

Section 8.3

Properties of Angles in Circles



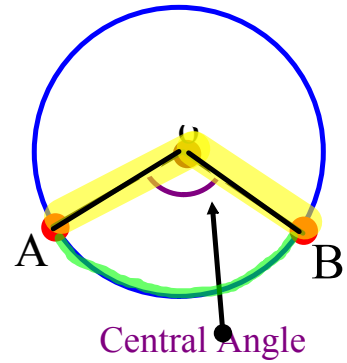


Central Angle:

The angle formed by joining the endpoints of a arc to the centre of a circle

(Made with 2 radii)

$$\angle AOB$$

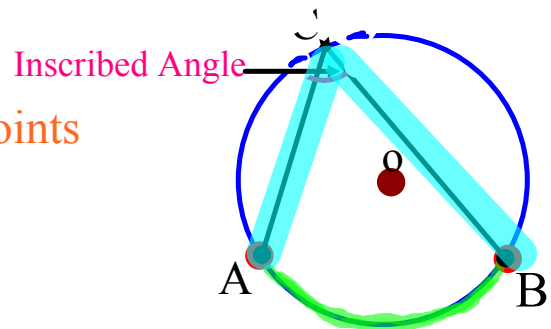


Inscribed Angle:

The angle formed by joining the endpoints of a arc to a point on the circle

(Made with two chords)

$$\angle ACB$$



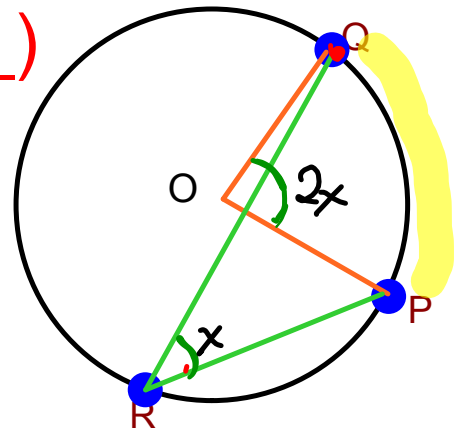
Inscribed and central angles are **SUBTENDED** by the **MINOR** arc

come from the same 'smaller arc'

Central Angle & Inscribed Angle Property

Property 1: (Ins/Cent \angle , $\overset{\frown}{QP}$)

In a circle, the measure of a **central angle** subtended by an arc is **TWICE** the measure of an **inscribed angle** subtended by the same arc.



$$\angle POQ = 2 \angle PRQ$$

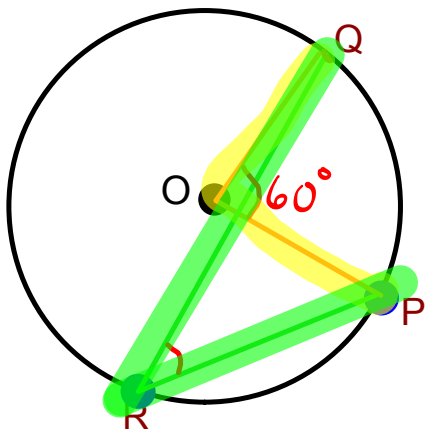
Central angle is twice the inscribed angle

or

$$\angle PRQ = \frac{1}{2} \angle POQ$$

Inscribed angle is half the center angle

Example:



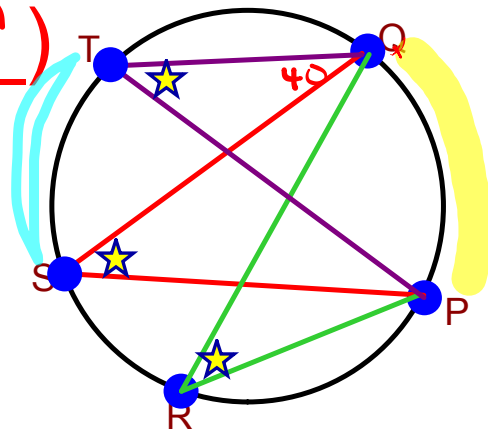
$$\angle QOP = 60^\circ$$

$$\angle QRP = 30^\circ \text{ (In/cent, } \overset{\frown}{QP} \text{)}$$

Inscribed Angle Property

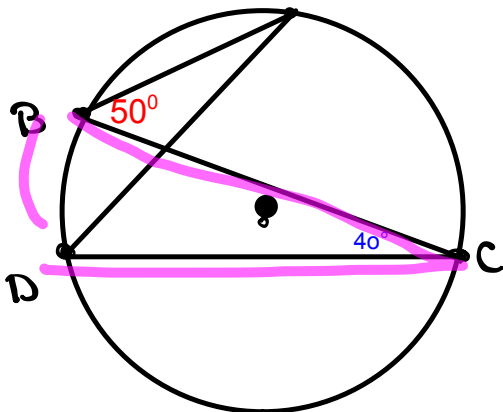
Property 2: (Ins \angle , $\widehat{\quad}$)

