

**Surface Area,
Volume &
Capacity**



Introduction to Chapter 6... page 222

• Math on the Job... page 224:

A standard roll of antique wallpaper measures 21" wide and 21' long, with the 21' length plastered vertically. Becky needs to completely paper the following walls:

Wall 1: 14 feet wide by 12 feet high
 Wall 2: 16 feet wide by 12 feet high
 Wall 3: 10 feet wide by 12 feet high
 Wall 4: 20 feet wide by 12 feet high

1. How many rolls will Becky need to cover each wall?
 2. What is the minimum number of rolls Becky will need to order to cover all of these walls?

$$21" \times \frac{1ft}{12"} = 1.75 ft$$

$$A = 1.75 \times 21 = 36.75 ft^2$$

As an irregular area of

SOLUTION

1. To calculate the number of wallpaper rolls needed, first calculate the surface area of one roll of wallpaper.

Convert the width to feet.
 $21 \text{ in} \div 12 \text{ in/ft} = 1.75 \text{ ft}$

SA = width \times length
 $SA = 1.75 \times 21$
 $SA = 36.75 \text{ sq. ft.}$

Calculate the area of each wall.

Wall 1:
 SA = width \times length
 $SA = 14 \times 12$
 $SA = 168 \text{ sq. ft.}$

Number of rolls to cover Wall 1:
 $168 \div 36.75 \approx 4.6$

Wall 2:
 SA = width \times length
 $SA = 16 \times 12$
 $SA = 192 \text{ sq. ft.}$

Number of rolls to cover Wall 2:
 $192 \div 36.75 \approx 5.2$

Wall 3:
 SA = width \times length
 $SA = 10 \times 12$
 $SA = 120 \text{ sq. ft.}$

Number of rolls to cover Wall 3:
 $120 \div 36.75 \approx 3.3$

Wall 4:
 SA = width \times length
 $SA = 20 \times 12$
 $SA = 240 \text{ sq. ft.}$

Number of rolls to cover Wall 4:
 $240 \div 36.75 \approx 6.5$

2. Total rolls:

$$4.6 + 5.2 + 3.3 + 6.5 = 19.6$$

Becky will need at least 20 rolls of wallpaper.

$$\#1 \frac{168 ft^2}{36.75 ft^2} = 4.6$$

$$\#2 \frac{192}{36.75} = 5.2$$

$$\#3 \frac{120}{36.75} = 3.3$$

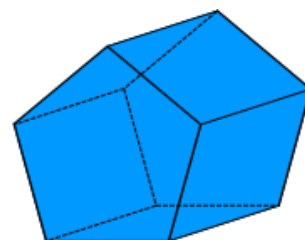
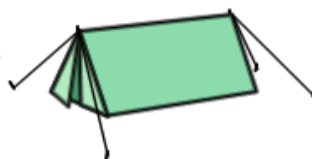
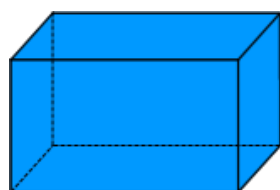
$$\#4 \frac{240}{36.75} = 6.5$$

$$\underline{\quad\quad\quad} 19.6$$

She needs 20 rolls.

3 Dimensional Shapes...

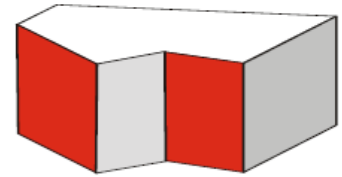
- **Prism** - a 3D shape with ends that are congruent polygons and with sides that are parallelograms.
ex: rectangular prism; triangular prism



- **Base** - one of the parallel faces of a prism
- **Lateral Face** - a face that connects the bases of a prism.

4.12.3: Right Prisms and Their Nets (Teacher)

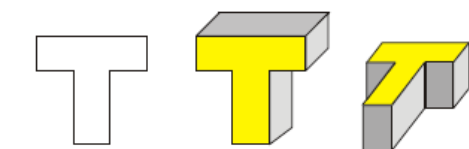
A right prism is a prism with two congruent polygon faces that lie directly above each other. The base is the face that “stacks” to create the prism. This face determines the name of the prism.



