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UNIT 1: ROOTS AND POWERS

SECTION 4.6: APPLYING THE EXPONENT LAWS

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NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

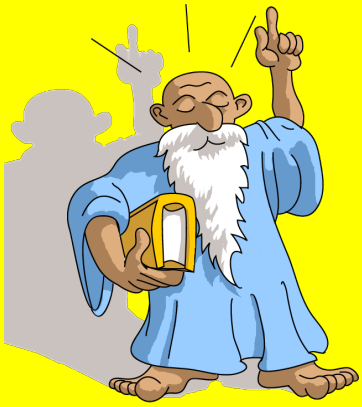
We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."

What does THAT mean???

SCO AN3 means that we will:

* **apply the 6 exponent laws you learned in grade 9:**
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* **use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$**
* **apply all exponent laws to evaluate a variety of expressions**
* **express powers with rational exponents as radicals and vice versa**
* **identify and correct errors in work that involves powers**



EXPONENT LAWS (separate sheet):

1. **Zero Exponent Law:** $a^0 = 1$ $(1 \times \square \times \cdots)^0 = 1$

2. **Product of Powers:** $(a^m)(a^n) = a^{m+n}$

3. **Quotient of Powers:** $a^m \div a^n = a^{m-n}$

4. **Power of a Power:** $(a^m)^n = a^{mn}$

5. **Power of a Product:** $(ab)^m = a^m b^m$

6. **Power of a Quotient:** $(a \div b)^n = a^n \div b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

7. MULTIPLICATION PROPERTY OF RADICALS:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\quad (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}}$

EX.: $\sqrt{24}$ **(Factors: 1, 2, 3, 4, 6, 8, 12, 24)**

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ **(MIXED RADICAL)**

EX.: $\sqrt[3]{24}$ **(ENTIRE RADICAL)**

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

$\sqrt{18}$

$\sqrt{9 \times 2}$

$9^{\frac{1}{2}} 2^{\frac{1}{2}}$

$\sqrt{9} \cdot \sqrt{2}$

$3\sqrt{2}$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$9^{\frac{1}{2}} = \sqrt[2]{9}$$

$$= \sqrt{9}$$

$$= 3$$

EX.:

$$8^{\frac{1}{3}}$$

$$= \sqrt[3]{8}$$

$$= 2$$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

INDEX

AND

EXPONENT

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

INDEX

EX.: Evaluate $16^{\frac{3}{2}}$.

$\frac{3}{2}$ (EXPONENT)

$16^{\frac{3}{2}}$ (INDEX)

$= \left(\sqrt[2]{16}\right)^3$

$= 4^3$

$= 64$

OR

$\frac{3}{2}$ (EXP.)

$16^{\frac{3}{2}}$ (INDEX)

$= \sqrt[2]{16^3}$

$= \sqrt{4096}$

$= 64$

$$(-125)^{\frac{2}{3}} = \left(\sqrt[3]{-125}\right)^2$$

$$= (-5)^2$$

$$= 25$$

10. POWERS WITH NEGATIVE EXPONENTS:

$$x^{-n} = \frac{1}{x^n} \quad AND \quad \frac{1}{x^{-n}} = x^n$$

EX.: 4^{-2}

$= \frac{1}{4^2}$ $\left(\frac{1}{4}\right)^2$

$= \frac{1}{16}$

EX.: $\frac{1}{5^{-2}}$

$= 5^2$

$= 25$

$$\left(\frac{3}{2}\right)^{-3} \rightarrow \left(\frac{2}{3}\right)^{3} \leftarrow \frac{3^{-3}}{2^{-3}} \leftarrow \frac{2^{3}}{3^{3}}$$

$$= \frac{8}{27}$$

Basically, remember to take the reciprocal of the ENTIRE base and change the negative exponent to a positive exponent.

