

September, 2017

UNIT 1: ROOTS AND POWERS

**SECTION 3.1:
FACTORS AND MULTIPLES
OF WHOLE NUMBERS**

K. Sears

NUMBERS, RELATIONS AND FUNCTIONS 10



WHAT'S THE POINT OF TODAY'S LESSON?

We will begin working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 1" OR "AN1" which states:

"Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root and cube root."



What does THAT mean???

SCO AN1 means that we will:

- * find the prime factors of whole numbers like 8 (2^3 or $2 \cdot 2 \cdot 2$)
- * determine the greatest common factor (GCF) of numbers like 28 and 49 (GCF = 7)
- * determine the least common multiple (LCM) of numbers like 6 and 9 (LCM = 18)
- * determine if a given whole number is a perfect square, like 25 ($5 \cdot 5$) or a perfect cube, like 8 ($2 \cdot 2 \cdot 2$)
- * determine the square root of perfect squares, like 36 ($\sqrt{36} = 6$), and the cube root of perfect cubes, like 64 ($\sqrt[3]{64} = 4$)



WARM UP:

Use prime factorization to determine the

a) GCF of 856, 1200 and 1368

b) LCM of 28, 42 and 63

WARM UP:

$$\begin{array}{c} 10 \times 12 \\ \swarrow \downarrow \\ 12 \times 10 \end{array}$$

a) GCF of 856, 1200 and 1368:

GCF = Greatest Common Factor

Method	856	1200	1368
I	1x 856	1x 1200	1x 1368
	2x 428	2x 600	2x 684
	4x 214	3x 400	3x 456
	8x 107	4x 300	4x 342
	⋮	5x 240	6x 228
	⋮	6x 200	8x 171
	⋮	8x 150	9x 152
	⋮	10x 120	
	continued		

Method II

$$\begin{array}{c} 856 \\ \wedge \\ 4 \times 214 \\ \wedge \quad \wedge \\ 2 \times 2 \times 2 \times 107 \\ 2^3 \times 107 \end{array}$$

$$\begin{array}{c} 1200 \\ \wedge \\ 4 \times 300 \\ \wedge \quad \wedge \\ 2 \times 2 \times 2 \times 150 \\ \wedge \quad \wedge \quad \wedge \\ 2 \times 2 \times 2 \times 2 \times 75 \\ \wedge \quad \wedge \quad \wedge \\ 2 \times 2 \times 2 \times 2 \times 25 \times 3 \\ \wedge \quad \wedge \quad \wedge \\ 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 3 \\ 2^4 \times 3 \times 5^2 \end{array}$$

$$\begin{array}{c} 1368 \\ \wedge \\ 8 \times 171 \\ \wedge \\ 2 \times 4 \times 3 \times 57 \\ \wedge \quad \wedge \quad \wedge \\ 2 \times 2 \times 2 \times 3 \times 57 \\ 2^3 \times 3 \times 57 \end{array}$$

$$\begin{aligned} \text{GCF} &= 2^3 \\ &= 8 \end{aligned}$$

WARM UP:

b) LCM of 28, 42 and 63:

LCM = Least Common Multiple

28

42

63

Method I

28 56 84 112 140 168 196 224 252

42 84 126 168 210 252

63 126 189 252

Method II

28
 \wedge
4x7
 \wedge
2x2x7
 \wedge
2²x7

42
 \wedge
6x7
 \wedge
2x3x7

63
 \wedge
7x9
 \wedge
7x3x3
 \wedge
3²x7

$$\begin{aligned} \text{LCM} &= 2^2 \times 3^2 \times 7 \\ &= 252 \end{aligned}$$

Choose higher power
for each number

WARM UP:

a) What is the GCF of 340 and 380?

b) Reduce the fraction $\frac{340}{380}$ to its simplest form.

c) What is the LCM of 340 and 380?

d) What is $\frac{15}{340} + \frac{10}{380}$?

$$\begin{array}{c} 340 \\ \wedge \\ 10 \times 34 \\ \wedge \quad \wedge \\ 2 \times 5 \times 2 \times 17 \\ \hline 2^2 \times 5 \times 17 \end{array}$$