Some right prisms and their nets:

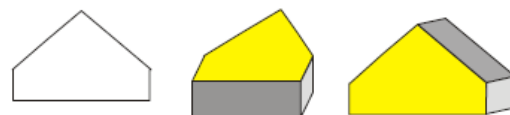
<p>Triangular prism:</p>	<p>Square prism (cube):</p>
<p>Rectangular prism:</p>	<p>Pentagon-based prism:</p>
<p>Hexagon-based prism:</p>	<p>Octagon-based prism:</p>
<p>Trapezoid-based prism:</p>	<p>Parallelogram-based prism:</p>

Right prisms with bases that are composite figures:



Composite figure

Right prism



Composite figure

Right prism

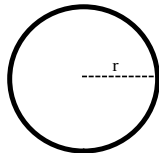
REVIEW: Area Formulas...

Rectangle or Square



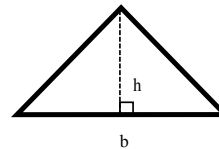
$$A = bh$$

Circle



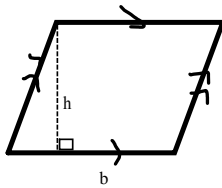
$$A = \pi r^2$$
$$C = 2\pi r$$

Triangle



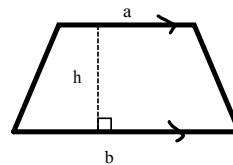
$$A = \frac{1}{2}bh$$

Parallelogram or Rhombus



$$A = bh$$

Trapezoid

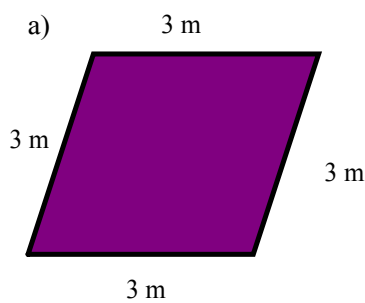


$$A = \frac{1}{2}h(a + b)$$

Perimeter and Circumference

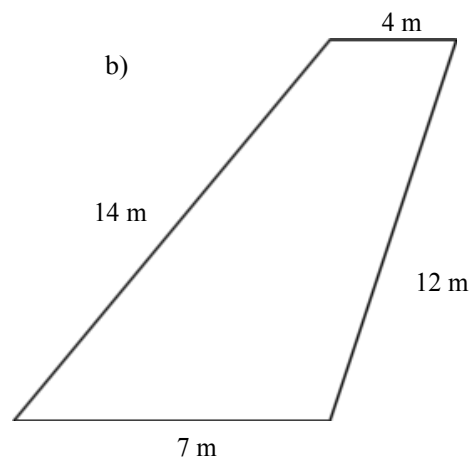
The perimeter is the distance around an object.

Ex: What is the perimeter of the following shapes?



$$P = 3 + 3 + 3 + 3$$

$$= 12 \text{ m}$$

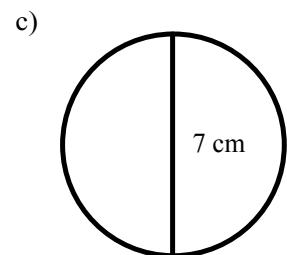


$$P = 14 + 4 + 12 + 7$$

$$= 37 \text{ m}$$

Circumference

$2\pi r$ OR πd



$$C = 2\pi r$$

$$= 2\pi(3.5)$$

$$= 21.99 \text{ cm}$$

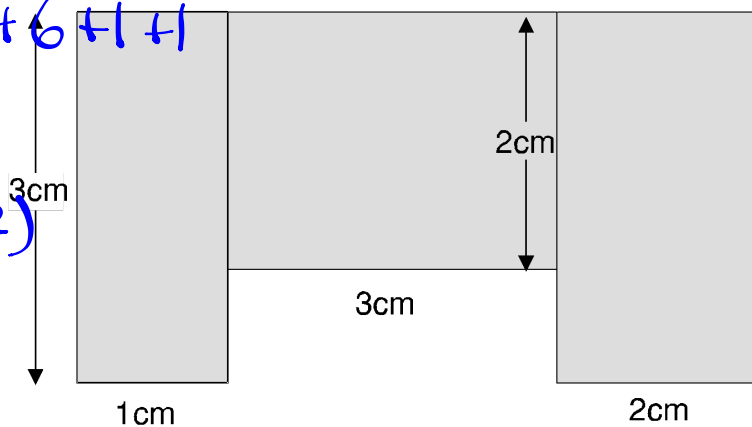
Perimeter and area



- 1) Find the perimeter of ~~each~~ ^{the} figure.
 2) Find the area of ~~each~~ ^{the} figure - they have been divided into rectangles for you.

