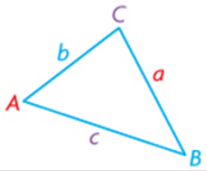
Trigonometry Summary AND 'The AMBIGUOUS Case'...



sine law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine law

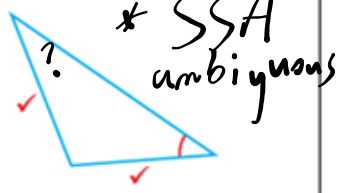
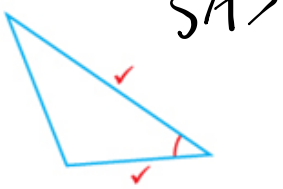
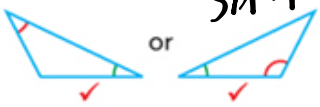
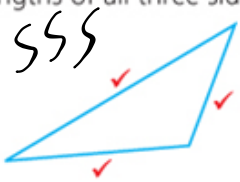
$$a^2 = b^2 + c^2 - 2bc \cos A$$

oblique triangle

A triangle that does not contain a 90° angle.

Need to Know

- The sine law and cosine law are used with obtuse triangles in the same way that they are used with acute triangles.

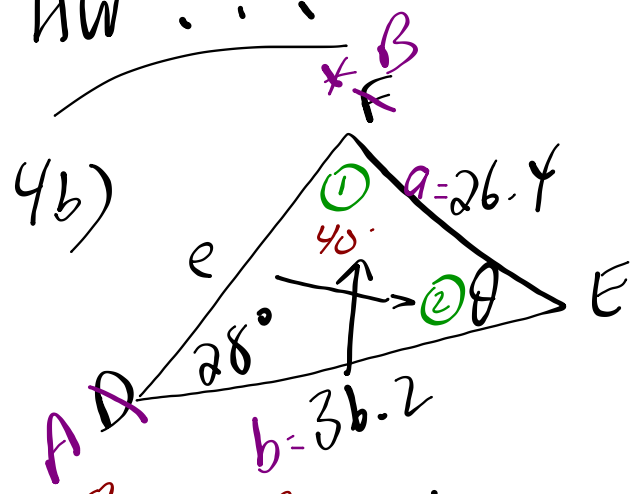
Use the sine law when you know ...	Use the cosine law when you know ...
- the lengths of two sides and the measure of the angle that is opposite a known side 	- the lengths of two sides and the measure of the contained angle 
- the measures of two angles and the length of any side 	- the lengths of all three sides 

Ambiguous Case

- Be careful when using the sine law to determine the measure of an angle. The inverse sine of a ratio always gives an acute angle, but the supplementary angle has the same ratio. You must decide whether the acute angle, θ, or the obtuse angle, $180° - \theta$, is the correct angle for your triangle.

HW ???

4b)



① $\frac{\sin F}{36.2} = \frac{36.2 \sin 28°}{26.4}$

$\sin F =$

36.2sin(28)/26.4
.6437450975
$\sin^{-1}$(Ans
40.07165227

$\angle F = 40°$

② $\theta = 180 - 40 - 28°$

$\theta = 112°$

③ $\frac{e}{\sin 112} = \frac{36.2 \sin 112}{\sin 40}$

$e = 52.2$

✓ * SSA
✓ - acute angle
- a < b

$h = 36.2 \sin 28°$

$h = 17$

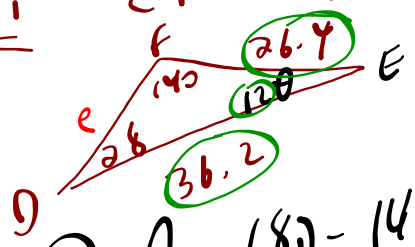
a vs h

26.4 > 17

* ambiguous!

0:2

$\angle F = 140°$



② $\theta = 180 - 140 - 28$

$\theta = 12°$

③ $e^2 = 36.2^2 + 26.4^2 - 2(36.2)(26.4)\cos 12°$

$e^2 =$

36.2²+26.4²-2*36
.2*26.4cos(12
137.8078019
√(Ans
11.73915678

$e =$

$e = 11.7$

Example 4: Solving a problem using the sine law

Martina and Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of 36.0° with the ground. Carl's rope is 5.9 m long. Assuming that Martina and Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina and Carl, to the nearest tenth of a metre?

P. 17

$h = 7.8 \sin 36°$

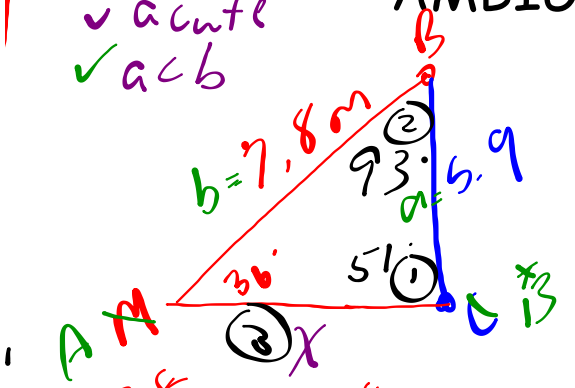
$h = 4.6$

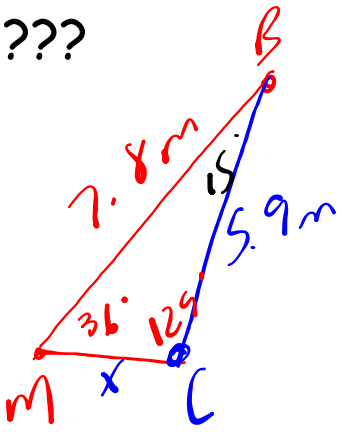
a vs h

$5.9 > 4.6$ *ambiguous

✓*SSA
✓acute
✓a<b

AMBIGUOUS???





