September 14, 2017

UNIT 1: ROOTS AND POWERS

SECTION 4.3: MIXED AND ENTIRE RADICALS

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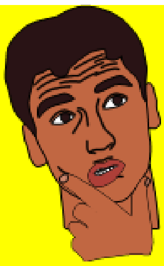
NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will begin working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."



What does THAT mean???

SCO AN3 means that we will:

* apply the 6 exponent laws you learned in grade 9:
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$
* apply all exponent laws to evaluate a variety of expressions
* express powers with rational exponents as radicals and vice versa
* identify and correct errors in work that involves powers

HOMEWORK QUESTIONS???
(page 211, #3 TO #5, #7, #8, #10 and #12 TO #14)

RADICALS CAN BE WRITTEN AS PRODUCTS: ...

EX.: $\sqrt{16 \cdot 9}$ **AND** $\sqrt{16} \cdot \sqrt{9}$

$= \sqrt{144}$ $= 4 \cdot 3$

$= 12$ $= 12$

JUST AS WITH FRACTIONS, RADICALS CAN BE WRITTEN AS EQUIVALENT EXPRESSIONS:

EX.: $\sqrt[3]{8 \cdot 27}$ **AND** $\sqrt[3]{8} \cdot \sqrt[3]{27}$

$= \sqrt[3]{216}$ $= 2 \cdot 3$

$= 6$ $= 6$

EXPONENT LAWS (separate sheet):

1. **Zero Exponent Law:** $a^0 = 1$ $2^0 = 1$

2. **Product of Powers:** $(a^m)(a^n) = a^{m+n}$

$3^2 \cdot 3^3 = 3^5$

3. **Quotient of Powers:** $a^m \div a^n = a^{m-n}$

4. **Power of a Power:** $(a^m)^n = a^{mn}$

5. **Power of a Product:** $(ab)^m = a^m b^m$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

6. **Power of a Quotient:** $(a \div b)^n = a^n \div b^n$

7. MULTIPLICATION ~~PROPERTY~~ OF RADICALS:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\quad (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}}$ (#5)

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ (MIXED RADICAL) like mixed fraction $2\frac{1}{2}$

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$
$$= \sqrt[3]{8}$$
$$= 2$$

$\sqrt{25}$

$= 25^{\frac{1}{2}}$

$= 5$

$9^{\frac{1}{2}}$

$= \sqrt{9}$

$= 3$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^{m} = \left(\sqrt[n]{x}\right)^{m}$$

INDEX

AND

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{m}\right)^{\frac{1}{n}} = \sqrt[n]{x^{m}}$$

INDEX

EX.: Evaluate $16^{\frac{3}{2}}$.

$16^{\frac{3}{2}}$ 3 (EXPONENT), 2 (INDEX)

$=\left(\sqrt[2]{16}\right)^{3}$

$= 4^3$

$= 64$

OR

$16^{\frac{3}{2}}$ 3 (EXP.), 2 (INDEX)

$=\sqrt[2]{16^{3}}$

$=\sqrt{4096}$

$= 64$

NOTE: THERE ARE SOME RADICALS THAT CANNOT BE SIMPLIFIED.

EX.: $\sqrt[4]{24}$ **(Factors: 1, 2, 3, 4, 6, 8, 12, 24)**

24 HAS NO FACTORS OTHER THAN 1 THAT CAN BE WRITTEN AS A FOURTH POWER; THEREFORE, IT CANNOT BE SIMPLIFIED (WRITTEN AS A MIXED RADICAL).

WE CAN ALSO USED PRIME FACTORIZATION TO SIMPLIFY A RADICAL.