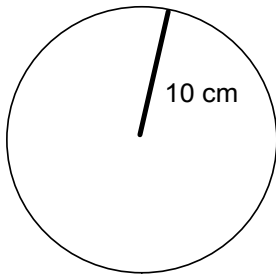
$$1. P = 3 + 1 + 3 + 2 + 3 + 6 + 1 + 1 \\ = 20 \text{ cm}$$

$$2. A = (3)(1) + (3)(2) \\ + (3)(2) \\ = 3 + 6 + 6 \\ = 15 \text{ cm}^2$$



Find the Perimeter and Area of each shape...

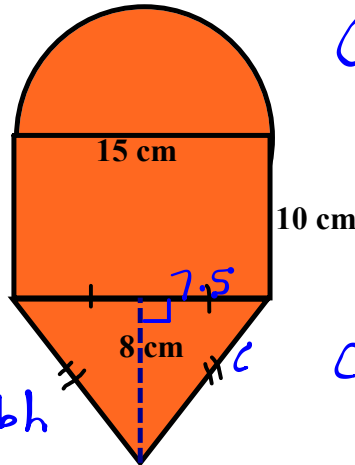
1)



$$\begin{aligned} P &= 2\pi r \\ &= 2\pi(10) \\ &= 62.8 \text{ cm} \end{aligned}$$

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(10)^2 \\ &= 100\pi \\ &= 314 \text{ cm}^2 \end{aligned}$$

2)



$$\begin{aligned} A &= \frac{\pi r^2}{2} + bh \\ &\quad + \frac{bh}{2} \\ &= \frac{\pi(7.5)^2}{2} + (10)(15) \\ &\quad + \frac{15(8)}{2} \\ &= 88.36 + 100 + 60 \\ &= 248.36 \text{ cm}^2 \end{aligned}$$

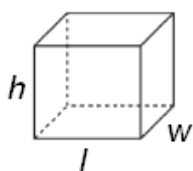
$$\begin{aligned} C &= \frac{2\pi r}{2} \\ &= 2\pi(7.5) \\ &= 23.56 \text{ cm} \end{aligned}$$

$$\begin{aligned} c^2 &= 8^2 + 7.5^2 \\ &= 64 + 56.25 \\ &= 120.25 \end{aligned}$$

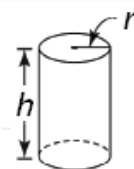
$$\begin{aligned} c &= \sqrt{120.25} \\ &= 10.97 \text{ cm} \end{aligned}$$

$$\begin{aligned} P &= 23.56 + 10 + 10 \\ &\quad + 2(10.97) \\ &= 65.5 \text{ cm} \end{aligned}$$

Activate Prior Learning:
**Surface Areas of Right Prisms
and Cylinders**



$$SA = 2wl + 2hl + 2hw$$



$$SA = 2\pi r^2 + 2\pi rh$$

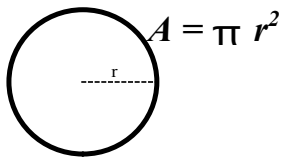


Area Formulas

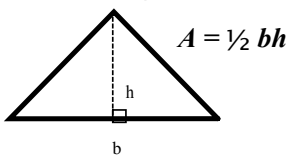
Rectangle or Square

$$A = bh$$

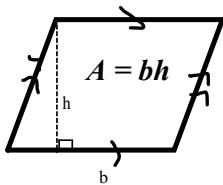
Circle



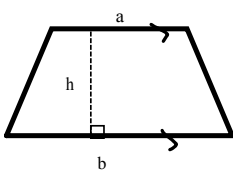
Triangle



Parallelogram or Rhombus

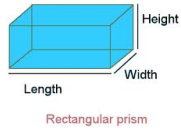


Trapaziod



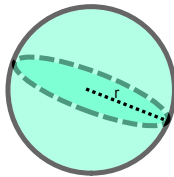
$$A = \frac{1}{2} h(a + b)$$

Surface Area Formulas



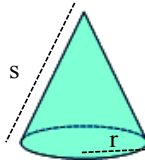
Rectangular Prism

SA = Area of all 6 rectangles added together



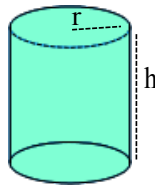
Sphere

$$SA = 4\pi r^2$$



Cone

$$SA = \pi r^2 + \pi rs$$

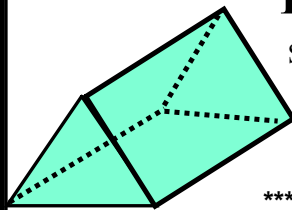


Cylinder

$$SA = 2\pi r^2 + 2\pi rh$$

Triangular Prism

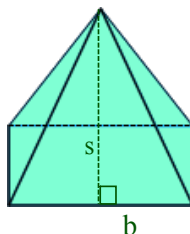
SA = Area of the three Rectangle Sides
+ Area of two Triangle sides



