MAY 29, 2017

UNIT 4: SYSTEMS OF LINEAR EQUATIONS

DISTANCE BETWEEN TWO POINTS ON A LINE SEGMENT / THE MIDPOINT OF A LINE SEGMENT

M. MALTBY INGERSOLL
NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will begin working on the NRF 10 Specific Curriculum Outcome (SCO) "Relations and Functions 8" OR "RF8" which states:

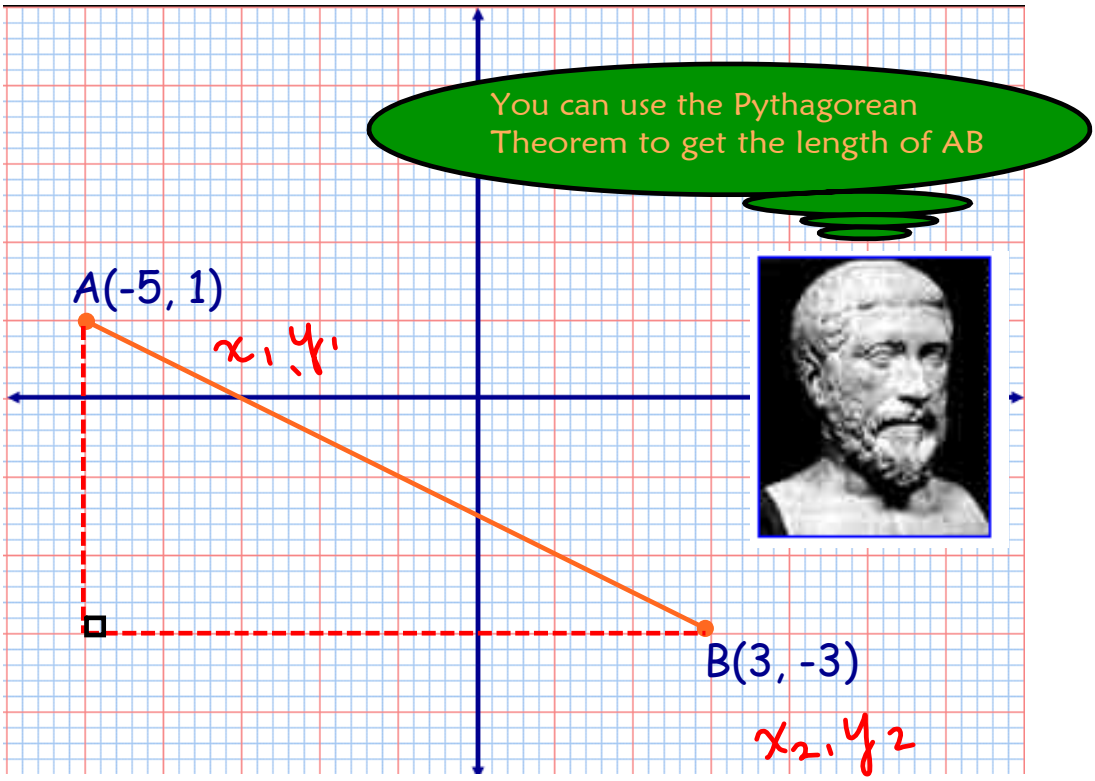
RF8: "Solve problems that involve the distance between two points and the midpoint of a line segment."

What does THAT mean???

SCO RF8 means that we will:

* **determine the distance between two points on a Cartesian plane using a variety of strategies**
* **determine the midpoint of a line segment given the endpoints of the segment using a variety of strategies**
* **determine an endpoint of a line segment given the other endpoint and the midpoint using a variety of strategies**
* **solve a contextual problem involving distance between two points or the midpoint of a line segment**

Distance Between Two Points



DISTANCE FORMULA:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{[3-(-5)]^2 + (-3-1)^2}$$

$$= \sqrt{8^2 + (-4)^2}$$

$$= \sqrt{64 + 16}$$

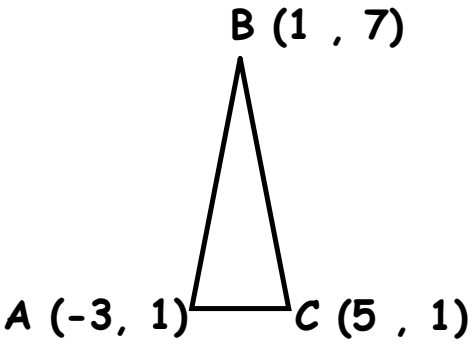
$$= \sqrt{80}$$

$$\doteq 8.9$$

EXAMPLE:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Use the distance formula to show that the triangle with vertices A (-3 , 1) , B (1 , 7) & C (5 , 1) is isosceles.



AB: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{[1-(-3)^2+(7-1)^2}$

$= \sqrt{4^2+6^2}$

$= \sqrt{16+36}$

$= \sqrt{52}$

$\doteq 7.2$

BC: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(1-5)^2+(7-1)^2}$

$= \sqrt{(-4)^2+6^2}$

$= \sqrt{16+36}$

$= \sqrt{52}$

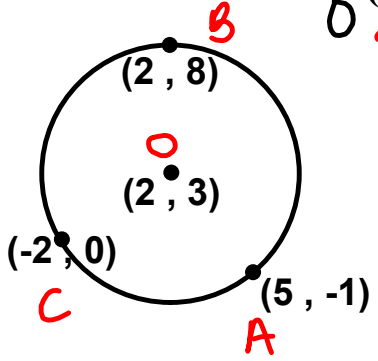
$\doteq 7.2$

AB = BC = 7.2 units, therefore $\triangle ABC$ is isosceles.