EX.: $\left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^{3}$

$= -\frac{64}{27}$

LAST MINUTE QUESTIONS???
(page 236, #1 to #8)

2. iv) $400^{1.5} = 400^{\frac{3}{2}}$

$= \left(\sqrt{400}\right)^3$

$= 20^3$

$= 8000$

v) $(-125)^{\frac{1}{3}}$

$= \sqrt[3]{-125}$

$= -5$

vi) $\left(\frac{8}{125}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{8}{125}}\right)^2$

$= \left(\frac{2}{5}\right)^2$

$= \frac{4}{25}$

8. $P = 5000(1.029)^{-3}$

$= \$4589.06$

4. $T = 17.5(85)^{\frac{1}{4}}$

$= 371.88$

QUIZ TIME!
(Sections 4.4 and 4.5 - 10 to 15 min.)

4.6 - APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$$(2^2)(2^6)$$
$$= 2^8$$
$$= 256$$

APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$$(2^2)(2^{-6})$$
$$= 2^{-4}$$
$$= \frac{1}{2^4}$$
$$= \frac{1}{16}$$

APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$$\left[\left(2^{2}\right)\left(2^{-6}\right)\right]^{\frac{-3}{2}}$$

$$=\left(2^{-4}\right)^{\frac{-3}{2}}$$

$$=\left(\frac{1}{2^{4}}\right)^{\frac{-3}{2}}$$

$$=\left(\frac{1}{16}\right)^{\frac{-3}{2}}$$

$$=16^{\frac{3}{2}}$$

$$=\left(\sqrt{16}\right)^{3}$$

$$=4^{3}$$

$$=64$$

EXAMPLE:

a) $0.3^{-3} \cdot 0.3^{5}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

c) $\frac{(1.4^{3})(1.4^{4})}{1.4^{-2}}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^{6}$

SOLUTIONS:

a) $0.3^{-3} \cdot 0.3^{5} = 0.3^{(-3)+5}$

$= 0.3^{2}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3} = \left(-\frac{3}{2}\right)^{-8} \cdot \left(-\frac{3}{2}\right)^{6}$

$= \left(-\frac{3}{2}\right)^{-2}$

$= \left(-\frac{2}{3}\right)^{2}$

c)

$$\frac{(1.4^3)(1.4^4)}{1.4^{-2}}$$

$$= \frac{1.4^{3+4}}{1.4^{-2}}$$

$$= \frac{1.4^7}{1.4^{-2}}$$

$$= 1.4^{7-(-2)}$$

$$= 1.4^9$$

d)

$$\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^6$$

$$= \left(\frac{7^{\frac{2}{3}}}{7^{\frac{6}{3}}}\right)^6$$

$$= \left(7^{\frac{2}{3}-\frac{6}{3}}\right)^6$$

$$= \left(7^{-\frac{4}{3}}\right)^6$$

$$= 7^{-\frac{24}{3}}$$

$$= 7^{-8}$$

$$= \frac{1}{7^8}$$

YOU TRY!

a) $0.8^{2} \cdot 0.8^{-7}$

b) $\left[\left(-\frac{4}{5}\right)^{2}\right]^{-3} \div \left[\left(-\frac{4}{5}\right)^{4}\right]^{-5}$

c) $\frac{(1.5^{-3})^{-5}}{1.5^{5}}$

d) $\frac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$

[Answers: a) $\frac{1}{0.8^5}$ b) $\left(-\frac{4}{5}\right)^{14}$

c) 1.5^{10} d) $9^{\frac{1}{4}}$]

EXAMPLE:

a) $(x^3y^2)(x^2y^{-4})$

b) $\dfrac{10a^5b^3}{2a^2b^{-2}}$

SOLUTIONS:

a) $(x^3y^2)(x^2y^{-4}) = x^3 \cdot y^2 \cdot x^2 \cdot y^{-4}$

$= x^3 \cdot x^2 \cdot y^2 \cdot y^{-4}$

$= x^{3+2} \cdot y^{2+(-4)}$

$= x^5 \cdot y^{-2}$

$= x^5 \cdot \frac{1}{y^2}$

$= \frac{x^5}{y^2}$

b) $$\frac{10a^5b^3}{2a^2b^{-2}} = \frac{10}{2} \cdot \frac{a^5}{a^2} \cdot \frac{b^3}{b^{-2}}$$

$$= 5 \cdot a^{5-2} \cdot b^{3-(-2)}$$

$$= 5 \cdot a^3 \cdot b^5$$

$$= 5a^3b^5$$

HOW I WOULD ACTUALLY SHOW MY WORK:

a) $(x^3y^2)(x^2y^{-4})$

b) $\dfrac{10a^5b^3}{2a^2b^{-2}}$

YOU TRY!

