p. 90: #3, 5, 7, 9, 13 [from today's lesson]

Questions?

13. Use the given information to determine the measures of

70°

110° 40° 30°

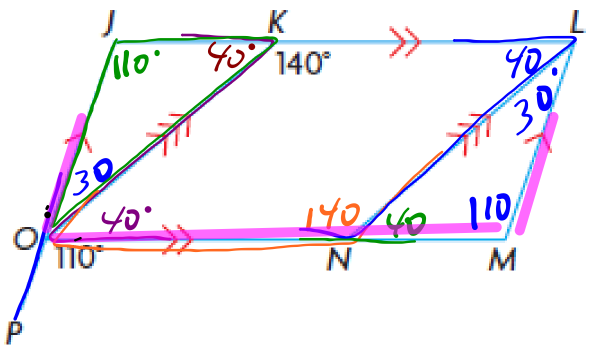
∠J, ∠JKO, ∠JOK, ∠KLM,

40° 110° 140° 40°

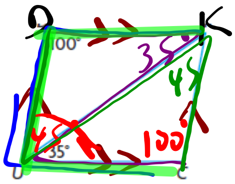
∠KLN, ∠M, ∠LNO, ∠LNM,

∠MLN, ∠NOK, and ∠JON.

30° 40° 70°

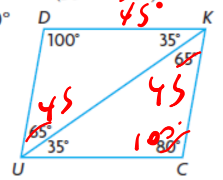


9. *DUCK* is a parallelogram. Benji determined the measures of the unknown angles in *DUCK*. Paula says he has made an error.



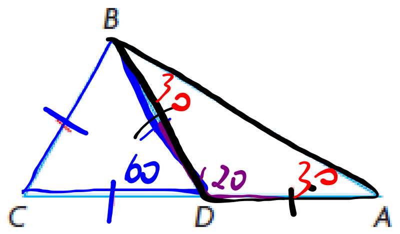
Benji's Solution

mistake

Statement	Justification
$\angle DKU = \angle KUC$	$\angle DKU$ and $\angle KUC$ are alternate interior angles.
$\angle DKU = 35°$	
$\angle UDK = \angle DUC$ — $\angle UDK + \angle DUC = 180°$	$\angle UDK$ and $\angle DUC$ are corresponding angles. co-interior
$\angle DUK + \angle KUC = \cancel{100°}$ $80°$	$\angle DUK$ and $\angle UKC$ are alternate interior angles.
$\angle DUK = \cancel{65°}$ 45	
$\angle UKC = \cancel{65°}$ $35°$	
$\angle UCK = 180° - (\angle KUC + \angle UKC)$	The sum of the measures of the angles in a triangle is 180°.
$\angle UCK = 180° - (35° + \cancel{65°})$ $45°$	
$\angle UCK = 80°$	
	I redrew the diagram, including the angle measures I determined.

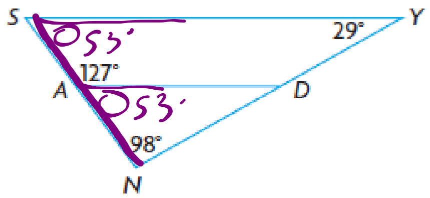
a) Explain how you know that Benji made an error.
b) Correct Benji's solution.

5. Prove: $\angle A = 30°$



Statement	Justification
$\angle BDC = 60°$	Equilateral
$\angle BDA = 120°$	SAT
$\angle A = 30°$	ITT & SATT

7. Prove: $SY \parallel AD$



8. Each vertex of a triangle has two

S	J
$\angle NSY = 53°$	SATT
$\angle ASY + \angle YSA = 180°$	Addition
$\therefore SY / AD$	CIA

or

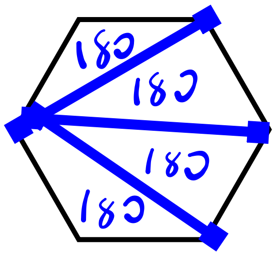
S	J
$\angle NSY = 53°$	SATT
$\angle NAD = 53°$	SAT
$\angle NSY = \angle NAD$	Transitive
$\therefore SY \parallel AD$	CA

2.4 Angle Properties in Polygons

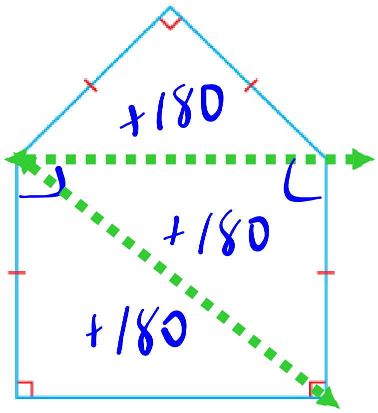
GOAL

Determine properties of angles in polygons, and use these properties to solve problems.

EXPLORE...



