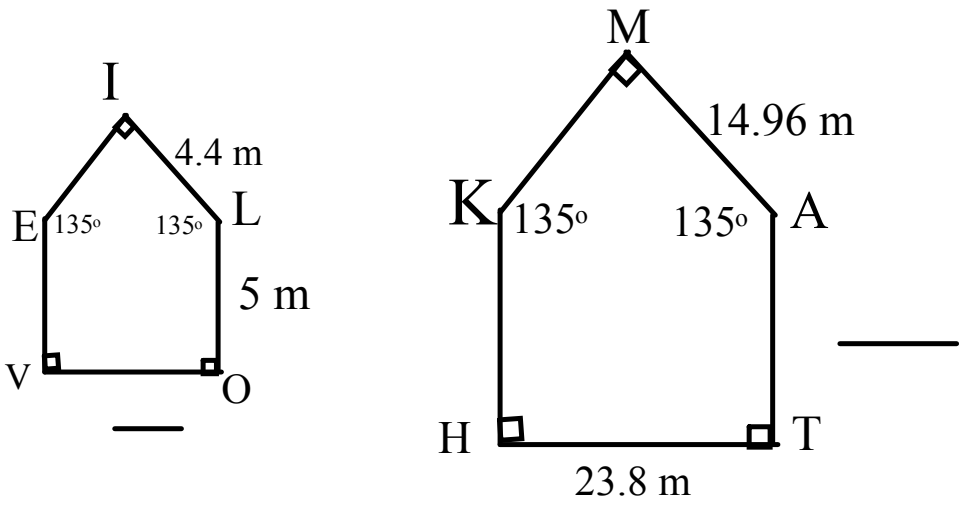
Compare Polygon EFGH and IJKL

Compare Polygon ABCD and IJKL

Solving Problems Using the Properties of Similar Polygons

These two polygons are similar.
a) Calculate the length of VO.
b) Calculate the length of AT

Use ratios



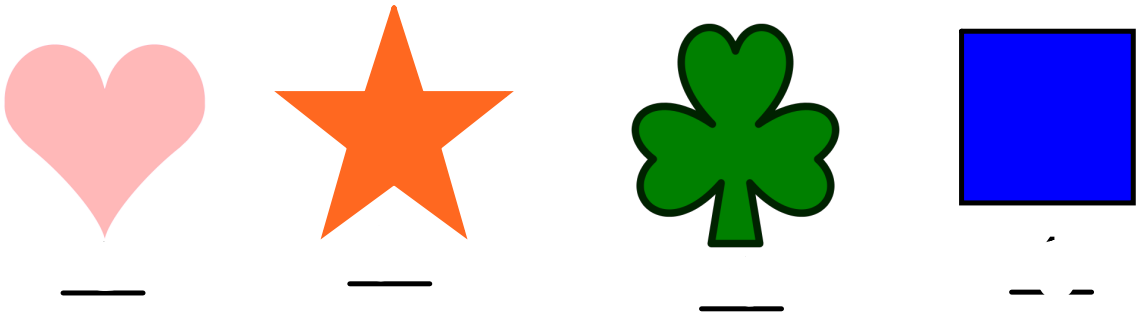
Set up 2 ratios of corresponding sides: $\frac{\text{figure 1 side}}{\text{figure 2 coressponding side}}$