In a circle, all inscribed angles subtended by the same arc are congruent (equal).



$$\angle PTQ = \angle PSQ = \angle PRQ$$

Example

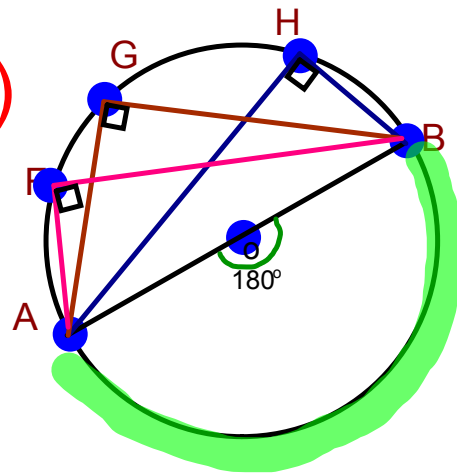


- $\angle ABC = 50^\circ$
 $\angle ADC = 50^\circ$ (ins \angle , \widehat{AC})
- $\angle BCD = 40$
 $\angle BAD = 40^\circ$ (ins \angle , \widehat{BD})

Angles in a Semicircle Property

Property 3: (Ins \angle , diam)

All inscribed angles subtended by a semicircle are right angles



Makes sense

Inscribed angles are always half the centre

Center Angle = 180° (Straight Line)

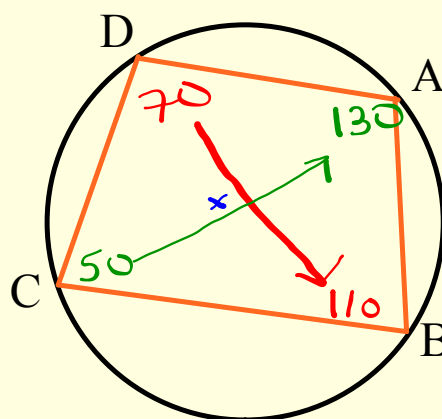
Inscribed angle is half the Central Angle

$$\begin{aligned} \text{Inscribed} &= \left(\frac{1}{2}\right) \text{central} \\ &= \left(\frac{1}{2}\right) 180^\circ \\ &= 90^\circ \end{aligned}$$

Cyclic Quadrilateral Angle Properties:

Property 4: (Cy Quad)

___ The opposite angles of an inscribed **quadrilateral** are supplementary.
(their sum is 180°)



ABCD is an inscribed quadrilateral.

$\angle A$ and $\angle C$ are opposite
therefore, $\angle A + \angle C = 180$

$\angle B$ and $\angle D$ are opposite
therefore, $\angle B + \angle D = 180$

Section 8. 3: Circle Properties reasoning

(ins/cent \leq , )

(ins \leq , )

(ins \leq , dia)

(CyQuad)

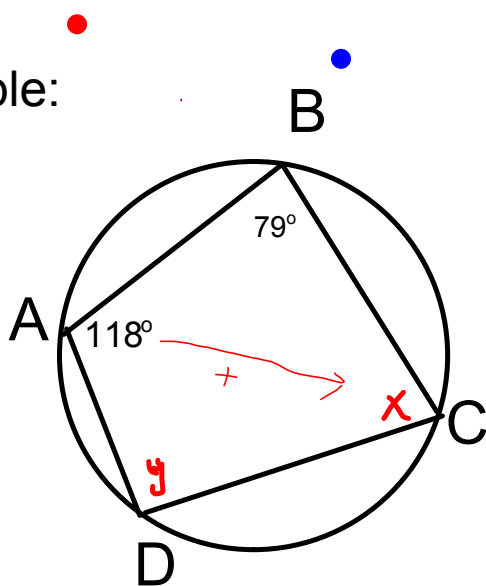
(SATT)

(ITT)

(SAT)

(CyAT)

Example:



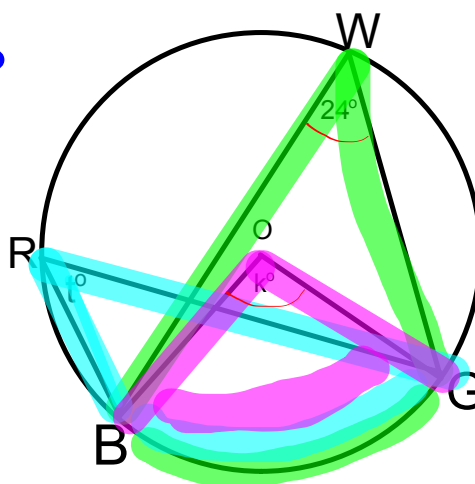
$$x \Rightarrow \angle BCD = \underline{62^\circ} \text{ (Cyclic Quad)}$$

$$y \Rightarrow \angle ADC = \underline{161^\circ} \text{ (Cyclic Quad)}$$

Example 1

Using Inscribed and Central Angles

Point O is the center of a circle.
Determine the values of k and t .

