**SEPTEMBER 19, 2017**

# UNIT 1: ROOTS AND POWERS

## SECTION 4.5: NEGATIVE EXPONENTS AND RECIPROCALS

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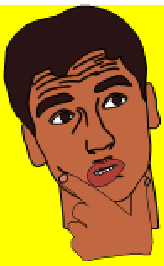
*NUMBERS, RELATIONS AND FUNCTIONS 10*

## WHAT'S THE POINT OF TODAY'S LESSON?

**We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:**

**"Demonstrate an understanding of powers with integral and rational exponents."**



# What does THAT mean???

**SCO AN3 means that we will:**

* apply the 6 exponent laws you learned in grade 9:
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$
* apply all exponent laws to evaluate a variety of expressions
* express powers with rational exponents as radicals and vice versa
* identify and correct errors in work that involves powers

(1) $2^{-3} = \frac{1}{2^3}$

Negative exponent → bottom

(2) $\frac{1}{3^{-2}} = 3^2$

Negative exponent → top

(3) $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}}$

$= \frac{3^2}{2^2}$

$= \left(\frac{3}{2}\right)^2$

## WARM-UP:

**Write the power below as a radical then evaluate.**

not recommended

$$\left(\left(\frac{64}{125}\right)^2\right)^{\frac{1}{3}}$$

$$\left(\frac{4096}{15625}\right)^{\frac{1}{3}}$$

$$\frac{16}{25}$$

$$\left(\frac{64}{125}\right)^{\frac{2}{3}}$$

$$=\left(\frac{4}{5}\right)^2$$

$$=\frac{16}{25}$$

**WHITE BOARD WARM-UP :**

**First, write the power below with a positive exponent. At this point, write the power as a radical then evaluate.**

$$\left(\frac{64}{125}\right)^{\frac{-2}{3}} = \left(\frac{125}{64}\right)^{\frac{2}{3}}$$

$$= \frac{25}{16}$$

# HOMEWORK QUESTIONS???
# (pages 227 / 228, #7, #8, #10, #11 and #15 TO #21)

17. $h = 35d^{\frac{2}{3}}$

$= 35(3.2)^{\frac{2}{3}}$

$= 76.0$ m

3.2 ^ (2/3)

ans * 35

## EXPONENT LAWS (separate sheet):

1. **Zero Exponent Law:** $a^0 = 1$
2. **Product of Powers:** $(a^m)(a^n) = a^{m+n}$
3. **Quotient of Powers:** $a^m \div a^n = a^{m-n}$
4. **Power of a Power:** $(a^m)^n = a^{mn}$
5. **Power of a Product:** $(ab)^m = a^m b^m$
6. **Power of a Quotient:** $(a \div b)^n = a^n \div b^n$

## 7. MULTIPLICATION PROPERTY OF RADICALS:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ (MIXED RADICAL)

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

## 8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

**EX.:**

$$8^{\frac{1}{3}}$$

$$= \sqrt[3]{8}$$

$$= 2$$

# 9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT → $m$

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

INDEX → $n$

**AND**

EXPONENT → $m$

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

INDEX → $n$

**EX.:** Evaluate $16^{\frac{3}{2}}$.

$16^{\frac{3}{2}}$ 3 (EXPONENT), 2 (INDEX)

$$= \left(\sqrt[2]{16}\right)^3$$
$$= 4^3$$
$$= 64$$

**OR**

$16^{\frac{3}{2}}$ 3 (EXP.), 2 (INDEX)

$$= \sqrt[2]{16^3}$$
$$= \sqrt{4096}$$
$$= 64$$

## 10. POWERS WITH NEGATIVE EXPONENTS:

$$x^{-n} = \frac{1}{x^n} \quad AND \quad \frac{1}{x^{-n}} = x^n$$

**EX.:** $4^{-2}$

$= \frac{1}{4^2}$

$= \frac{1}{16}$

**EX.:** $\frac{1}{5^{-2}}$

$= 5^2$

$= 25$

## VOCABULARY:

1. **RECIPROCAL:** Two numbers whose product is 1.

EX.: 2 and ½ are reciprocals.

$\frac{2}{3} \times \frac{3}{2} = \frac{6}{6}$

$= 1$

**We build on our understanding of powers to work with negative exponents.**

**For example:**

$5^{-2} \quad 5^{2}$

$= 5^{-2+2}$

$= 5^{0}$

$= 1$

**This means that $5^{-2}$ and $5^{2}$ are RECIPROCALS!**
**(Their product equals 1...)**

If...

$5^{-2} \quad 5^{2} = 1$

... then...

$5^{-2} \quad 25 = 1$

... and this must actually mean...

$\frac{1}{25} \quad 25 = 1$

... so...

$5^{-2}$ must be equal to $\frac{1}{25}$ or $\frac{1}{5^{2}}$ !!!

