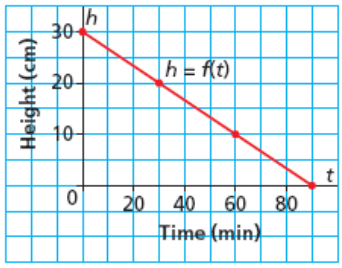
6.5 Slope-Point Form of the Equation for a Linear Function

LESSON FOCUS Relate the graph of a linear function to its equation in slope-point form.

Make Connections

This graph shows the height of a candle as it burns.
How would you write an equation to describe this line?
Suppose you could not identify the h-intercept.
How could you write an equation for the line?



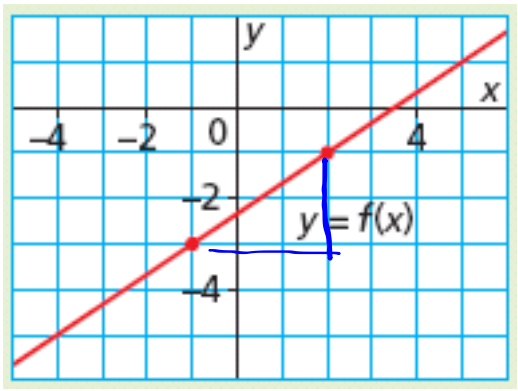
$y = mx + b$ $y_{int} = 30$

$m = \frac{rise}{run} = \frac{-2}{3}$

$y = mx + b$

$y = -\frac{2}{3}x + 30$

How about this one using y = mx + b?



$m = \frac{rise}{run} = \frac{-2}{-3} = \frac{2}{3}$

$P_1 = (-1, -3)$

$P_2 = (2, -1)$

~~$y = mx + b$~~

$y - y_1 = m(x - x_1)$

$y + 1 = \frac{2}{3}(x - 2)$

$3(y+1) = 3\left(\frac{2}{3}(x-2)\right)$

$3y + 3 = 2(x - 2)$

$3y + 3 = 2x - 4$

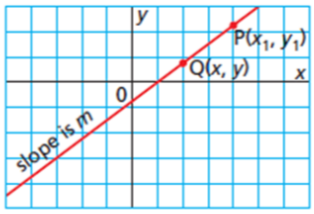
$-2x + 3y + 3 + 4 = 0$

$-2x + 3y + 7 = 0$

$2x - 3y - 7 = 0$

We can use this strategy to develop a formula for the slope-point form for the equation of a line.

This line has slope m and passes through the point $P(x_1, y_1)$.
Another point on the line is $Q(x, y)$.



The slope, m, of the line is:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y - y_1}{x - x_1}$$ Multiply each side by $(x - x_1)$.

$$m(x - x_1) = (x - x_1)\left(\frac{y - y_1}{x - x_1}\right)$$ Simplify.

$$m(x - x_1) = y - y_1$$

$$y - y_1 = m(x - x_1)$$

Slope-Point Form of the Equation of a Linear Function

The equation of a line that passes through $P(x_1, y_1)$ and has slope m is:

$$y - y_1 = m(x - x_1)$$

6.5 Slope-Point Form of the Equation for a Linear Function

$m = \frac{y_2 - y_1}{x_2 - x_1}$

equation 1) equal sign
2) variables x, y
3) + - optional
4 numbers

$m = \frac{y - y_1}{x - x_1}$

$(x - x)\, m = \frac{y - y_1}{x - x_1}(x - x_1)$

$(x - x_1)m = y - y_1$

Slope - Point Formula...

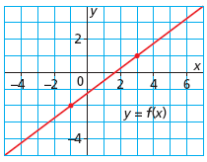
$$y - y_1 = m(x - x_1)$$

YOU NEED... 1) slope & 2) a point on the line

Example 2 Writing an Equation Using a Point on the Line and Its Slope

a) Write an equation in slope-point form for this line.

b) Write the equation in part a in slope-intercept form. What is the *y*-intercept of this line?



a) $m = \frac{\text{rise}}{\text{run}} = \frac{-3}{-4} = \frac{3}{4}$ $P_1 = (3,1)$ $y - y_1 = m(x - x_1)$ $y - 1 = \frac{3}{4}(x - 3)$

b) $4(y-1) = 3(x-3)$ $4y - 4 = 3x - 9$ $4y = 3x - 9 + 4$ $\frac{4y}{4} = \frac{3x}{4} - \frac{5}{4}$ $y = \frac{3}{4}x - \frac{5}{4}$

$y - 1 = \frac{3}{4}x - \frac{9}{4}$ $y = \frac{3}{4}x - \frac{9}{4} + 1$ $= \frac{3}{4}x - \frac{9}{4} + \frac{4}{4}$ $= \frac{3}{4}x - \frac{5}{4}$

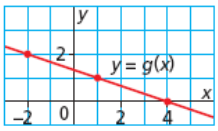
y-int $= -\frac{5}{4}$

6.5 Slope-Point Form of the Equation for a Linear Function

YOUR TURN...

2. a) Write an equation in slope-point form for this line.

b) Write the equation in part a in slope-intercept form. What is the y-intercept of this line?



a) $y-y_1=m(x-x_1)$

$m=\frac{\text{rise}}{\text{run}}$ $P_1(-2,2)$

$=\frac{-1}{3}$ $P_2(1,1)$

$P_3(4,0)$

$y-2=-\frac{1}{3}(x+2)$

$3(y-2)=-1(x+2)$

$3y-6=-x-2$

$3y=-x-2+6$

$\frac{3y}{3}=\frac{-x}{3}+\frac{4}{3}$

$y=-\frac{1}{3}x+\frac{4}{3}$

$y_{int}=\frac{4}{3}$ $(0,\frac{4}{3})$

EXAMPLE #3:

Determine the equation of the line that passes through (-1, 4) & (3, -12).

~~$y=mx+b$~~ no y-int

$y-y_1=m(x-x_1)$

$m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{4-(-12)}{-1-3}$

$=\frac{16}{-4}$

$=-4$

$y-y_1=m(x-x_1)$

$y-4=-4(x+1)$

$y-4=-4x-4$

$4x+y-4+4=0$

$4x+y=0$ General Form