Warm Up

Describe the 3 C's for Ambiguous Case...

Criteria:
- ✓ – SSA
- ✓ – given angle is acute
- ✓ – $a < b$

Calculation: $h = b\sin A$

Cases:

① $a < h$
No Solution

② $a = h$
1 Right Triangle

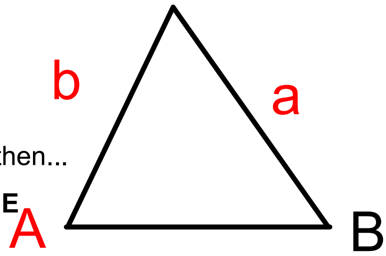
* ③ $a > h$
2 possible triangles
↳ Acute (Calc)
* ↳ Obtuse $(180° - \theta)$

MUST MEMORIZE THESE NOTES IN ORDER TO KNOW AMBIGUOUS CASE

Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b

*** If ALL 3 criteria are met, then...



CALCULATE THE ALTITUDE

$\boxed{\text{alt} = b \sin A}$

CASE 1: a < altitude; there is <u>NO SOLUTION</u>

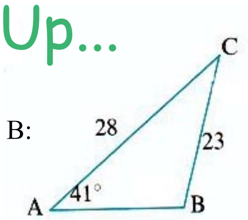
CASE 2: a = altitude; there is <u>ONE SOLUTION</u> [Right Triangle]

CASE 3: a >altitude; this is the <u>'AMBIGUOUS CASE'...TWO SOLUTIONS</u>

1) Acute Triangle (angle, θ, is found with Law of Sines)
2) Obtuse Triangle (angle is 180º - θ)

Back to the Warm-Up...

Determine the measure of the obtuse angle B:



4.3 The Ambiguous Case of the Sine Law

GOAL

Analyze the ambiguous case of the sine law, and solve problems that involve the ambiguous case.

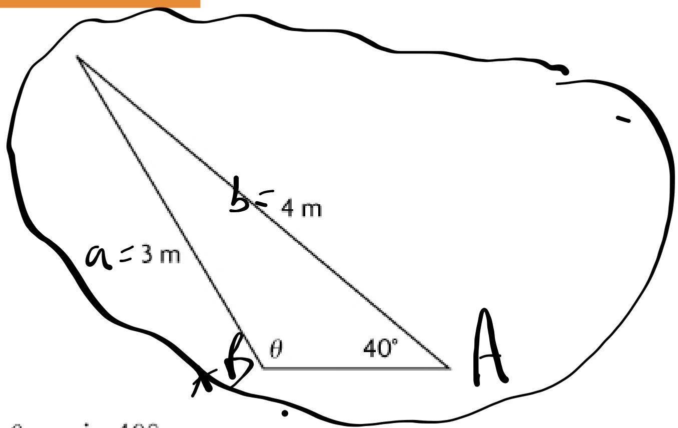
ambiguous case of the sine law

A situation in which two triangles can be drawn, given the available information; the ambiguous case may occur when the given measurements are the lengths of two sides and the measure of an angle that is not contained by the two sides (*SSA*).

EXPLORE...

- Two sides in an obtuse triangle are 3 m and 4 m in length. The angle that is opposite the 3 m side measures 40°. Determine the measure of the angle that is opposite the 4 m side.

SAMPLE ANSWER



$$\frac{\sin \theta}{4} = \frac{\sin 40°}{3}$$

$$\sin \theta = 0.8570\ldots$$

$$\theta = 58.9869\ldots°$$

This angle is not obtuse, so determine the supplementary angle.

$$\theta = 180 - 58.9869\ldots°$$

$$\theta = 121.0130\ldots°$$

The angle measure is about 121°.

✓ – SSA

✓ – acute

✓ – $a < b$

$\boxed{h = b \sin A}$

$h = 4 \sin 40°$

$h = 2.57$

$a < h$ $a = h$ $a > h$

2 solns

$3 > 2.57$

Notes - Ambiguous Case.pdf

In Summary

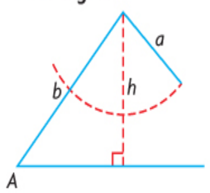
Key Idea

- The ambiguous case of the sine law may occur when you are given two side lengths and the measure of an angle that is opposite one of these sides. Depending on the measure of the given angle and the lengths of the given sides, you may need to construct and solve zero, one, or two triangles.

