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UNIT 2: POWERS AND EXPONENT LAWS

SECTION 2.5: EXPONENT LAWS II

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MATH 9

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the Math 9 Specific Curriculum Outcome (SCO) "Numbers 1" OR "N1" which states:

"Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by: representing repeated multiplication using powers; using patterns to show that a power with an exponent of zero is equal to one; solving problems involving powers."

We will also continue working on the Math 9 Specific Curriculum Outcomes (SCOs) "Numbers 2" and "Numbers 4" OR "N2" and "N4" which state:

"Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents."

AND

"Explain and apply the order of operations, including exponents, with and without technology."

What does THAT mean???

SCO N1 means that we will learn about the two parts of a power (the base, or "the big number", and the exponent, or "the little number"). We will show what a power means when we write it out using multiplication (ex: $3^2 = 3 \times 3$), and we will use patterns to prove, for example, that $3^0 = 1$. Finally, we will use what we know about powers to solve problems.

SCO N2 means that we will learn rules to work with powers with integer bases (other than 0) and exponents of 0 or higher.

SCO N4 means that we will use order of operations (as always) to solve problems that include powers both with and without calculators.

WARM UP:

1. Write each product as a power, then evaluate the power.

a) $5^3 \times 5^4$ b) $(-2)^3 \times (-2)^2$

c) $3^2 \times 3^3 \times 3^1$ d) $-10^4 \times 10^0$

2. Write each quotient as a power, then evaluate the power.

a) $7^5 \div 7^3$ b) $(-10)^9 \div (-10)^3$

c) $\dfrac{8^4}{8^2}$ d) $-\dfrac{6^7}{6^4}$

HOMEWORK QUESTIONS???
(pages 77 / 78, #7, 8, 10, 13 and 15 TO 19)

SECTION 2.5: EXPONENT LAWS II
(Powers of Powers, Products, and Quotients)

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A power indicates repeated multiplication.

$2^3 \cdot 2^3$

What is the standard form of $(2^3)^2$?

$= 2^3 \text{ x } 2^3$

$= (2 \text{ x } 2 \text{ x } 2) \text{ x } (2 \text{ x } 2 \text{ x } 2)$

$= 2^6$

$= 64$

Copy and complete this table:

Question	Repeated Multiplication	Product of Factors	Power
$(2^4)^3$	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	$2^4 \times 2^4 \times 2^4$	2^{12}
$(3^2)^4$			3^8
$[(-4)^3]^2$			$(-4)^6$
$[(-5)^3]^5$			$(-5)^{15}$

We can raise a power to a power. For example, $(3^2)^4$ means 3^2 x 3^2 x 3^2 x 3^2.

Using the exponent law for the product of powers, we know that we can add the exponents, so:

$$3^2 \times 3^2 \times 3^2 \times 3^2$$
$$= 3^8$$

The exponent in 3^8 is the product of the exponents in $(3^2)^4$. That is:

$$(3^2)^4$$
$$= 3^{2 \times 4}$$
$$= 3^8$$

This leads us to our fourth exponent law...

4. **EXPONENT LAW FOR POWER OF A POWER:** To raise a power to a power, multiply the exponents. We express this law as:

* $(a^m)^n = a^{mn}$ *

("mn" means "m x n")

where "a" is any integer other than 0, and "m" and "n" are any whole numbers.

Ex.: $(2^4)^5 = 2^{\boxed{20}}$

$[(-4)^5]^3 = (-4)^{\boxed{15}}$

The base of a power may be a product; for example, $(2 \times 3)^4$.

What is the standard form of $(2 \times 3)^4$? Grade 10

6^4 $\quad (2 \times 3)^4 \quad 2^4 \times 3^4 \quad (x^2y^3)^5 = x^{10}y^{15}$

$= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3)$

$= 2^4 \times 3^4$

$= 16 \times 81$

$= 1296$

What is the standard form of $(2^2 \times 3^3)^4$?