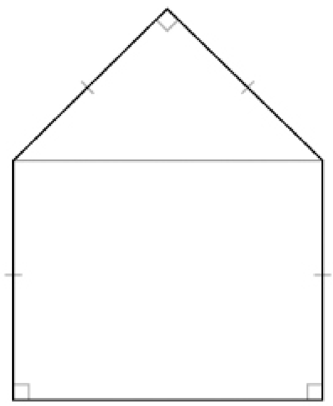
- A pentagon has three right angles and four sides of equal length, as shown. What is the sum of the measures of the angles in the pentagon?



TOTAL +540

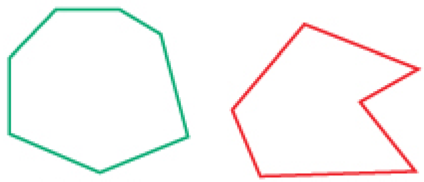
SAMPLE ANSWER

I drew a diagonal joining the two angles that are not right angles. This cut the pentagon into a rectangle and a triangle. I knew that the quadrilateral was a rectangle, not a trapezoid, because the two right angles share an arm, so their other arms must be parallel. As well, the other arms are equal length. I knew that the sum of the measures of the angles in a rectangle is 360° and the sum of the measures of the angles in a triangle is 180°, so the sum of the measures of the angles in the pentagon must be 540°.



convex polygon

A polygon in which each interior angle measures less than 180°.



convex non-convex (concave)

This is my conjecture: The sum of the measures of the interior angles in a polygon, $S(n)$, is:

$S(n) = 180°(n - 2)$ OR $\text{Sum} = 180°(n-2)$

Function Notation → $S(6) = 180°(6-2)$

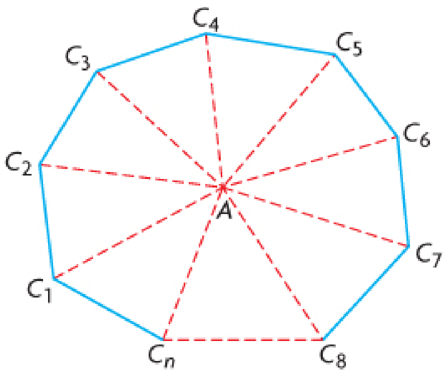
$= 720°$

APPLY the Math Deriving the formula...

EXAMPLE 1 Reasoning about the sum of the interior angles of a polygon

Prove that the sum of the measures of the interior angles of any n-sided **convex polygon** can be expressed as $180°(n - 2)$.

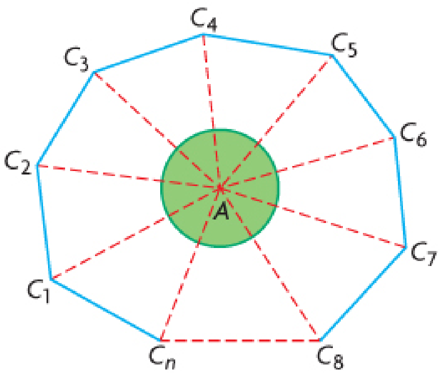
Viktor's Solution



I drew an n-sided polygon. I represented the nth side using a broken line. I selected a point in the interior of the polygon and then drew line segments from this point to each vertex of the polygon. The polygon is now separated into n triangles.

The sum of the measures of the angles in each triangle is 180°.

The sum of the measures of the angles in n triangles is $n(180°)$.



Two angles in each triangle combine with angles in the adjacent triangles to form two interior angles of the polygon.

Each triangle also has an angle at vertex A. The sum of the measures of the angles at A is 360° because these angles make up a complete rotation. These angles do not contribute to the sum of the interior angles of the polygon.

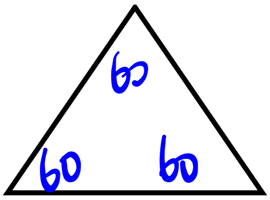
The sum of the measures of the interior angles of the polygon, $S(n)$, where n is the number of sides of the polygon, can be expressed as:

$S(n) = 180°n - 360°$

$S(n) = 180°(n - 2)$

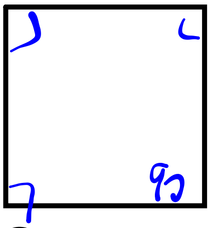
The sum of the measures of the interior angles of a convex polygon can be expressed as $180°(n - 2)$.