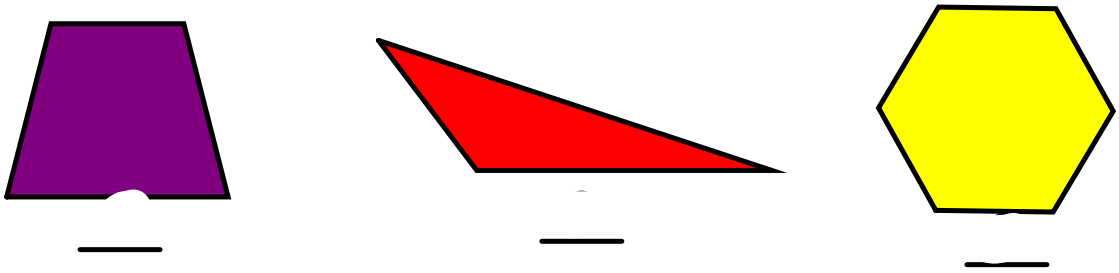
Then set them equal and cross multiply

SECTION 7.5

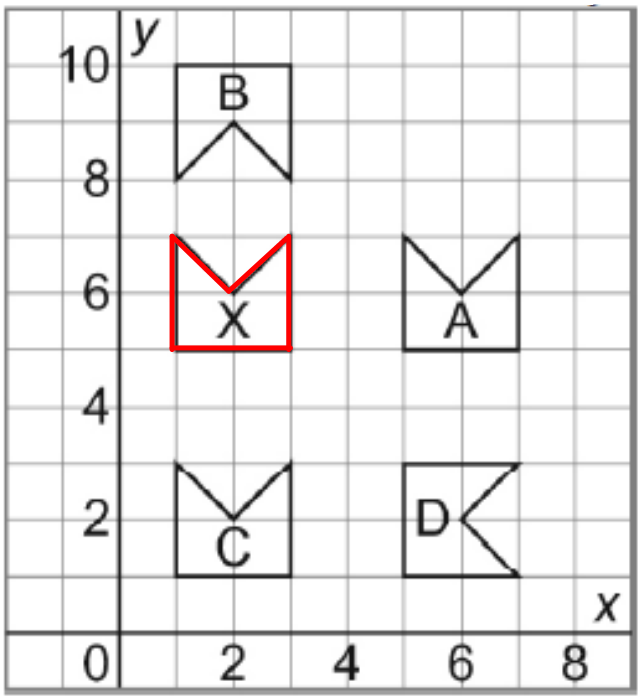
REFLECTIONS AND LINE SYMMETRY

How many lines of symmetry are in the following figures?





Identify the shapes that are related to the shape X by a line of **reflection**. Describe the symmetry in each case.



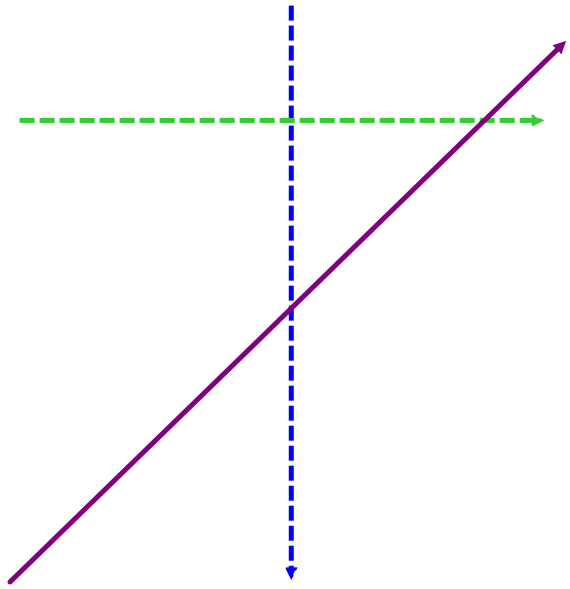
A:

B:

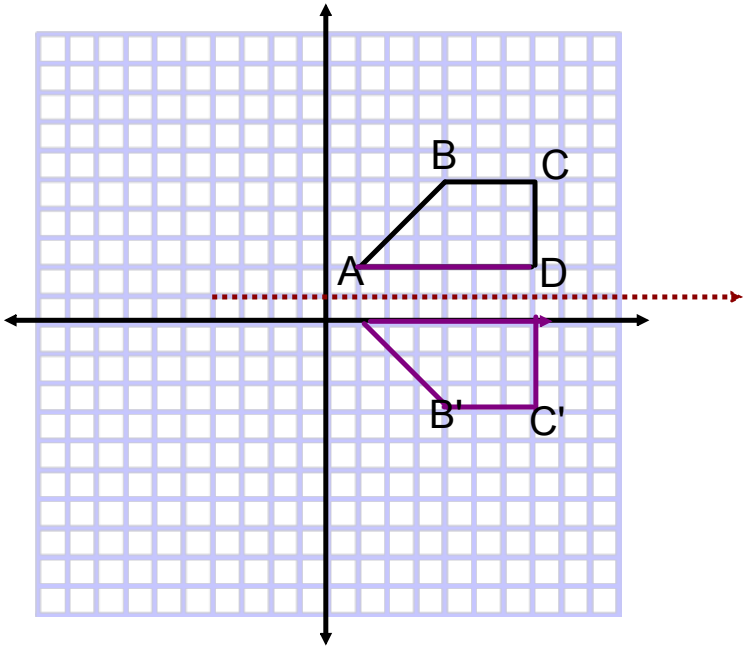
C:

D:

Oblique just means a slanted line



Draw a reflection in the horizontal line through 1 on the y-axis.