****not all three rectangles have to be the same*

Pyramid

SA = Area of the base + all triangular faces



Surface Area

Surface area is the total area of all of the faces of the object.

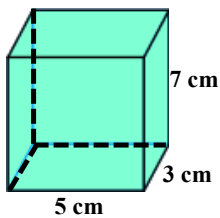
Steps needed to find Surface area are...

- 1. Draw all of the faces with dimensions displayed on them.**
 - 2. Find the area of each face.**
 - 3. Then add up the areas of all of the faces.**
- .

EXAMPLES...

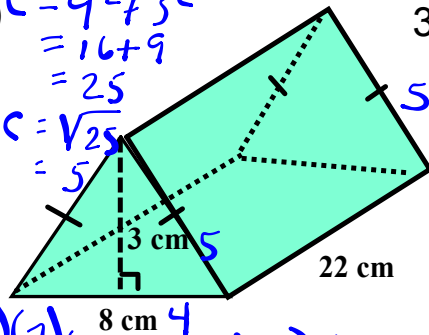
Determine the surface area of each of the following...

1)



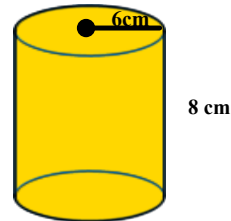
$$\begin{aligned}
 A &= (5)(3)(2) + (3)(7)(2) \\
 &\quad + (5)(7)(2) \\
 &= 30 + 42 + 70 \\
 &= 142 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 2) \quad c^2 &= 4^2 + 3^2 \\
 &= 16 + 9 \\
 &= 25 \\
 c &= \sqrt{25} \\
 &= 5
 \end{aligned}$$



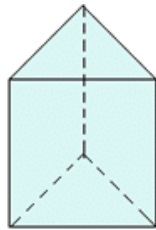
$$\begin{aligned}
 A &= \frac{(8)(3)(2)}{2} \\
 &\quad + (22)(5)(2) \\
 &\quad + 22(8) \\
 &= 24 + 220 + 176 \\
 &= 420 \text{ cm}^2
 \end{aligned}$$

3)

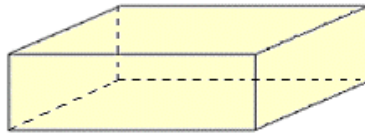


$$\begin{aligned}
 A &= 2\pi r^2 + 2\pi rh \\
 &= 2\pi(6^2) + 2\pi(6)(8) \\
 &= 72\pi + 96\pi \\
 &= 168\pi \\
 &\approx 527.79 \text{ cm}^2
 \end{aligned}$$

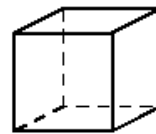
Regular Polygonal Prism



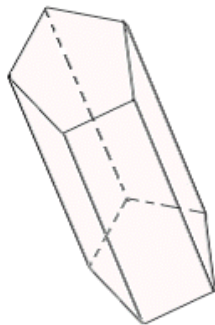
Triangular Prism



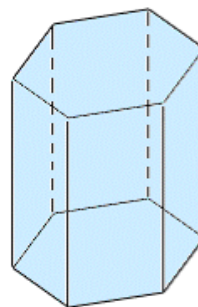
Rectangular Prism



Cube



Pentagonal Prism



Hexagonal Prism

DISCUSS THE IDEAS

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CALCULATING SURFACE AREA OF A STAINED GLASS LANTERN

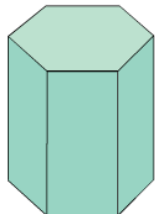
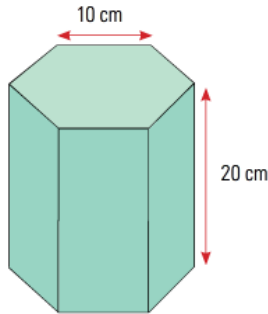
A three-dimensional object is a **prism** if it contains two parallel polygons that are congruent.