**Another scenario based on exponent laws:**

$$5^{-2} \cdot \frac{1}{5^{-2}}$$

$$= \frac{5^{-2}}{5^{-2}}$$

$$= 5^{-2 - (-2)}$$

$$= 5^{-2+2}$$

$$= 5^{0}$$

$$= 1$$

**This means that $5^{-2}$ and $\frac{1}{5^{-2}}$ are also RECIPROCALS!**

**(Their product also equals 1...)**

**"THE OLD FASHIONED WAY"... :)**

**The way we used to teach the negative exponent rule:**

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

**EXAMPLE:**

a) $3^{-2}$

$= \frac{1}{9}$

b) $0.3^{-4}$

$= \left(\frac{3}{10}\right)^{-4}$

$= \left(\frac{10}{3}\right)^{4}$

$= \frac{10000}{81}$

**Basically, remember to take the reciprocal of the ENTIRE base and change the negative exponent to a positive exponent.**

**EX.:**

$$\left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^{3}$$

$$= -\frac{64}{27}$$

**YOU TRY!**

Evaluate each power.

a) $7^{-2}$

$= \left(\frac{1}{7}\right)^2$

$= \frac{1}{49}$

b) $\left(\frac{10}{3}\right)^{-3}$

$= \left(\frac{3}{10}\right)^3$

$= \frac{27}{1000}$

c) $(-1.5)^{-3}$

$= \left(-\frac{3}{2}\right)^{-3}$

$= \left(\frac{-2}{3}\right)^3$

$= -\frac{8}{27}$

**EXAMPLE:**

Evaluate each power without using a calculator.

a) $8^{-\frac{2}{3}}$

$\left(\left(8^{\frac{1}{3}}\right)^2\right)^{-1}$

$\left(2^2\right)^{-1}$

$4^{-1}$

$\frac{1}{4}$

b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$ $= \left(\frac{3}{4}\right)^{-3}$

$= \left(\frac{4}{3}\right)^3$

$= \frac{64}{27}$

**YOU TRY!**

Evaluate each power without using a calculator.

**a)** $16^{-\frac{5}{4}}$

$= \left(\frac{1}{16}\right)^{\frac{5}{4}}$

$= \left(\frac{1}{2}\right)^{5}$

$= \frac{1}{32}$

**b)** $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$

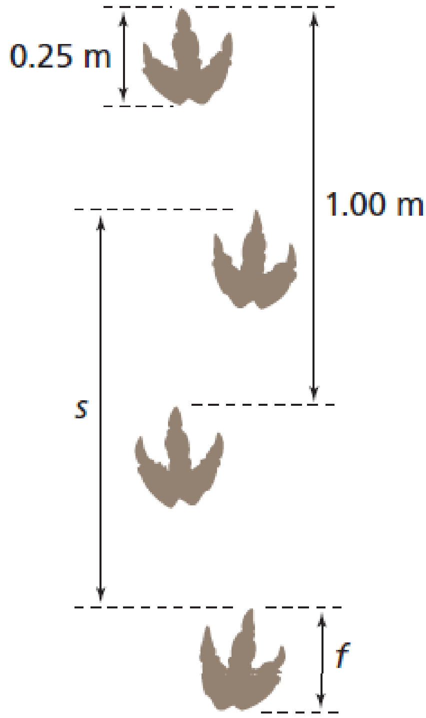
$= \left(\frac{36}{25}\right)^{\frac{1}{2}}$

$= \frac{6}{5}$

# EXAMPLE:

Paleontologists use measurements from fossilized dinosaur tracks and the formula $v = 0.155\,s^{\frac{5}{3}}f^{-\frac{7}{6}}$ to estimate the speed at which the dinosaur travelled. In the formula, $v$ is the speed in metres per second, $s$ is the distance between successive footprints of the same foot, and $f$ is the foot length in metres.

Use the measurements in the diagram to estimate the speed of the dinosaur.




## SOLUTION

Use the formula: $v = 0.155\,s^{\frac{5}{3}}f^{-\frac{7}{6}}$

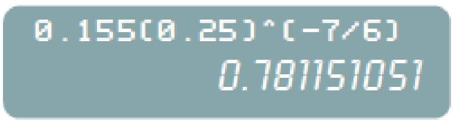
Substitute: $s = 1$ and $f = 0.25$

$v = 0.155\,(1)^{\frac{5}{3}}\,(0.25)^{-\frac{7}{6}}$

$v = 0.155\,(0.25)^{-\frac{7}{6}}$

$v = 0.7811\ldots$

The dinosaur travelled at approximately 0.8 m/s.



**YOU TRY!**

Use the formula $v = 0.155\, s^{\frac{5}{3}} f^{-\frac{7}{6}}$ to estimate the speed of a dinosaur when $s = 1.5$ and $f = 0.3$.

Answer: approximately 1.2 m/s

# CONCEPT REINFORCEMENT:

***FPCM 10*:**

**Page 233: #3 TO #14**

**Page 234: #15 TO #17ab and #18 TO #20**

**QUIZ PREPARATION - SECTIONS 4.4 & 4.5: (Fractional Exponents and Radicals; Negative Exponents and Reciprocals)**

***FPCM 10*:**
**Page 236: #1 to #8 (ALL!)**