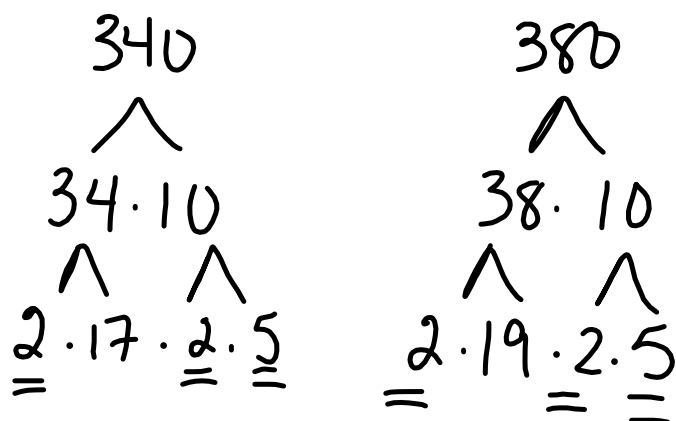
$$\begin{array}{c} 380 \\ \wedge \\ 10 \times 38 \\ \wedge \quad \wedge \\ 2 \times 5 \times 2 \times 19 \\ \hline 2^2 \times 5 \times 19 \end{array}$$

$$\begin{aligned} \text{GCF} &= 2^2 \times 5 \\ &= 4 \times 5 \\ &= 20 \end{aligned}$$

$$\begin{aligned} \text{LCM} &= 2^2 \times 5 \times 17 \times 19 \\ &= 6460 \end{aligned}$$

$$\begin{array}{l} 340 = 2 \times 2 \times 5 \times 17 \\ 380 = 2 \times 2 \times 5 \times 19 \end{array}$$

a) GCF



$$\begin{aligned} \text{GCF} &= 2 \cdot 2 \cdot 5 \\ &= 4 \cdot 5 \\ &= 20 \end{aligned}$$

$$\begin{aligned} b) \quad & \frac{340}{380} \quad \div 20 \\ & \div 20 \\ & = \frac{17}{19} \end{aligned}$$

c) LCM

$$\begin{array}{c}
 340 \\
 \wedge \\
 34 \cdot 10 \\
 \wedge \quad \wedge \\
 2 \cdot 17 \cdot 2 \cdot 5
 \end{array}$$

$$\begin{array}{c}
 380 \\
 \wedge \\
 38 \cdot 10 \\
 \wedge \quad \wedge \\
 2 \cdot 19 \cdot 2 \cdot 5
 \end{array}$$

$$2^2 \cdot 5 \cdot 17$$

$$2^2 \cdot 5 \cdot 19$$

$$\begin{aligned}
 \text{LCM} &= 2^2 \cdot 5 \cdot 17 \cdot 19 \\
 &= 4 \cdot 5 \cdot 17 \cdot 19 \\
 &= 20 \cdot 17 \cdot 19 \\
 &= 340 \cdot 19 \\
 &= 6460
 \end{aligned}$$

$$d) \quad \frac{15}{340} \begin{array}{l} \times 19 \\ \times 19 \end{array} + \frac{10}{380} \begin{array}{l} \times 17 \\ \times 17 \end{array}$$

$$= \frac{285}{6460} + \frac{170}{6460}$$

$$= \frac{455}{6460} \div 5$$

$$= \frac{91}{1292}$$

VOCABULARY:

prime number: a whole number with exactly two factors, itself and 1.

EX.: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29 are the first 10 prime numbers.

composite number: a number with three or more factors.

EX.: 8 is a composite number; it has four factors (1, 2, 4 and 8).

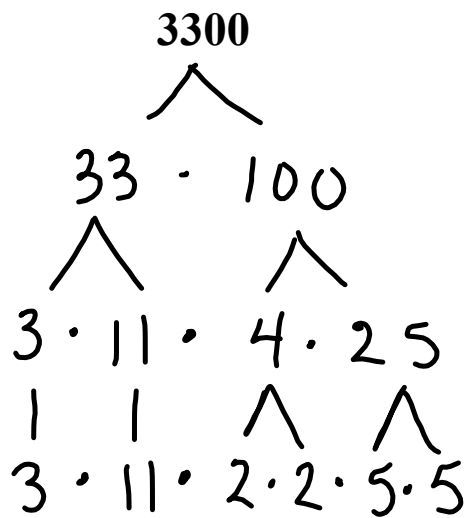
prime factor: a prime number that is a factor of a number.

