Geometric Proofs... The 'Two-Column Proof'

Key Terms (in your notes)...

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deductive reasoning
Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

proof
A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

transitive property
If two quantities are equal to the same quantity, then they are equal to each other.
If $a = b$ and $b = c$, then $a = c$.

two-column proof
A presentation of a logical argument involving deductive reasoning in which the statements of the argument are written in one column and the justifications for the statements are written in the other column.

STATEMENT	JUSTIFICATION

***ADD this one to your notes...

converse

A statement that is formed by switching the premise and the conclusion of another statement.

Premise Conclusion

EXAMPLES...

Conjecture: If it is raining outside, then the grass is wet.

CONVERSE: If the grass is wet, then it is raining.

THEOREM: If you have parallel lines, then the corresponding angles are equal.

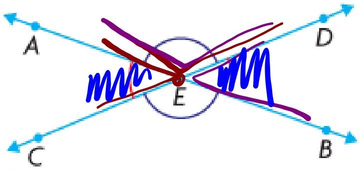
CONVERSE: If the corresponding angles are equal, then the lines are parallel.

Prove Parallel Lines

p. 29

EXAMPLE 4 Using deductive reasoning to prove a geometric conjecture

Prove that when two straight lines intersect, the vertically opposite angles are equal.



Jose's Solution: Reasoning in a two-column proof

Statement	Justification
$\angle AEC + \angle AED = 180°$	Supplementary angles SAT
$\angle AEC = 180° - \angle AED$	Subtraction property
$\angle BED + \angle AED = 180°$	Supplementary angles SAT
$\angle BED = 180° - \angle AED$	Subtraction property
$\angle AEC = \angle BED$	**Transitive property**

NOTE: There are 3 ways to Prove Parallel Lines..

1) CA (F Rule)
2) AIA (Z Rule)
3) CIA (C Rule)

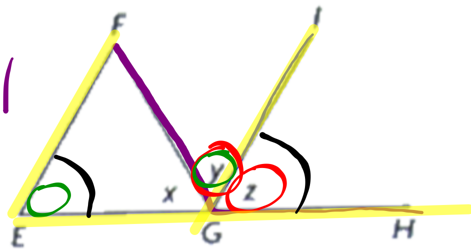
Example #2:

2 equal pieces

parallel

In ΔEFG, GI bisects $\angle FGH$

$a)$ *If* $\angle E = \angle y$, *then prove that* $EF \parallel GI$

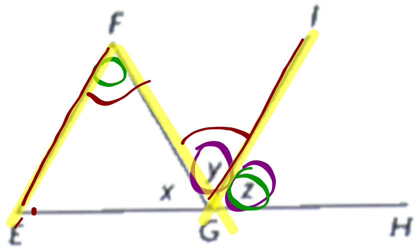


Statement	Justification
$\angle y = \angle z$	Given -> Bisect
$\angle E = \angle y$	Given
$\angle E = \angle z$	Transitive
$\therefore EF \parallel GI$	CA

Therefore

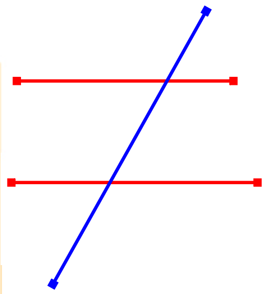
In ΔEFG, GI bisects $\angle FGH$

$b)$ *If* $\angle F = \angle z$, *then prove that* $EF \parallel GI$



Statement	Justification
$\angle y = \angle z$	Bisect
$\angle F = \angle z$	Given
$\angle y = \angle F$	Transitive
$\therefore EF \parallel GI$	AIA

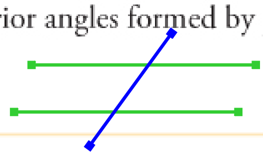
APPLY the Math



Pull for Lesson Notes

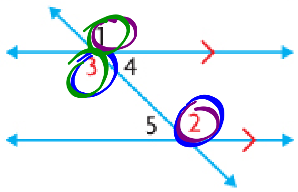
EXAMPLE 1 p. 75 — Reasoning about conjectures involving angles formed by transversals

Make a conjecture that involves the interior angles formed by parallel lines and a transversal. Prove your conjecture.



Tuyet's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the **alternate interior angles** are equal.



I drew two parallel lines and a transversal as shown, and I numbered the angles. I need to show that $\angle 3 = \angle 2$.

Statement	Justification
$\angle 1 = \angle 2$	Corresponding angles
$\angle 1 = \angle 3$	Vertically opposite angles
$\angle 3 = \angle 2$	Transitive property

My conjecture is proved.

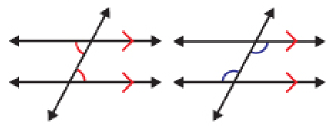
Since I know that the lines are parallel, the corresponding angles are equal.

When two lines intersect, the opposite angles are equal.

$\angle 2$ and $\angle 3$ are both equal to $\angle 1$, so $\angle 2$ and $\angle 3$ are equal to each other.

alternate interior angles

Two non-adjacent interior angles on opposite sides of a transversal.

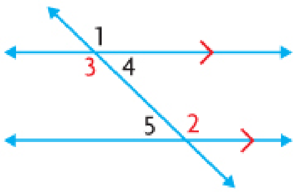


EXAMPLE 1 Reasoning about conjectures involving angles formed by transversals

Make a conjecture that involves the interior angles formed by parallel lines and a transversal. Prove your conjecture.

Ali's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the interior angles on the same side of the transversal are supplementary.



I need to show that $\angle 3$ and $\angle 5$ are supplementary.

$\angle 1 = \angle 2$

Since the lines are parallel, the corresponding angles are equal.

$\angle 2 + \angle 5 = 180°$

These angles form a straight line, so they are supplementary.

$\angle 1 + \angle 5 = 180°$

Since $\angle 2 = \angle 1$, I could substitute $\angle 1$ for $\angle 2$ in the equation.

$\angle 1 = \angle 3$

Vertically opposite angles are equal. Since $\angle 1 = \angle 3$, I could substitute $\angle 3$ for $\angle 1$ in the equation.

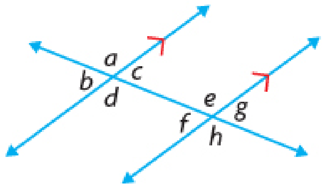
$\angle 3 + \angle 5 = 180°$

My conjecture is proved.

In Summary p. 78

Key Idea

- When a transversal intersects two parallel lines,
  - i) the corresponding angles are equal.
  - ii) the alternate interior angles are equal.
  - iii) the alternate exterior angles are equal.
  - iv) the interior angles on the same side of the transversal are supplementary.



i) $a = e, b = f$
$c = g, d = h$

ii) $c = f, d = e$

iii) $a = h, b = g$

iv) $c + e = 180°$
$d + f = 180°$

Need to Know

- If a transversal intersects two lines such that
  - i) the corresponding angles are equal, or
  - ii) the alternate interior angles are equal, or
  - iii) the alternate exterior angles are equal, or
  - iv) the interior angles on the same side of the transversal are supplementary,

  then the lines are parallel.

Homework...

p. 72: #4-6

p. 78: #2, 8, 10, 12, 20

Proof

* Thurs?

p. 104 1, 2

p. 106 1-5

Attachments

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