① $\frac{\sin C}{7.8} = \frac{\sin 36°}{5.9}$

$\sin C =$ 7.8sin(36)/5.9 → .7770720285

$\angle C$ sin⁻¹(Ans → 50.99326934

*$\angle C = 51°$

or

$\angle C = 180 - 51°$

① $\angle C = 129°$

② $\angle B = 93°$

② $\angle B = 15°$

③ $\frac{x}{\sin 93°} = \frac{5.9}{\sin 36°}$ sins

③ $\frac{x}{\sin 15°} = \frac{5.9}{\sin 36°}$ sins

$x = 10.0$ m

$x = 2.6$m

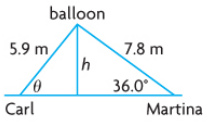
Example 4: Solving a problem using the sine law

Martina and Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of 36.0° with the ground. Carl's rope is 5.9 m long. Assuming that Martina and Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina and Carl, to the nearest tenth of a metre?

Sandra's Solution: Using the sine law and then the cosine law

Let h represent the height of the weather balloon.
Let θ represent the angle for Carl's rope.

Situation 1:

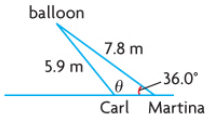


$$\sin 36.0 = \frac{h}{7.8}$$
$$7.8(\sin 36.0) = 7.8\left(\frac{h}{7.8}\right)$$
$$4.5847\ldots = h$$

I drew the triangle.

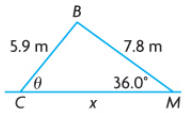
I noticed that this is a *SSA* situation. I had to determine the height of the triangle to determine if this is an ambiguous case.

Situation 2:



Carl's rope is longer than the height and shorter than Martina's rope, so there are two possible triangles. I drew the second triangle.

Situation 1:



$$\frac{\sin\theta}{7.8} = \frac{\sin 36^\circ}{5.9}$$
$$\sin\theta = \frac{7.8\sin 36^\circ}{5.9}$$
$$\sin\theta = 0.7770\ldots$$
$$\theta = \sin^{-1}(0.7770\ldots)$$
$$\theta = 50.9932\ldots^\circ$$

I substituted the side lengths and angles (including θ) into the formula for the sine law and isolated θ.

$$\angle B = 180^\circ - 36.0^\circ - 50.9932\ldots^\circ$$
$$\angle B = 93.0067\ldots^\circ$$

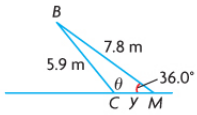
The measures of the angles in a triangle sum to 180°.

$$x^2 = 5.9^2 + 7.8^2 - 2(5.9)(7.8)\cos 93.0067\ldots^\circ$$
$$x^2 = 100.4777\ldots$$
$$x = 10.0238\ldots$$

In Situation 1, Martina and Carl are 10.0 m apart.

I used the cosine law to determine the distance, x, between Martina and Carl. I substituted the known measurements into the cosine law.

Situation 2:



I also considered the situation in which Carl is closer to Martina.

$$\frac{\sin\theta}{7.8} = \frac{\sin 36^\circ}{5.9}$$
$$\sin\theta = \frac{7.8\sin 36^\circ}{5.9}$$
$$\sin\theta = 0.7770\ldots$$
$$\theta = \sin^{-1}(0.7770\ldots)$$
$$\theta = 50.9932\ldots^\circ$$

I used the sine law to determine θ.

$$\theta = 180^\circ - 50.9932\ldots^\circ$$
$$\theta = 129.0067\ldots^\circ$$

I determined the measure of the supplementary angle, which is suitable for this situation.

$$\angle B = 180^\circ - 36.0^\circ - 129.0067\ldots^\circ$$
$$\angle B = 14.9932\ldots^\circ$$

The measures of the angles in a triangle sum to 180°.

$$y^2 = 5.9^2 + 7.8^2 - 2(5.9)(7.8)\cos 14.9932\ldots^\circ$$
$$y^2 = 6.7433\ldots$$
$$y = 2.5968\ldots$$

I can use $\angle B$ in the cosine law to determine the distance, y, between Martina and Carl.

I substituted the measure of $\angle B$ and the given side lengths into the cosine law.

In the second situation, Martina and Carl are 2.6 m apart.
Martina and Carl are either 10.0 m apart or 2.6 m apart.

HOMEWORK...

Worksheet - Ambiguous Case.pdf

#5, 6
Page 184: #7, 8

Attachments

Worksheet - Ambiguous Case.pdf