EX.: The prime factors of 30 are 2, 3, and 5.

prime factorization: writing a number as a product of its prime factors.

EX.: The prime factorization of 20 is $2 \cdot 2 \cdot 5$ or $2^2 \cdot 5$.

USING FACTOR TREES FOR PRIME FACTORIZATION:



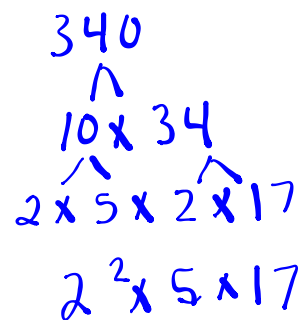
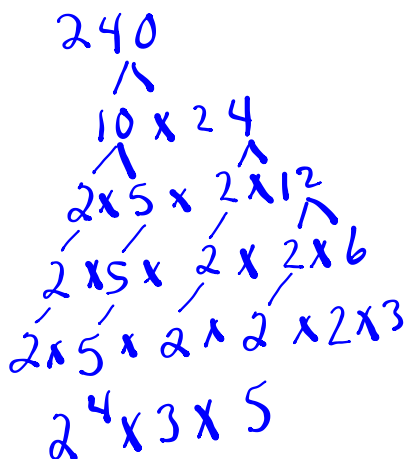
$$P.F. = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 11$$

$$\begin{array}{c} \text{OR} \\ 2^2 \cdot 3 \cdot 5^2 \cdot 11 \end{array}$$

Warm Up

Find the GCF and LCM of the following numbers using prime factorization (factor tree).

180, 240, 340



$$\begin{aligned}
 \text{GCF} &= 2^2 \times 5 \\
 &= 20
 \end{aligned}$$

$$\begin{aligned}
 \text{LCM} &= 2^4 \times 3^2 \times 5 \times 17 \\
 &= 12240
 \end{aligned}$$

$$2 \times 2 \times \left(\begin{array}{c} 2 \\ 2 \\ 2 \end{array} \right) \times \left(\begin{array}{c} 2 \\ 2 \end{array} \right) \times 3 \times 3 \times \left(\begin{array}{c} 5 \\ 5 \\ 5 \end{array} \right) \times 17$$

USING FACTOR TREES FOR PRIME FACTORIZATION:

YOU TRY! :)

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DETERMINING THE GREATEST COMMON FACTOR:

EX.: Determine the GCF of 12 and 15.

$$\begin{array}{l} 12: 1, 2, 3, 4, 6, 12 \\ 15: 1, 3, 5, 15 \end{array} \left. \vphantom{\begin{array}{l} 12: 1, 2, 3, 4, 6, 12 \\ 15: 1, 3, 5, 15 \end{array}} \right\} \text{GCF} = 3$$

DETERMINING THE GREATEST COMMON FACTOR:

EX.: You try! Determine the GCF of 18 and 24.

$$\begin{array}{l} 18: 1, 2, 3, 6, 9, 18 \\ 24: 1, 2, 3, 4, 6, 8, 12, 24 \end{array} \left. \vphantom{\begin{array}{l} 18: 1, 2, 3, 6, 9, 18 \\ 24: 1, 2, 3, 4, 6, 8, 12, 24 \end{array}} \right\} \text{GCF} = 6$$

**DETERMINING THE GREATEST COMMON FACTOR
FOR LARGER NUMBERS: PRIME FACTORIZATION**

EX.: Determine the GCF of 138 and 198.

**Once you have the prime factorization of the numbers,
multiply all the prime factors they have in common to
determine their GCF.**

**DETERMINING THE GREATEST COMMON FACTOR
FOR LARGER NUMBERS: PRIME FACTORIZATION**

EX.: You try! Determine the GCF of 126 and 144.

DETERMINING THE LOWEST COMMON MULTIPLE:

EX.: Determine the LCM of 8 and 12.