a) $m^4 n^{-2} \cdot m^2 n^3$

b) $\dfrac{6x^4 y^{-3}}{14xy^2}$

[Answers: a) m^6n b) $\frac{3x^3}{7y^5}$]

EXAMPLE:

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\dfrac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

SOLUTIONS:

a) $(8a^3b^6)^{\frac{1}{3}} = 8^{\frac{1}{3}} \cdot a^{3\left(\frac{1}{3}\right)} \cdot b^{6\left(\frac{1}{3}\right)}$

$= (2^3)^{\frac{1}{3}} \cdot a^1 \cdot b^2$

$= 2ab^2$

$$\begin{aligned}\textbf{b)}\ (x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1}) &= x^{\frac{3}{2}} \cdot x^{\frac{1}{2}} \cdot y^2 \cdot y^{-1} \\ &= x^{\frac{3}{2}+\frac{1}{2}} \cdot y^{2+(-1)} \\ &= x^2 y\end{aligned}$$

$$\text{c)}\ \frac{4a^{-2}b^{\frac{2}{3}}}{2a^{2}b^{\frac{1}{3}}} = \frac{4}{2}\cdot\frac{a^{-2}}{a^{2}}\cdot\frac{b^{\frac{2}{3}}}{b^{\frac{1}{3}}}$$

$$= 2\cdot a^{(-2)-2}\cdot b^{\frac{2}{3}-\frac{1}{3}}$$

$$= 2\cdot a^{-4}\cdot b^{\frac{1}{3}}$$

$$= \frac{2b^{\frac{1}{3}}}{a^{4}}$$

$$\textbf{d)}\left(\frac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}=\left(\frac{100}{25}\cdot\frac{a^1}{a^5}\cdot\frac{1}{b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$$

$$=\left(4\cdot a^{1-5}\cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$=\left(4\cdot a^{-4}\cdot b^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$=4^{\frac{1}{2}}\cdot a^{(-4)\left(\frac{1}{2}\right)}\cdot b^{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$$

$$=2\cdot a^{-2}\cdot b^{\frac{1}{4}}$$

$$=\frac{2b^{\frac{1}{4}}}{a^2}$$

HOW I WOULD ACTUALLY SHOW MY WORK:

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

HOW I WOULD ACTUALLY SHOW MY WORK:

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^{2}b^{\frac{1}{3}}}$

d) $\left(\dfrac{100a}{25a^{5}b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

YOU TRY!

a) $(25a^4b^2)^{\frac{3}{2}}$

b) $(x^3y^{-\frac{3}{2}})(x^{-1}y^{\frac{1}{2}})$

c) $\dfrac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}}$

d) $\left(\dfrac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}}$

[Answers: a) $125a^6b^3$ b) $\frac{x^2}{y}$

c) $\frac{4y^3}{x^{\frac{11}{2}}}$ d) $\frac{5}{xy^{\frac{3}{2}}}$]

CONCEPT REINFORCEMENT:

***FPCM 10*:**
Page 241: #3 to #6
Page 242: #7 to #11, #14 to #17 & #19
Page 243: #21 & #22

UNIT 1 TEST PREPARATION

***FPCM 10*:**

Page 197:	**Skills Summary (3.1 / 3.2)**
Page 198:	**Review Questions (3.1 / 3.2)**
Page 201:	**Practice Test (#1 & #3)**

Page 244:	**Study Guide**
Page 245:	**Skills Summary**
Pages 246 to 248:	**Review Questions**
Page 249:	**Practice Test**