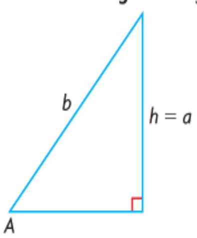
Need to Know

- In $\triangle ABC$ below, where h is the height of the triangle, $\angle A$ and the lengths of sides a and b are given, and $\angle A$ is acute, there are four possibilities to consider:

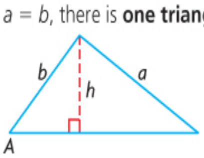
If $\angle A$ is acute and $a < h$, there is **no triangle.**



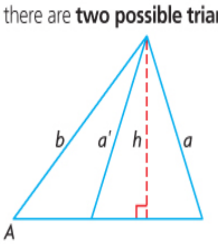
If $\angle A$ is acute and $a = h$, there is **one right triangle.**



If $\angle A$ is acute and $a > b$ or $a = b$, there is **one triangle.**

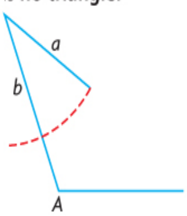


If $\angle A$ is acute and $h < a < b$, there are **two possible triangles.**

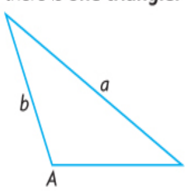


- If $\angle A$, a, and b are given and $\angle A$ is obtuse, there are two possibilities to consider:

If $\angle A$ is obtuse and $a < b$ or $a = b$, there is **no triangle.**



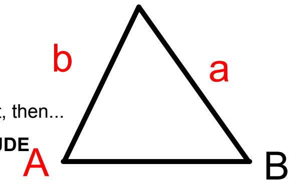
If $\angle A$ is obtuse and $a > b$, there is **one triangle.**



Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b

*** If ALL 3 criteria are met, then...



CALCULATE THE ALTITUDE

$\boxed{\text{alt} = b \sin A}$

CASE 1: a < altitude; there is NO SOLUTION

CASE 2: a = altitude; there is ONE SOLUTION [Right Triangle]

CASE 3: a >altitude; this is the 'AMBIGUOUS CASE'...TWO SOLUTIONS

1) Acute Triangle (angle, θ, is found with Law of Sines)
2) Obtuse Triangle (angle is 180º - θ)

EXAMPLE 1 Connecting the *SSA* situation to the number of possible triangles

p.177

✓- SSA
✓- acute
✓- $a < b$

Given each *SSA* situation for $\triangle ABC$, determine how many triangles are possible.

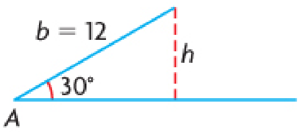
a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m
b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m
c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m
~~d)~~ $\angle A = 30°$, $a = 15$ m, and $b = 12$ m

$h = 12 \sin 30$
$h = 6$

d) 1 triangle

a) $a < h$ No solution b) $a = h$ 1 right △ c) $a > h$ 2 solutions

Saskia's Solution



I drew the beginning of a triangle with a 30° angle and a 12 m side.

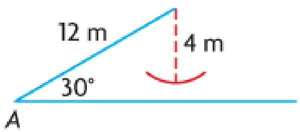
$$\sin 30° = \frac{h}{12}$$

I used the sine ratio to calculate the height of the triangle.

$$12 \sin 30° = h$$
$$6 \text{ m} = h$$

I can use this height as a benchmark to decide on side lengths opposite the 30° angle that will result in zero, one, or two triangles.

a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

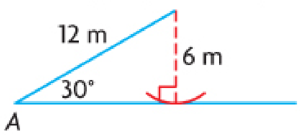


No triangles are possible.

Since $a < b$ and $a < h$, I knew that no triangles are possible.

I used a compass to be certain. I set the compass tips to represent 4 m. I placed one tip of the compass at the open end of the 12 m side and swung the pencil tip toward the other side. The pencil couldn't reach the base, so a 4 m side could not close the triangle.

b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m

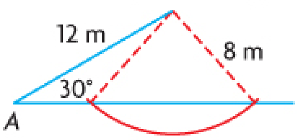


One triangle is possible.

Since $a < b$ and $a = h$, there is only one possible triangle, a right triangle.

A compass arc intersects the base at only one point.

c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m

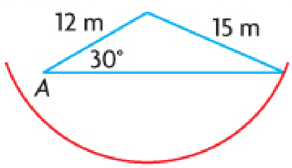


Two triangles are possible.

Since $a < b$ and $a > h$, there are two possible triangles.

A compass arc intersects the base at two points.

d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m



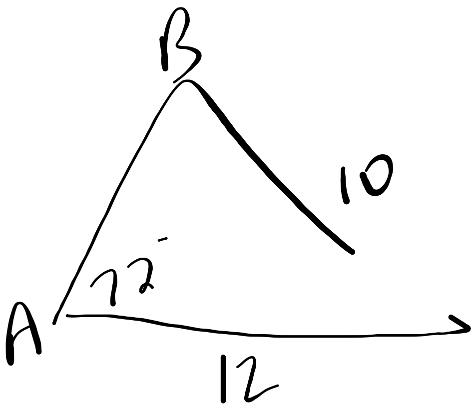
One triangle is possible.