8: 8, 16, 24, 32, 40 }
12: 12, 24, 36, 48 } LCM = 24

DETERMINING THE LOWEST COMMON MULTIPLE:

EX.: You try! Determine the LCM of 6 and 10.

6: 6, 12, 18, 24, 30, 36
10: 10, 20, 30, 40, 50

} LCM = 30

**DETERMINING THE LOWEST COMMON MULTIPLE
FOR LARGER NUMBERS OR GROUPS OF NUMBERS:
PRIME FACTORIZATION**

EX.: Determine the LCM of 18, 20 and 30.

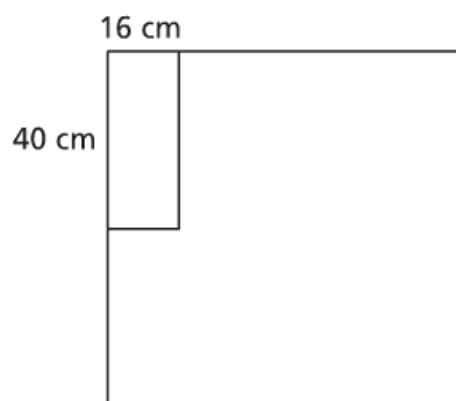
**Once you have the prime factorization of the numbers,
multiply the greatest power of each of the prime factors to
determine the LCM.**

**DETERMINING THE LOWEST COMMON MULTIPLE
FOR LARGER NUMBERS OR GROUPS OF NUMBERS:
PRIME FACTORIZATION**

EX.: You try! Determine the LCM of 28, 42 and 63.

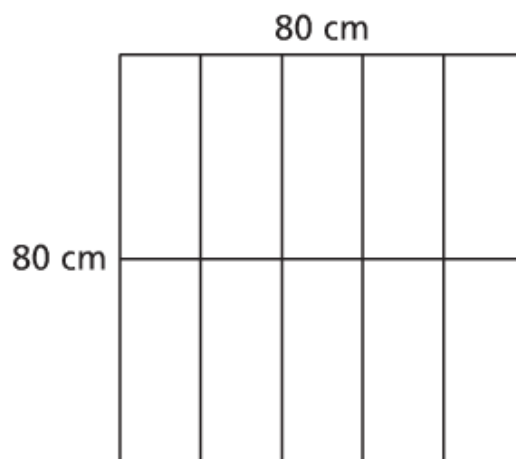
SOLVING PROBLEMS THAT INVOLVE GCF AND LCM:

EX.: What is the side length of the **SMALLEST** square that could be tiled with rectangles that measure 16 cm by 40 cm? (Assume the tiles cannot be cut.) Sketch the square and the rectangular tiles.



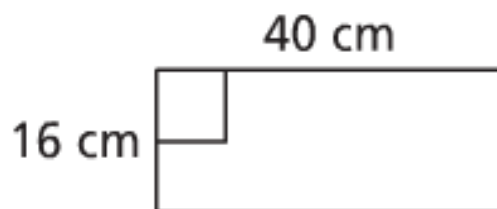
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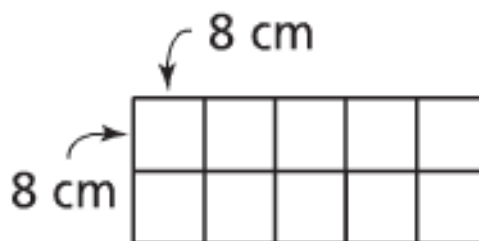
SOLVING PROBLEMS THAT INVOLVE GCF AND LCM:

EX.: What is the side length of the **LARGEST** square that could be used to tile a rectangle that measures 16 cm by 40 cm? (Assume the tiles cannot be cut.) Sketch the rectangle and the square tiles.



SOLVING PROBLEMS THAT INVOLVE GCF AND LCM:

EX.: What is the side length of the **LARGEST** square that could be used to tile a rectangle that measures 16 cm by 40 cm? (Assume the tiles cannot be cut.) Sketch the rectangle and the square tiles.



CONCEPT REINFORCMENT:

**"FOUNDATIONS AND PRE-CALCULUS -
MATHEMATICS 10" (FPCM 10)**

pages 140 / 141: #3 TO #13, #15 TO #19a and #20