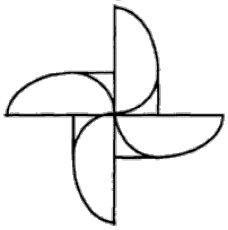
b) Write the coordinates of the shape formed.

A( ) B( ) C( )

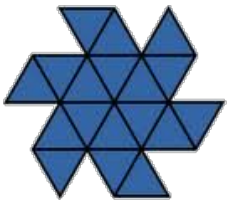
A'( ) B'( ) C'( )

c) Describe the new shape and its symmetry.

Section 7.6

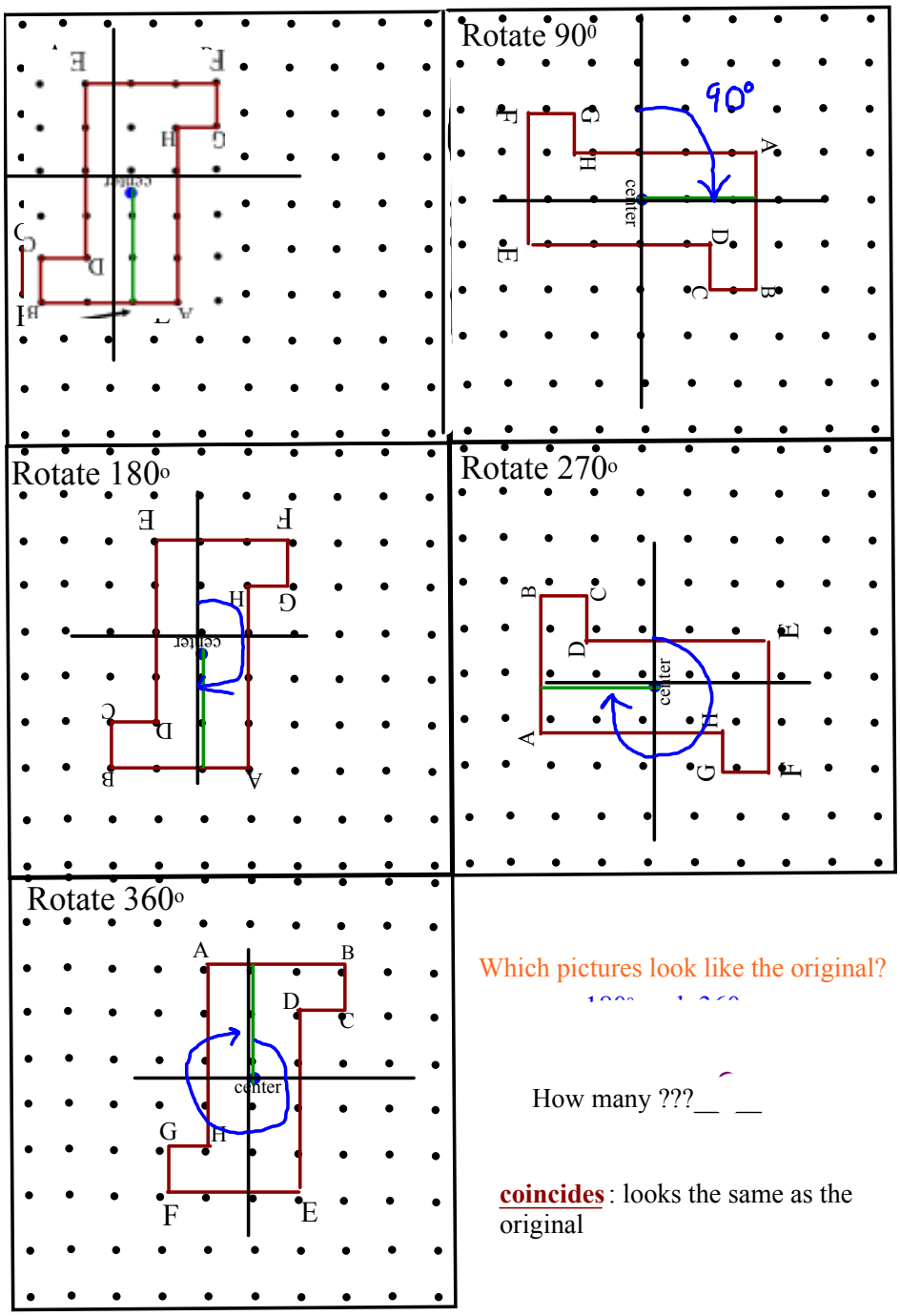


Rotations & Rotational Symmetry



Lets rotate this object about its center

On your copy draw the rotated figure



Which pictures look like the original?