EX.: Simplify each radical.

a) $\sqrt{80}$

$2^2=4$
$3^2=9$
$4^2=16$

$\sqrt{4\times20}$

$\sqrt{4\times4\times5}$

$4\sqrt{5}$

$\sqrt{16\times5}$

$4\sqrt{5}$

b) $\sqrt[3]{144}$

$2^3=8$

$\sqrt[3]{8\times18}$

$2\sqrt[3]{18}$

$2\sqrt[3]{2\times9}$

$2\sqrt[3]{2\times3\times3}$

$2\sqrt[3]{18}$

c) $\sqrt[4]{162}$

$\sqrt[4]{2\times81}$

$\sqrt[4]{2\times9\times9}$

$\sqrt[4]{2\times3\times3\times3\times3}$

$3\sqrt[4]{2}$

YOU TRY!

EX.: Simplify each radical.

a) $\sqrt{63}$

$\sqrt{7 \cdot 9}$

$\sqrt{7 \cdot 3 \cdot 3}$

$3\sqrt{7}$

b) $\sqrt[3]{108}$

$\sqrt[3]{9 \times 12}$

$\sqrt[3]{3 \times 3 \times 3 \times 4}$

$3\sqrt[3]{4}$

c) $\sqrt[4]{128}$

$= \sqrt[4]{2 \cdot 64}$

$= \sqrt[4]{2 \cdot 2 \cdot 32}$

$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 16}$

$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 8}$

$2\sqrt[4]{8}$

LET'S TRY TO SIMPLIFY RADICALS WITHOUT USING PRIME FACTORIZATION, IF POSSIBLE.

EX.: Write each radical in simplest form, if possible.

$2^3 = 8$ **a)** $\sqrt[3]{40}$ **b)** $\sqrt{26}$ **c)** $\sqrt[4]{32}$ $2^4 = 16$

$\sqrt[3]{8 \cdot 5}$ $\sqrt{13 \cdot 2}$ $\sqrt[4]{16 \cdot 2}$

$2\sqrt[3]{5}$ $\sqrt{26}$ $2\sqrt[4]{2}$

YOU TRY TO SIMPLIFY RADICALS WITHOUT USING PRIME FACTORIZATION, IF POSSIBLE.

EX.: Write each radical in simplest form, if possible. $2^3=8$

$2^4=16$

a) $\sqrt{30}$ b) $\sqrt[3]{32}$ c) $\sqrt[4]{48}$

$\sqrt{3\cdot 10}$ $\sqrt[3]{8\cdot 4}$ $\sqrt[4]{3\cdot 16}$

$\sqrt{3\cdot 5\cdot 2}$ $2\sqrt[3]{4}$ $2\sqrt[4]{3}$

$\sqrt{30}$

WRITING MIXED RADICALS AS ENTIRE RADICALS:

EX.: Write each mixed radical as an entire radical.

a) $4\sqrt{3}$ **b)** $3\sqrt[3]{2}$ **c)** $2\sqrt[5]{2}$

$= \sqrt{4^2 \cdot 3}$ $\sqrt[3]{3^3 \cdot 2}$ $\sqrt{2^5 \cdot 2}$

$= \sqrt{48}$ $\sqrt[3]{54}$ $\sqrt{64}$

YOU TRY WRITING MIXED RADICALS AS ENTIRE RADICALS:

EX.: Write each mixed radical as an entire radical.

a) $7\sqrt{3}$

$\sqrt{7 \times 7 \times 3}$

$= \sqrt{147}$

b) $2\sqrt[3]{4}$

$\sqrt{2^3 \cdot 4}$

$\sqrt{32}$

c) $2\sqrt[5]{3}$

$\sqrt[5]{2^5 \cdot 3}$

$\sqrt[5]{96}$

CONCEPT REINFORCEMENT:

***FPCM 10*:**

Page 211: 3, 4, 11
Page 218: 4, 5, 10 - 12, 14, 15, 16, 18
Page 219: 21, 22b, 24

QUIZ PREPARATION:

***FPCM 10*:**
Page 221: #1, #3, #4, #6a, #7b, #8, #9 & #11

https://mathmalfunctions.wordpress.com/exponential-functions/exponent-laws/