Congruent polygons

- have the same shape and size
- have sides and angles in the same positions

Each of the parallel faces is called a **base**, and the faces connecting the bases are called **lateral faces**.

If the lateral faces are perpendicular to the bases, the prism is a right prism.



Right hexagonal prism



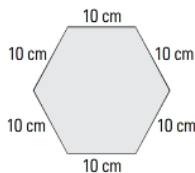
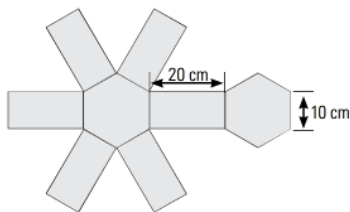
Oblique prism

Shabina makes stained-glass garden lanterns in the shape of hexagonal prisms. The bases (top and bottom) are hexagons and are made of metal. The faces are coloured glass.

1. Look at the lateral faces of the right hexagonal prism above. Draw a net of the right hexagonal prism.
2. If each edge of the hexagonal base is 10 cm long, and the lanterns are 20 cm high, what is the area of each piece of glass used as a lateral face?
3. What is the total surface area of glass that Shabina needs for one lantern? Can you think of two ways to calculate the total surface area?

SOLUTIONS

1. The faces are rectangles.



2. The hexagonal lantern will have six sides. Each side will be rectangular and have a width of 10 cm (the edge length of the hexagonal base) and a length of 20 cm.

$$A = \ell \times w$$

$$A = 20 \times 10$$

$$A = 200$$

The area of each piece of stained glass will be 200 cm^2 .

3. The total area is the sum of areas of all the lateral faces. A hexagonal prism has 6 lateral faces.

$$\text{Total area} = 6 \times 200$$

$$\text{Total area} = 1200 \text{ cm}^2$$

DISCUSS THE IDEAS Page 230

SURFACE AREA OF CABINETS

- Karl is building a set of cabinets. He makes the first one 40 cm long \times 40 cm deep \times 70 cm high. When the cabinet is fully closed, it is the shape of a rectangular prism. What is the surface area of the cabinet?
- Karl makes the second cabinet 2 times as long, but with the same depth and height. What is the surface area of the second cabinet?
- Karl makes the third cabinet 2 times as long and 2 times as high as the first one, but with the same depth. What is the surface area of the third cabinet?
- Examine the following table showing how the surface area changes when the dimensions are changed.

SURFACE AREA RATIOS					
Cabinet	Length	Depth	Height	Surface area	$\frac{\text{(Surface Area)}}{\text{(Surface Area of Cabinet 1)}}$
1	L	D	H	14 400	1.00
2	2L	D	H	23 200	1.61
3	2L	D	2H	40 000	2.78
4	2L	2D	2H	?	?

What do you notice about the surface area ratio when the length is doubled? How do the surface areas compare when length and height are doubled?

- Calculate the surface area of a cabinet where all three dimensions are double those of cabinet one, and calculate the ratio of surface areas. What can you conclude about the relationship between the scale factor used to create the new dimensions and the ratio of the surface areas?

CONCLUSION...

If ALL dimensions are doubled, then the total surface area will be quadrupled.

SOLUTIONS

- The surface area of a rectangular prism can be expressed by the following equation:

$$SA = 2 \times (\text{length} \times \text{depth}) + 2 \times (\text{length} \times \text{height}) + 2 \times (\text{depth} \times \text{height})$$

The factor of 2 is introduced because the prism has three sets of identical pairs of faces.

$$SA = 2 \times (40 \times 40) + 2 \times (40 \times 70) + 2 \times (40 \times 70)$$

$$SA = 14\,400 \text{ cm}^2$$

- $SA = 2 \times (80 \times 40) + 2 \times (80 \times 70) + 2 \times (40 \times 70)$

$$SA = 23\,200 \text{ cm}^2$$

- $SA = 2 \times (80 \times 40) + 2 \times (80 \times 140) + 2 \times (40 \times 140)$

$$SA = 40\,000 \text{ cm}^2$$

- When the length is doubled, the surface area ratio increases by more than half the original ratio. The surface area does not double when one dimension is doubled. When two dimensions are doubled, such as the length and the height, the surface area more than doubles, measuring 40 000 cm².
- $SA = 2 \times (80 \times 80) + 2 \times (80 \times 140) + 2 \times (80 \times 140)$

$$SA = 57\,600 \text{ cm}^2$$

$$\text{Ratio of surface areas} = \frac{57\,600}{14\,400}$$

$$\text{Ratio of surface areas} = 4.0$$

Because the total surface area is a sum of several areas, and not all the terms of the sum are affected when one or two dimensions are doubled, the effect on surface area of changing only one or two dimensions is not an integer. When all three dimensions are doubled, all terms of the surface area sum are affected, and the total surface area scales by the square of the multiplication factor.