How many ???__ __

coincides: looks the same as the original

LOOK AT THE NEXT SLIDE THEN COME BACK TO THIS

This object has Rotational symmetry of ord 2 .

Angle of Rotaional Symmetry _____ _____

= _____

=

Rotations

A shape has rotational symmetry when it coincides with itself after a rotation of less than 360° about its centre.

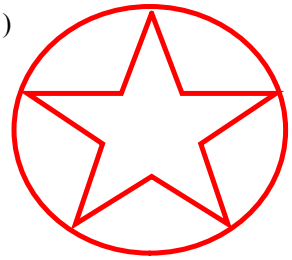
Order of Rotation is the number of times a shape coincides with itself during a 360° rotation

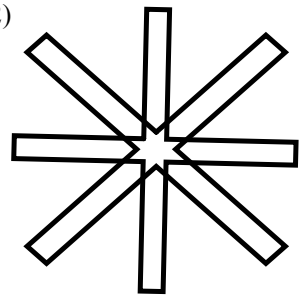
How to state this?
rotational symmetry of order___

Angle of Rotational Symmetry $\frac{360^\circ}{\text{the order of rotation}}$

Look at the web book video in rotations www.mathmakessense.ca

Determine if the following shapes have rotational symmetry. If so state the order of rotation and the angle of rotation symmetry.

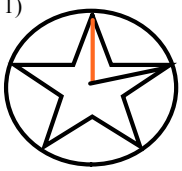
1) 

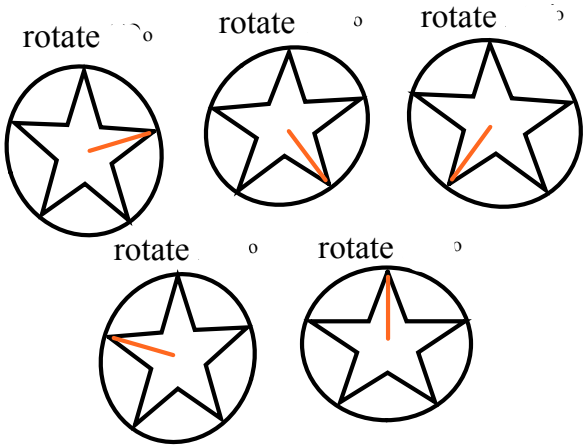
2) 

3) 

Determine if the following shapes have rotational symmetry. If so state the order of rotation and the angle of rotation symmetry.

1)

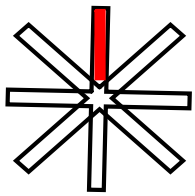


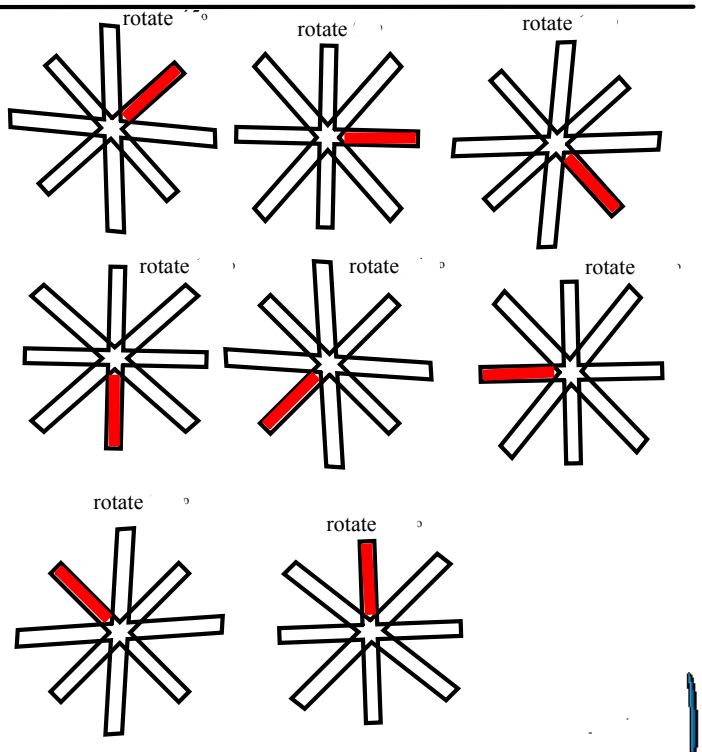


Rotational symmetry of order

Angle of rotation: $\frac{360^o}{\quad} = \quad^o$

2)