Since $a > b$, only one triangle is possible.

A compass arc intersects the base at only one point.

Example 2:

Solve the triangle ABC if $a = 10$, $b = 12$ and angle A = 72°.



* SSA

✓ – acute

✓ – $a < b$

$h = 12 \sin 72^{\circ}$

$h = 11.4$

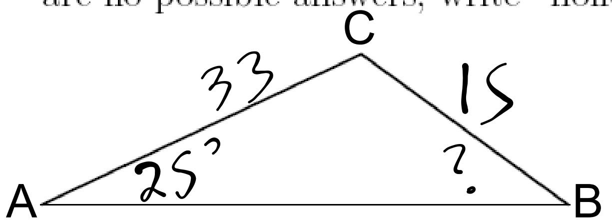
a vs h

$10 < 11.4$

No Solution

Example 3:

Given that A $= 25°$, $a = 15$, and $b = 33$, find the measure of angle B to the nearest degree. If there are two answers, give both of them. If there are no possible answers, write "none".



$$\frac{33 \sin B}{\cancel{33}} = \frac{33 \sin 25}{15}$$

$$\sin^{-1} \sin B = \sin^{-1}(0.9298)$$

$$\boxed{\begin{array}{c} \measuredangle B = 68° \\ \text{or} \\ \angle B = 180 - 68 \\ \angle B = 112° \end{array}}$$

*SSA

✓- acute angle

✓ $a < b$

$h = 33 \sin 25$

$h = 13.9$

a vs h

$15 > 13.9$

$\measuredangle$ ambiguous

HOMEWORK...

Worksheet - Ambiguous Case.pdf

Do questions #1, 2 & 4

MEMORIZE!!! → Quiz

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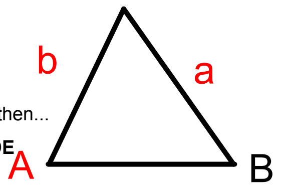

$\boxed{\text{alt} = b \sin A}$

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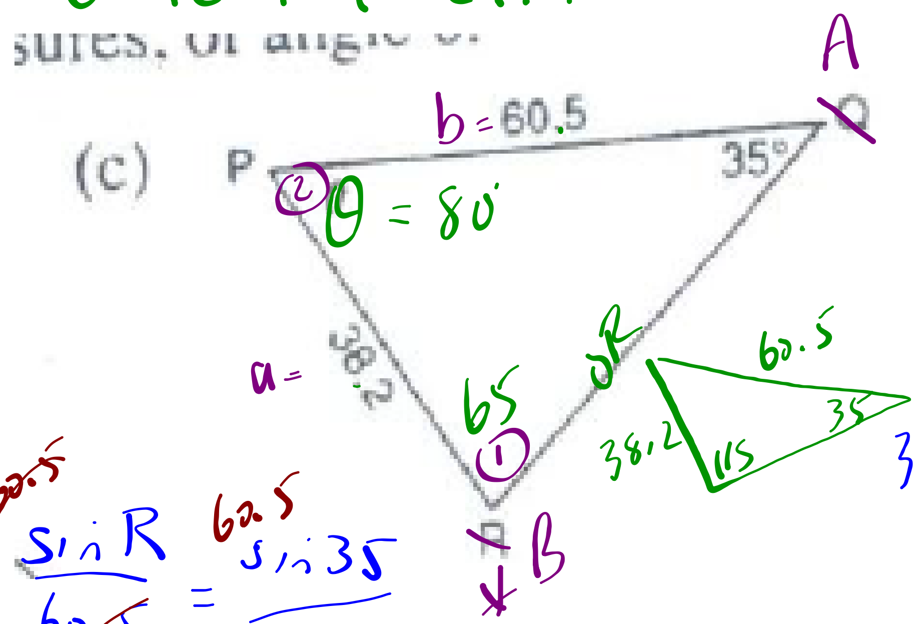
One More...



✓SSA
✓acute
✓$a < b$

$h = 60.5 \sin 35°$

$h = 34.7$

a vs h

$38.2 > 34.7$

∡ambiguous

$$\frac{\sin R}{60.5} = \frac{\sin 35}{38.2}$$

$$\sin R = 0.9084$$

$$\sin^{-1}(0.9084)$$

$\measuredangle R = 65° \rightarrow$ $\boxed{\theta = 80°}$

OR

$\angle R = 115° \rightarrow$ $\boxed{\theta = 30°}$

Attachments

Notes - Ambiguous Case.pdf

Worksheet - Ambiguous Case.pdf