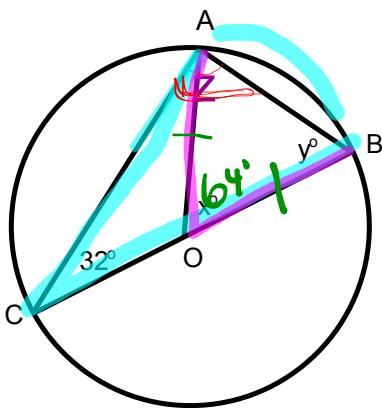


$$k \Rightarrow \angle \underline{BOG} = \underline{48^\circ} \text{ (Ins/cent, } \widehat{BG} \text{)}$$

$$t \Rightarrow \angle \underline{BRG} = \underline{24^\circ} \text{ (Ins } \angle, \widehat{BG} \text{)}$$

Example 2

Applying the Property of an Angle Inscribed in a Semicircle



Point O is the center of the circle.
Determine the value of x° and y° .

$$\sphericalangle \angle CAB = 90^\circ \text{ (Ins, dia)}$$

For Y°

$$y = 58^\circ \text{ (Itt)}$$

or
(SATT)

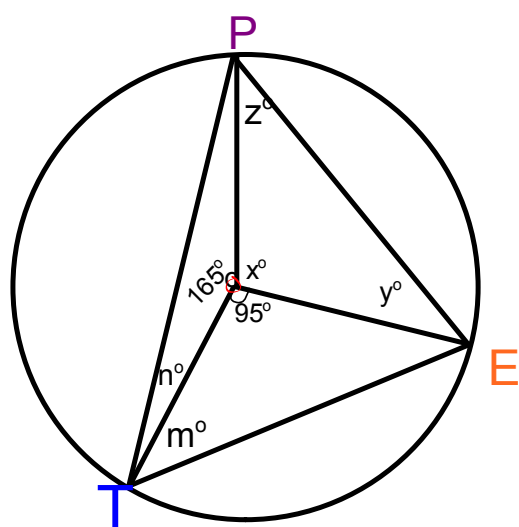
For X°

$$\angle AOB = 64^\circ \text{ (Ins/cent, } \widehat{AB})$$

Example 3

Determining Angles in an Inscribed Triangle

Determining the values of x° , y° , z° , m° , n°

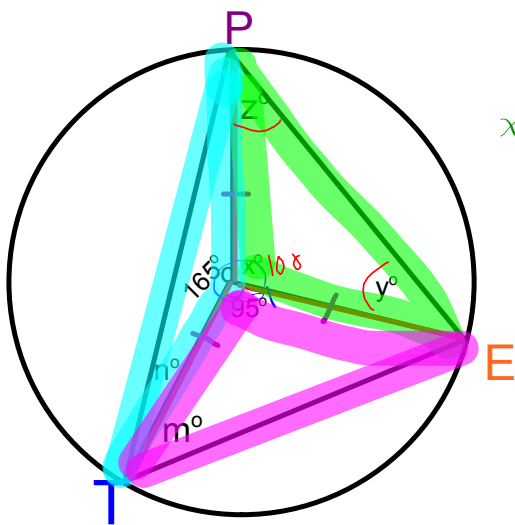


Hint: LOOK AT CENTER ANGLES and Complete the circle for x

Example 3

Determining Angles in an Inscribed Triangle

Determining the values of x° , y° , z° , m° , n°



Hint: LOOK AT CENTER ANGLES and Complete the circle for x

$$x = \angle POE = 100^\circ \text{ (Cy AT) }$$

$$y = \angle OEP = 40^\circ \text{ (I t t) }$$

$$z = \angle OPE = 40^\circ \text{ (I t t) }$$

$$m = \angle OTE = 42.5^\circ \text{ (I t t) }$$

$$n = \angle OTP = 7.5^\circ \text{ (I t t) }$$

Attachments

Worksheet - Angles in a Circle.doc