Rotational symmetry of order

Angle of rotation: $\frac{360^o}{8} = \quad^o$

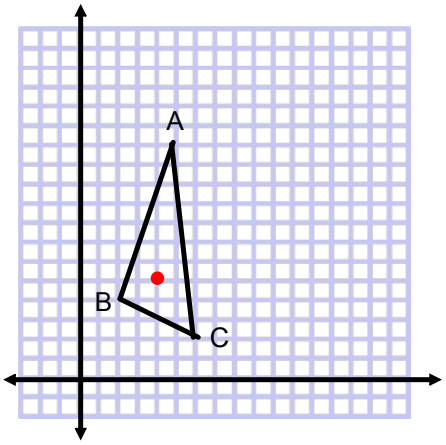
3)

Is rotated one complete turn before it coincides. It DOES NOT have rotational symmetry.

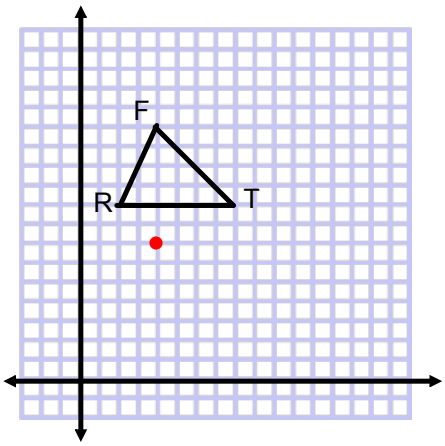
Draw and label the rotated image for each triangle. Label the center of rotation (given in the question)

Step 1) Trace the shape on your own paper and rotate that shape holding your finger or pencil at the rotation center.

1) Rotation 180° counterclockwise, center R(4, 5)

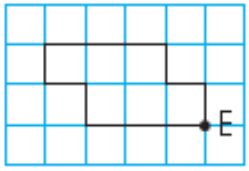


2) Rotation 90° counterclockwise, center R(4, 7)

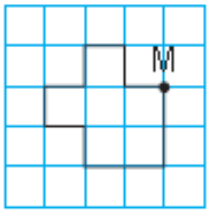


9. Copy each shape on grid paper. Draw the rotation image after each given rotation.

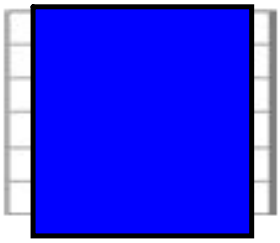
a) 90° clockwise about E



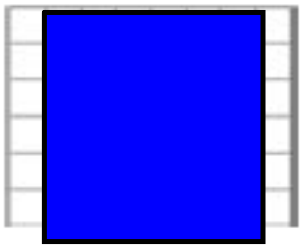
b) 180° about M



a)

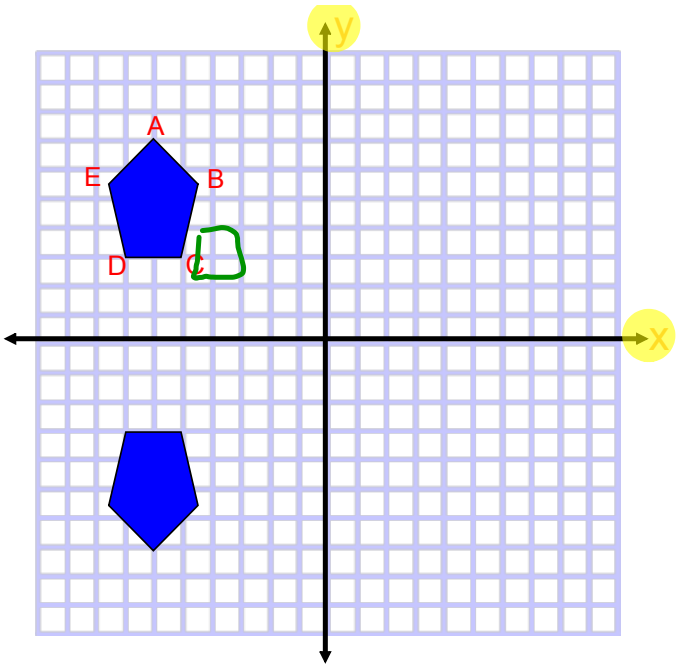


b)