YOU TRY!

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Show that the points (5 , -1) , (2 , 8) & (-2 , 0) lie on a circle whose center is (2 , 3) .



O x_1, y_1 A B C

AO: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-5)^2+[3-(-1)]^2}$

$= \sqrt{(-3)^2+4^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

BO: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-2)^2+(3-8)^2}$

$= \sqrt{(-5)^2}$

$= \sqrt{25}$

$= 5$

CO: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{[2-(-2)^2+(3-0)^2}$

$= \sqrt{4^2+3^2}$

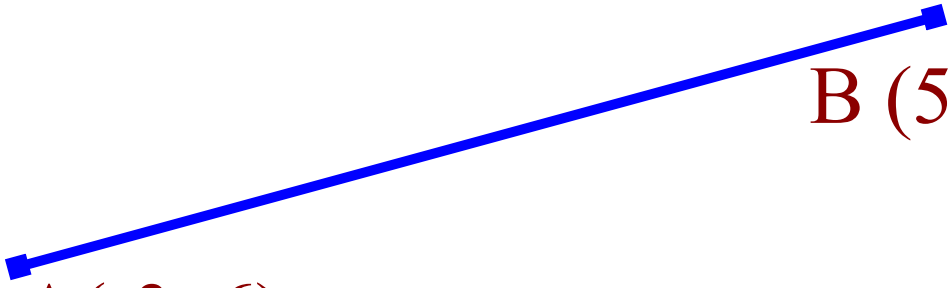
$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

AO = BO = CO = 5 units they are points that lie on this circle. They are all 5 units from the centre.

Midpoint of a Line



$$midpoint\,(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-3+5}{2}, \frac{6+14}{2} \right)$$

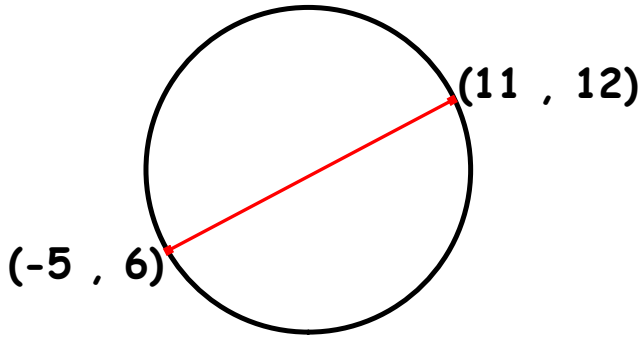
$$= \left(\frac{2}{2}, \frac{20}{2} \right)$$

$$= (1, 10)$$

EXAMPLE:

$$midpoint\,(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The endpoints of a diameter of a circle are (-5 , 6) and (11 , 12). Find the coordinates of the center of the circle.



$$= \left(\frac{-5+11}{2}, \frac{6+12}{2}\right)$$

$$= \left(\frac{6}{2}, \frac{18}{2}\right)$$

$$= (3, 9)$$

EXAMPLE:

$$midpoint\,(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

One endpoint of a line segment is (-4 , 3). The midpoint is (-3 , 6). Find the other endpoint.

x y

$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

$$-3 = \frac{-4 + x}{2} \qquad 6 = \frac{3 + y}{2}$$

$$-6 = -4 + x \qquad 12 = 3 + y$$

$$-2 = x \qquad 9 = y$$

The second endpoint of the line was (-2, 9).

YOU TRY!

$$midpoint\,(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Find the midpoint of the line with endpoints (2 , 1) & (8 , -13) .

$$= \left(\frac{2+8}{2}, \frac{1-13}{2}\right)$$

$$= \left(\frac{10}{2}, \frac{-12}{2}\right)$$

$$= (5, -6)$$

YOU TRY!

$$midpoint\,(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

A line has endpoint (10 , 6) & midpoint (4 , -2) . Find the other endpoint.

x y

$$x = \frac{x_1 + x_2}{2}$$

$$4 = \frac{10 + x}{2}$$

$$8 = 10 + x$$

$$-2 = x$$

x y

$$y = \frac{y_1 + y_2}{2}$$

$$-2 = \frac{6 + y}{2}$$

$$-4 = 6 + y$$

$$-10 = y$$

2nd endpoint is $(-2, -10)$.

CONCEPT REINFORCEMENT:

WORKSHEETS:

"Distance Formula - Worksheet 1": #1 TO #10

"The Midpoint Formula": Left Side (#1, #3, #5... TO #29)

Attachments

Worksheet - Review of Coordinate Geometry (Math 10).doc

area of a triangle.doc

coord geom review.doc

Puzzle Worksheet - Graphing #2 (Coffee).pdf

Puzzle Worksheet - Graphing #1 (Cow).pdf

Puzzle Worksheet - Slope Point (given both).pdf

Puzzle Worksheet - Slope Point (given two points).pdf

Worksheet - Equation of a Line.pdf

Worksheet Solutions - Equation of a Line.pdf

Worksheet - Distance_Midpoint(2).pdf

Review - Coordinate Geometry.pdf