$(2^2 \times 3^3)^4$ $2^8 \times 3^{12}$

$= (2^2 \times 3^3) \times (2^2 \times 3^3) \times (2^2 \times 3^3) \times (2^2 \times 3^3)$

$= (2^2 \times 2^2 \times 2^2 \times 2^2) \times (3^3 \times 3^3 \times 3^3 \times 3^3)$

$= 2^8 \times 3^{12}$

$= 256 \times 531\,441$

$= 136\,048\,896$

Copy and complete this table:

Question	Repeated Multiplication	Product of Factors	Product of Powers
$(2 \times 5)^3$	10^3		$2^3 \times 5^3$
$(3 \times 4)^2$	12^2		$3^2 \times 4^2$
$(4 \times 2)^5$	8^5		$4^5 \times 2^5$
$(5 \times 3)^4$	15^4		$5^4 \times 3^4$

We can raise a product to a power. For example, $(3 \times 4)^5$ means:

$$
\begin{aligned}
& (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \times (3 \times 4) \\
= \; & (3 \times 3 \times 3 \times 3 \times 3) \times (4 \times 4 \times 4 \times 4 \times 4) \\
= \; & 3^5 \times 4^5
\end{aligned}
$$

This leads us to our fifth exponent law...

5. EXPONENT LAW FOR POWER OF A PRODUCT: **To raise a product to a power, distribute the exponent to each part of the product. We express this law as:**

$$(ab)^m = a^m b^m$$

where "a" and "b" are any integers other than 0, and "m" is any whole number.

Ex.: $(2 \times 4)^5 = 2^{\boxed{5}} \times 4^{\boxed{5}}$

$[(-7) \times 5]^2 = (-7)^{\boxed{2}} \times 5^{\boxed{2}}$

$(2^2 \times 3^3)^4 = 2^{\boxed{8}} \times 3^{\boxed{12}}$

The base of a power may be a quotient; for example, $\left(\frac{5}{6}\right)^3$.

What is the standard form of $\left(\frac{5}{6}\right)^3$?

$$\frac{5\cdot 5\cdot 5}{6\cdot 6\cdot 6}$$

will not reduce.

$$\frac{5^3}{6^3}$$

$$\frac{125}{216}$$

What is the standard form of $\left(\frac{5^2}{6^3}\right)^4$?

$$= \frac{5^8}{6^{12}}$$

$$= \frac{390\,625}{2\,176\,782\,336}$$

We can use this to write our sixth (and final!) exponent law...

6. EXPONENT LAW FOR POWER OF A QUOTIENT: **To raise a quotient to a power, distribute the exponent to each part of the quotient. We express this law as:**

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

where "a" and "b" are any integers other than 0, and "n" is any whole number.

Ex.: $\left(\frac{1}{3}\right)^4 = \frac{1^{\boxed{4}}}{3^{\boxed{4}}}$

$$\left(\frac{5^2}{6^3}\right)^4 = \frac{5^{\boxed{8}}}{6^{\boxed{12}}}$$

Examples:

Write as a power.

a) $[(-7)^3]^2$

$= (-7)^6$

b) $-(2^4)^5$

$= -(2^{20})$

c) $(6^2)^7$

$= 6^{14}$

Examples:

Evaluate.

a) $[(-7) \times 5]^2$

$= (-35)^2$

$= 1225$

b) $[24 \div (-6)]^4$

$= (-4)^4$

$= 256$

c) $-(3 \times 2)^2$

$= -6^2$

$= -36$

Examples:

Simplify, then evaluate each expression.

a) $(3^2 \times 3^3)^3 - (4^3 \times 4^2)^2$

$(3^5)^3 - (4^5)^2$

$3^{15} - 4^{10}$

$14\,348\,907 - 1\,048\,576$

$13\,300\,331$

b) $(6 \times 7)^2 + (3^8 \div 3^6)^3$

$= (42)^2 + (3^2)^3$

$= 1764 + 3^6$

$= 1764 + 729$

$= 2493$

c) $[(-5)^3 + (-5)^4]^0$

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WARM-UP: **Simplify (as much as possible using exponent laws) then evaluate.**

$$\frac{\left(4^2\right)^4 \times \left(5^3\right)^2}{\left(5^2\right)^1 \times \left(4^3\right)^2} \times \frac{\left(4^3\right)^5 \times \left(5^3\right)^4}{\left(4^2\right)^6 \times \left(5^2\right)^5}$$

CONCEPT REINFORCEMENT:

MMS9:
PAGE 84: #4, 5, 6, 7, 8, 9, 11, 12, 13, and 14
PAGE 85: #16, 17, 19, and 21

TEST PREPARATION:

MMS9:
PAGE 86: Study Guide
PAGE 87: #1, 3, 4, 6, 8, and 9
PAGE 88: #12, 13, 14, and 17
PAGE 89: #18, 19, 20, 21, 22, 23, 24, 26, and 27
PAGE 90: Practice Test (#1 to #6)

RULE OF THUMB: When you see an exponent law possibility, use it; otherwise, follow BEDMAS.