SEPTEMBER 18, 2017

UNIT 1: ROOTS AND POWERS

SECTION 4.4: FRACTIONAL EXPONENTS AND RADICALS

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NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."

1. $a^0 = 1$

2. $a^m a^n = a^{m+n}$

3. $\frac{a^m}{a^n} = a^{m-n}$

4. $(a^m)^n = a^{mn}$

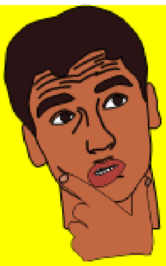
5 $a^{\frac{1}{n}} = \sqrt[n]{a}$

6. $\longleftarrow$ $a^n \cdot b^n = (ab)^n$

7. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

8. $x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m$

9. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$



What does THAT mean???

SCO AN3 means that we will:

* apply the 6 exponent laws you learned in grade 9:
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$
* apply all exponent laws to evaluate a variety of expressions
* express powers with rational exponents as radicals and vice versa
* identify and correct errors in work that involves powers

ANY QUESTIONS BEFORE THE QUIZ???
(page 221, #1, #3, #4, #6a, #7b, #8, #9 & #11)

TIME FOR THE QUIZ! :)

(Use your list of the first 20 perfect squares, perfect cubes and perfect 4th powers if you have it. Otherwise, use your calculator.)

	Perfect Squares	Perfect Cubes	Perfect Fourth Powers
1	1	1	1
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625
6	36	216	1296
7	49	343	2401
8	64	512	4096
9	81	729	6561
10	100	1 000	10 000
11	121	1 331	14 641
12	144	1 728	20 736
13	169	2 197	28 561
14	196	2 744	38 416
15	225	3 375	50 625
16	256	4 096	65 536
17	289	4 913	83 521
18	324	5 832	104 976
19	361	6 859	130 321
20	400	8 000	160 000

EXPONENT LAWS (separate sheet):

1. Zero Exponent Law: $a^0 = 1$
2. Product of Powers: $(a^m)(a^n) = a^{m+n}$
3. Quotient of Powers: $a^m \div a^n = a^{m-n}$
4. Power of a Power: $(a^m)^n = a^{mn}$
5. Power of a Product: $(ab)^m = a^m b^m$
6. Power of a Quotient: $(a \div b)^n = a^n \div b^n$

7. MULTIPLICATION PROPERTY OF RADICALS:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ (MIXED RADICAL)

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

LET'S COPY AND COMPLETE THESE TABLES TOGETHER.

$\sqrt{x}$

x	$x^{\frac{1}{2}}$ $(x^{0.5})$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$

$\sqrt[3]{x}$

x	$x^{\frac{1}{3}}$ $[x^{(1 \div 3)}]$
1	$1^{\frac{1}{3}} = 1$
8	$8^{\frac{1}{3}} = 2$
27	$27^{\frac{1}{3}} = 3$
64	$64^{\frac{1}{3}} = 4$
125	$125^{\frac{1}{3}} = 5$

25 y^x (1/2)

WHAT DO YOU THINK THE EXPONENT $\frac{1}{2}$ MEANS?

Need 2 of something multiplied together

WHAT DO YOU THINK THE EXPONENT $\frac{1}{3}$ MEANS?

Need 3 of something multiplied together

WHAT DO YOU THINK $a^{\frac{1}{4}}$ AND $a^{\frac{1}{5}}$ MEAN? (TEST YOUR PREDICTIONS WITH A CALCULATOR.)

WHAT DO YOU THINK $a^{\frac{1}{n}}$ MEANS?

In grade 9, you learned that for powers with integral bases and whole number exponents:

$a^m \cdot a^n = a^{m+n}$

WE CAN EXTEND THIS LAW TO POWERS WITH FRACTIONAL EXPONENTS WITH A NUMERATOR OF 1:

$5^{\frac{1}{2}} \cdot 5^{\frac{1}{2}}$

$5^{\frac{1}{2}+\frac{1}{2}}$

5^1

$= 5$

$\sqrt{5} \cdot \sqrt{5}$

$\sqrt{25}$

5

$5^{\frac{1}{2}}$ and $\sqrt{5}$ are equivalent expressions; that is, $5^{\frac{1}{2}} = \sqrt{5}$.

SIMILARLY...

$$5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{1}{3}}$$

$$\sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5}$$

$5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$

$= 5^{\frac{3}{3}}$

$= 5$

SO...

$$5^{\frac{1}{3}} = \sqrt[3]{5}$$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$
$$= \sqrt[3]{8}$$
$$= 2$$

EXAMPLE:

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$

$= 3$

b) $0.49^{\frac{1}{2}}$

$\left(\frac{49}{100}\right)^{\frac{1}{2}}$

$\frac{7}{10}$

0.7

c) $(-64)^{\frac{1}{3}}$

$= -4$

d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

$= \frac{2}{3}$

YOU TRY!

Evaluate each power without using a calculator.

a) $1000^{\frac{1}{3}} = 10$

b) $0.25^{\frac{1}{2}} = 0.5$ $\left(\frac{25}{100} \quad \frac{5}{10}\right)$

c) $(-8)^{\frac{1}{3}} = -2$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{2}{3}$

NOTE: Because a fraction can be written as a terminating or repeating decimal, we can interpret powers with decimal exponents.

EX.:

$$32^{0.2}$$

$$=32^{\frac{2}{10}}$$

$$=32^{\frac{1}{5}}$$

$$=\sqrt[5]{32}$$

$$= \mathbf{2}$$

$32^{\frac{2}{10}}$

$= 32^{\frac{1}{5}}$

$= 2$

YOU TRY!

