

CALCULATING SURFACE AREA OF A STAINED GLASS LANTERN

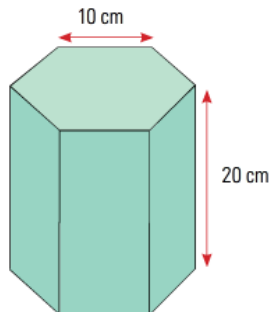
A three-dimensional object is a **prism** if it contains two parallel polygons that are congruent.

Congruent polygons

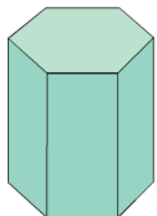
- have the same shape and size
- have sides and angles in the same positions

Each of the parallel faces is called a **base**, and the faces connecting the bases are called **lateral faces**.

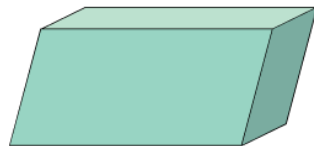
If the lateral faces are perpendicular to the bases, the prism is a right prism.



Page 226



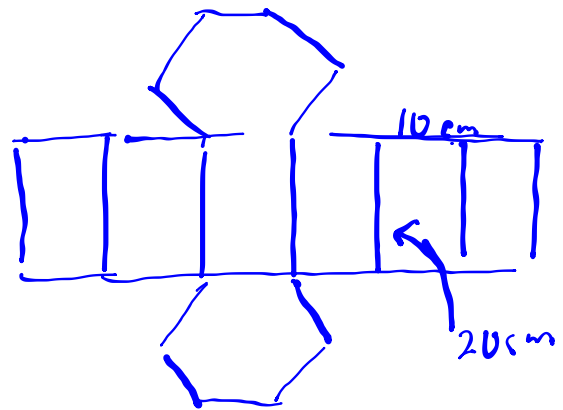
Right hexagonal prism



Oblique prism

Shabina makes stained-glass garden lanterns in the shape of hexagonal prisms. The bases (top and bottom) are hexagons and are made of metal. The faces are coloured glass.

1. Look at the lateral faces of the right hexagonal prism above. Draw a net of the right hexagonal prism.
2. If each edge of the hexagonal base is 10 cm long, and the lanterns are 20 cm high, what is the area of each piece of glass used as a lateral face?
3. What is the total surface area of glass that Shabina needs for one lantern? Can you think of two ways to calculate the total surface area?

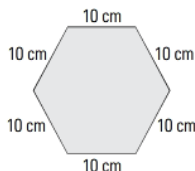
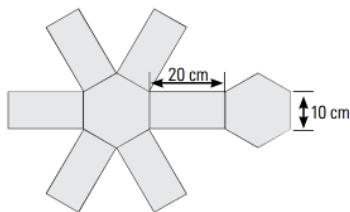


2. $A = 10(20) = 200 \text{ cm}^2$

3. $A = 200(6) = 1200 \text{ cm}^2$

SOLUTIONS

1. The faces are rectangles.



2. The hexagonal lantern will have six sides. Each side will be rectangular and have a width of 10 cm (the edge length of the hexagonal base) and a length of 20 cm.

$$A = \ell \times w$$

$$A = 20 \times 10$$

$$A = 200$$

The area of each piece of stained glass will be 200 cm^2 .

3. The total area is the sum of areas of all the lateral faces. A hexagonal prism has 6 lateral faces.

$$\text{Total area} = 6 \times 200$$

$$\text{Total area} = 1200 \text{ cm}^2$$

DISCUSS THE IDEAS Page 230

SURFACE AREA OF CABINETS

1. Karl is building a set of cabinets. He makes the first one 40 cm long × 40 cm deep × 70 cm high. When the cabinet is fully closed, it is the shape of a rectangular prism. What is the surface area of the cabinet?
2. Karl makes the second cabinet 2 times as long, but with the same depth and height. What is the surface area of the second cabinet?
3. Karl makes the third cabinet 2 times as long and 2 times as high as the first one, but with the same depth. What is the surface area of the third cabinet?
4. Examine the following table showing how the surface area changes when the dimensions are changed.

↙ 2x

$$A = 2lw + 2lh + 2wh$$

SURFACE AREA RATIOS					
Cabinet	Length	Depth	Height	Surface area	$\frac{\text{(Surface Area)}}{\text{(Surface Area of Cabinet 1)}}$
1	L	D	H	14 400	1.00
2	2L	D	H	23 200	1.61
3	2L	D	2H	40 000	2.78
4	2L	2D	2H	57 600	4

What do you notice about the surface area ratio when the length is doubled? How do the surface areas compare when length and height are doubled?

5. Calculate the surface area of a cabinet where all three dimensions are double those of cabinet one, and calculate the ratio of surface areas. What can you conclude about the relationship between the scale factor used to create the new dimensions and the ratio of the surface areas?

CONCLUSION...

If ALL dimensions are doubled, then the total surface area will be quadrupled.

SOLUTIONS

1. The surface area of a rectangular prism can be expressed by the following equation:

$$SA = 2 \times (\text{length} \times \text{depth}) + 2 \times (\text{length} \times \text{height}) + 2 \times (\text{depth} \times \text{height})$$

The factor of 2 is introduced because the prism has three sets of identical pairs of faces.

$$SA = 2 \times (40 \times 40) + 2 \times (40 \times 70) + 2 \times (40 \times 70)$$

$$SA = 14\,400 \text{ cm}^2$$

2. $SA = 2 \times (80 \times 40) + 2 \times (80 \times 70) + 2 \times (40 \times 70)$

$$SA = 23\,200 \text{ cm}^2$$

3. $SA = 2 \times (80 \times 40) + 2 \times (80 \times 140) + 2 \times (40 \times 140)$

$$SA = 40\,000 \text{ cm}^2$$

4. When the length is doubled, the surface area ratio increases by more than half the original ratio. The surface area does not double when one dimension is doubled. When two dimensions are doubled, such as the length and the height, the surface area more than doubles, measuring 40 000 cm².


5. $SA = 2 \times (80 \times 80) + 2 \times (80 \times 140) + 2 \times (80 \times 140)$

$$SA = 57\,600 \text{ cm}^2$$

$$\text{Ratio of surface areas} = \frac{57\,600}{14\,400}$$

$$\text{Ratio of surface areas} = 4.0$$

Because the total surface area is a sum of several areas, and not all the terms of the sum are affected when one or two dimensions are doubled, the effect on surface area of changing only one or two dimensions is not an integer. When all three dimensions are doubled, all terms of the surface area sum are affected, and the total surface area scales by the square of the multiplication factor.

 Review prior knowledge for 6.1
Due...April 3

Review - Prior Knowledge for Section 6.1(22774).pdf