HOMWORK...

Review - Prior Knowledge for Section 6.1.pdf



BLACKLINE MASTER 6.9: SOLUTIONS

Order of Operations

- $$5^2 \times 3 - (84 - 37)$$

$$= 25 \times 3 - 47$$

$$= 75 - 47$$

$$= 28$$
- $$(22 - 25)^3 \div [(13 - 7) + 3]$$

$$= (-3)^3 \div (6 + 3)$$

$$= -27 \div 9$$

$$= -3$$
- $$\left(\frac{36}{9}\right)^2 \times 2 - 15 \div (-3)$$

$$= 4^2 \times 2 - 15 \div (-3)$$

$$= 16 \times 2 - (-5)$$

$$= 32 + 5$$

$$= 37$$
- $$(-4)^3 + (5 - 11)^2 \div 12 + 20$$

$$= -64 + (-6)^2 \div 12 + 20$$

$$= -64 + 36 \div 12 + 20$$

$$= -64 + 3 + 20$$

$$= -41$$

Finding the Area of Composite Figures

- $$A = \ell w$$

$$A = (10.5)(4.5)$$

$$A = 47.25 \text{ in}^2$$
- $$A = wh$$

$$A = (12)(18)$$

$$A = 216 \text{ cm}^2$$
- $$A = \pi r^2$$

$$A = \pi(3.5)^2$$

$$A \approx 38.48 \text{ yd}^2$$
- $$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(5)(2.9)$$

$$A = 7.25 \text{ ft}^2$$

Working with Formulas

- $$4\pi r^2 \quad (r = 3.4)$$

$$= 4\pi(3.4)^2$$

$$\approx 145.27$$
- $$\frac{1}{3}\pi r^2 h \quad (r = 5.2, h = 8)$$

$$= \frac{1}{3}\pi(5.2)^2(8)$$

$$\approx 226.53$$
- $$\pi rs + \pi r^2 \quad (r = 3, s = 4.3)$$

$$= \pi(3)(4.3) + \pi(3)^2$$

$$\approx 40.53 + 28.27$$

$$\approx 68.8$$
- $$2\pi r^2 + 2\pi rh \quad (r = 6.7, h = 12.3)$$

$$= 2\pi(6.7)^2 + 2\pi(6.7)(12.3)$$

$$\approx 282.05 + 517.80$$

$$\approx 799.85$$

Converting Measurements Within and Between the SI and Imperial Systems

- 4.56 km; metres

$$1 \text{ km} = 1000 \text{ m}$$

$$4.56 \text{ km} = 4560 \text{ m}$$
- 56.64 yd; inches (1 yard = 36 inches)

$$1 \text{ yard} = 36 \text{ inches}$$

$$56.64 \text{ yards} = 2039.04 \text{ inches}$$
- 27.2 feet; cm (1 foot \approx 30.48 cm)

$$1 \text{ foot} \approx 30.48 \text{ cm}$$

$$27.2 \text{ feet} \approx 829.056 \text{ cm}$$
- 89.2 miles; km (1 mile = 1.609344 km)

$$1 \text{ mile} = 1.609344 \text{ km}$$

$$89.2 \text{ miles} \approx 143.55 \text{ km}$$

Homework...

6.1 Worksheet - Surface Area of Prisms, Pyramids and Cylinders.pdf



Remember...

$SA_{\text{prism}} =$ Add the area of all the faces

$SA_{\text{pyramid}} =$ Add the area of a base and the area of the triangular faces (note: $A_{\text{triangle}} = bh/2$)

$SA_{\text{cylinder}} = 2\pi r^2 + 2\pi rh$

Homework...

p. 232: #1 - 7

6.1 - Build Your Skills Solutions.pdf



Attachments

Section 6.1 Surface Area of Prisms.pdf

Review - Prior Knowledge for Section 6.1.pdf

6.1 Worksheet - Surface Area of Prisms, Pyramids and Cylinders.pdf

6.1 - Build Your Skills Solutions.pdf