Section 7.7

Symmetry on the Cartesian Plane

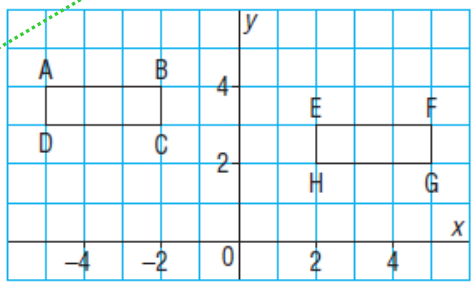


Describe any symmetry you can see...
be specific!!

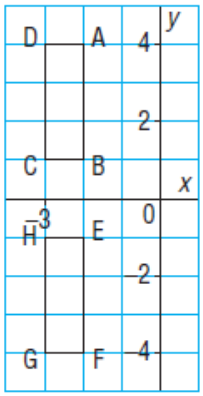
for a
Line of reflection
or
Rotation

For each pair of rectangles ABCD and EFGH, determine whether they are related by symmetry.

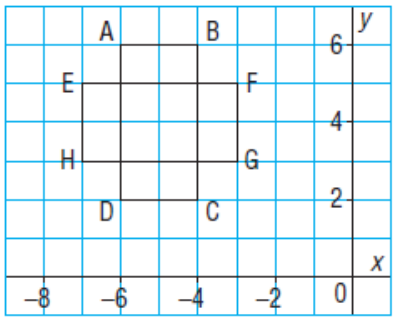
a)



b)

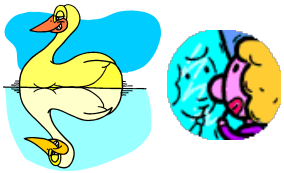


c)



Be specific when you describe the symmetry.

Reflection



What do you need?

Translation

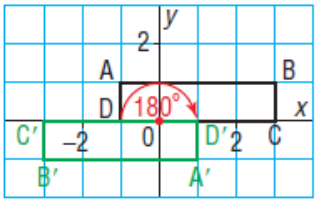
What do you need?

Rotation

What do you need?

a) Use tracing paper to rotate ABCD 180° about the origin.

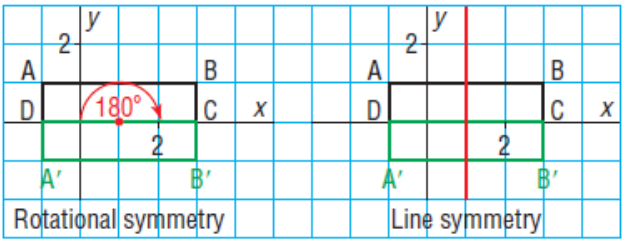
Point	Image
A(−1, 1)	A′(1, −1)
B(3, 1)	B′(−3, −1)
C(3, 0)	C′(−3, 0)
D(−1, 0)	D′(1, 0)



The octagon ABCD′A′B′C′D, formed by both rectangles together, has rotational symmetry of order 2 about the origin, and no line symmetry.

Reflect ABCD in the x-axis.

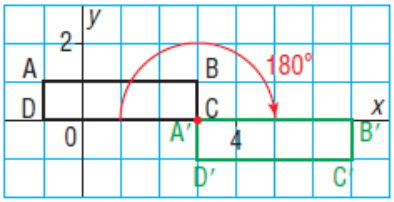
Point	Image
A(−1, 1)	A′(−1, −1)
B(3, 1)	B′(3, −1)
C(3, 0)	C(3, 0)
D(−1, 0)	D(−1, 0)



The rectangle ABB′A′, formed by both rectangles, has rotational symmetry of order 2 about the point (1, 0). It also has 2 lines of symmetry: the x-axis and the vertical line through 1 on the x-axis.

Translate ABCD 4 units right and 1 unit down.

Point	Image
A(−1, 1)	A′(3, 0)
B(3, 1)	B′(7, 0)
C(3, 0)	C′(7, −1)
D(−1, 0)	D′(3, −1)



The two rectangles do not form a shape; but they have a common vertex at C (or A′).
The two rectangles are related by rotational symmetry of order 2 about the point C(3, 0).
There is no line of symmetry relating the rectangles.

Textbook Questions

Page 377-380

Questions:
3,57,8,9,10,11,12,14,15