Regular Polygon → all angles / sides are equal

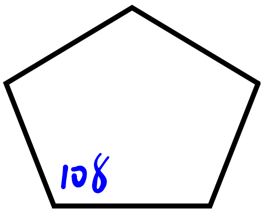


Equilateral

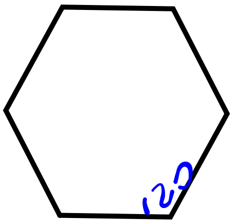
Triangle



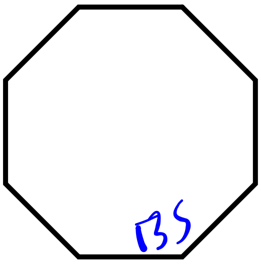
Square



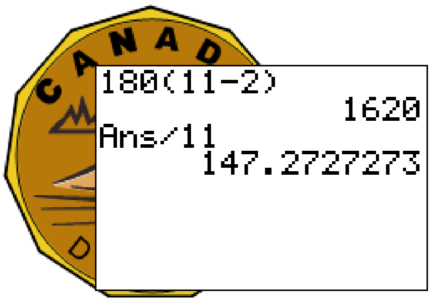
Pentagon



Hexagon



Octagon



Undecagon

[11 sided]

EXAMPLE 2 Reasoning about angles in a regular polygon

Outdoor furniture and structures like gazebos sometimes use a regular hexagon in their building plan. Determine the measure of each interior angle of a regular hexagon.

Nazra's Solution

Let $S(n)$ represent the sum of the measures of the interior angles of the polygon, where n is the number of sides of the polygon.

$$S(n) = 180°(n - 2)$$

A hexagon has six sides, so $n = 6$.

$$S(6) = 180°[(6) - 2]$$
$$S(6) = 720°$$
$$\frac{720°}{6} = 120°$$

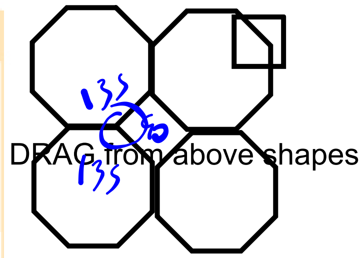
Since the measures of the angles in a regular hexagon are equal, each angle must measure $\frac{1}{6}$ of the sum of the angles.

The measure of each interior angle of a regular hexagon is 120°.

Tiling Using Regular Polygons...

Regular Polygon	Measure of Interior Angle (degrees)
Equilateral Triangle	60
Square	90
Pentagon	108
Hexagon	120
Heptagon (7 sided)	128.3
Octagon	135
Nonagon (9 sided)	140
Decagon (10 sided)	144

EXAMPLE 3 Visualizing tessellations



A floor tiler designs custom floors using tiles in the shape of regular polygons. Can the tiler use congruent regular octagons and congruent squares to tile a floor, if they have the same side length?

Vanessa's Solution

$S(n) = 180°(n - 2)$
$S(8) = 180°[(8) - 2]$
$S(8) = 1080°$
$\frac{1080°}{8} = 135°$

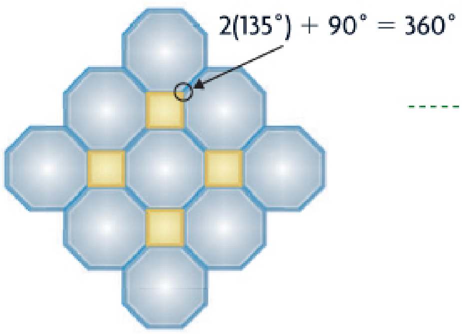
Since an octagon has eight sides, $n = 8$.

First, I determined the sum of the measures of the interior angles of an octagon. Then I determined the measure of each interior angle in a regular octagon.

The measure of each interior angle in a regular octagon is 135°.
The measure of each internal angle in a square is 90°.

Two octagons fit together, forming an angle that measures:
$2(135°) = 270°$.
This leaves a gap of 90°.
$2(135°) + 90° = 360°$
A square can fit in this gap if the sides of the square are the same length as the sides of the octagon.

I knew that three octagons would not fit together, as the sum of the angles would be greater than 360°.

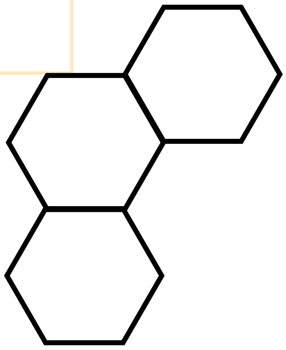


I drew what I had visualized using dynamic geometry software.

The tiler can tile a floor using regular octagons and squares when the polygons have the same side length.

Your Turn

Can a tiling pattern be created using regular hexagons and equilateral triangles that have the same side length? Explain.



Answer

In Summary

Key Idea

- You can prove properties of angles in polygons using other angle properties that have already been proved.

Need to Know

- The sum of the measures of the interior angles of a convex polygon with n sides can be expressed as $180°(n - 2)$.
- The measure of each interior angle of a regular polygon is $\frac{180°(n - 2)}{n}$.
- The sum of the measures of the exterior angles of any convex polygon is 360°.

HOMEWORK...

Page 99: 1, 3, 4, 5, 10, 11, 16

HISTORY on Buckyball Do A, B and C

Attachments

2s4e3 finalt.mp4