Evaluate $100^{0.5}$.

$100^{\frac{5}{10}}$

$= 100^{\frac{1}{2}}$

$= 10$

To give meaning to a power such as $8^{\frac{2}{3}}$,

we use the exponent law $(a^m)^n = a^{mn}$.

EX.:

$$8^{\frac{2}{3}}$$
$$= 8^{\frac{1}{3}\cdot 2}$$
$$= \left(8^{\frac{1}{3}}\right)^2$$
$$= \left(\sqrt[3]{8}\right)^2$$
$$= 2^2$$
$$= 4$$

EX.:

$$8^{\frac{2}{3}}$$
$$= 8^{2\cdot\frac{1}{3}}$$
$$= \left(8^2\right)^{\frac{1}{3}}$$
$$= \sqrt[3]{8^2}$$
$$= \sqrt[3]{64}$$
$$= 4$$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT → m; INDEX → n

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

AND

EXPONENT → m; INDEX → n

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

EX.: Evaluate $16^{\frac{3}{2}}$.

$16^{\frac{3}{2}}$ 3 (EXPONENT), 2 (INDEX)

$= \left(\sqrt[2]{16}\right)^3$

$= 4^3$

$= 64$

EXAMPLE:

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

$(\sqrt[3]{40})^2$ $\sqrt[3]{40^2}$

b) Write $\sqrt{3^5}$ and $(\sqrt[3]{25})^2$ in exponent form.

$3^{\frac{5}{2}}$ $25^{\frac{2}{3}}$

SOLUTION:

a) $40^{\frac{2}{3}} = (\sqrt[3]{40})^2$ or $\sqrt[3]{40^2}$

b) $\sqrt{3^5} = 3^{\frac{5}{2}}$ **AND** $(\sqrt[3]{25})^2 = 25^{\frac{2}{3}}$

YOU TRY!

a) Write $26^{\frac{2}{5}}$ in radical form in 2 ways. $\left(\sqrt[5]{26}\right)^2$ $\sqrt[5]{26^2}$

b) Write $\sqrt{6^5}$ and $\left(\sqrt[4]{19}\right)^3$ in exponent form.

$6^{\frac{5}{2}}$ $19^{\frac{3}{4}}$

SOLUTION:

a) $\left(\sqrt[5]{26}\right)^2$ or $\sqrt[5]{26^2}$

b) $6^{\frac{5}{2}}, 19^{\frac{3}{4}}$

EXAMPLE:

Evaluate.

$\sqrt{\left(\frac{4}{100}\right)}^3 = \left(\frac{2}{10}\right)^3 \rightarrow \frac{8}{1000}$

a) $0.04^{\frac{3}{2}}$ 0.008 b) $27^{\frac{4}{3}}$ $\left(27^{\frac{1}{3}}\right)^4 = 3^4$

$= 81$

c) $(-32)^{0.4}$ d) $1.8^{1.4}$

$-32^{\frac{4}{10}}$

$-32^{\frac{2}{5}}$

$(-2)^2$

4

$= 2.27\ldots$

$1\frac{4}{10}$

$\frac{14}{10}$

$\frac{7}{5}$ Exponent

SOLUTION:

a) $0.04^{\frac{3}{2}} = \left(0.04^{\frac{1}{2}}\right)^3$

$= \left(\sqrt{0.04}\right)^3$

$= 0.2^3$

$= 0.008$

c) The exponent $0.4 = \frac{4}{10}$ or $\frac{2}{5}$

So, $(-32)^{0.4} = (-32)^{\frac{2}{5}}$

$$= \left[(-32)^{\frac{1}{5}}\right]^2$$

$$= \left(\sqrt[5]{-32}\right)^2$$

$$= (-2)^2$$

$$= 4$$

d) $1.8^{1.4}$

Use a calculator.

```
1.8^1.4
        2.277096874
```

$1.8^{1.4} = 2.2770\ldots$

YOU TRY!

Evaluate.

$\left(\left(\frac{1}{100}\right)^{\frac{1}{2}}\right)^3 \to \left(\frac{1}{10}\right)^3$

a) $0.01^{\frac{3}{2}} = \frac{1}{1000} = 0.001$

b) $(-27)^{\frac{4}{3}} = 81$

c) $81^{\frac{3}{4}} = 27$

d) $0.75^{1.2}$ 0.75 y^x 1.2 0.708...

SOLUTION:

a) 0.001 b) 81 c) 27 d) 0.7080…

EXAMPLE:

Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass m kilograms. Estimate the brain mass of each animal.

a) a husky with a body mass of 27 kg

b) a polar bear with a body mass of 200 kg

SOLUTION:

Use the formula $b = 0.01m^{\frac{2}{3}}$.

a) Substitute: $m = 27$

$b = 0.01(27)^{\frac{2}{3}}$

$b = 0.01\left(\sqrt[3]{27}\right)^2$

$b = 0.01(3)^2$

$b = 0.01(9)$

$b = 0.09$

The brain mass of the husky is approximately 0.09 kg.

Use the order of operations.
Evaluate the power first.

b) Substitute: $m = 200$

$b = 0.01\,(200)^{\frac{2}{3}}$

Use a calculator.

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0.01(200)^(2/3)
            0.341995189
```

The brain mass of the polar bear is approximately 0.34 kg.

YOU TRY!

Use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass of each animal.

a) a moose with a body mass of 512 kg

b) a cat with a body mass of 5 kg

SOLUTION:

a) approximately 0.64 kg

b) approximately 0.03 kg

CONCEPT REINFORCEMENT:

***FPCM 10*:**
Page 227: #3 to #16
Page 228: #17 to #21