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UNIT 1: ROOTS AND POWERS

SECTION 4.6: APPLYING THE EXPONENT LAWS

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NUMBERS, RELATIONS AND FUNCTIONS 10

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."

What does THAT mean???

SCO AN3 means that we will:

* **apply the 6 exponent laws you learned in grade 9:**
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* **use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$**
* **apply all exponent laws to evaluate a variety of expressions**
* **express powers with rational exponents as radicals and vice versa**
* **identify and correct errors in work that involves powers**

EXPONENT LAWS (separate sheet):

1. **Zero Exponent Law:** $a^0 = 1$ $(1 + \square + \ddot{\cup})^0 = 1$
2. **Product of Powers:** $(a^m)(a^n) = a^{m+n}$
3. **Quotient of Powers:** $a^m \div a^n = a^{m-n}$
4. **Power of a Power:** $(a^m)^n = a^{mn}$
5. **Power of a Product:** $(ab)^m = a^m b^m$
6. **Power of a Quotient:** $(a \div b)^n = a^n \div b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

7. MULTIPLICATION PROPERTY OF RADICALS:

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\quad (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}}$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$= \sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6}$

$= 2\sqrt{6}$ (MIXED RADICAL)

$\sqrt{18}$

$\sqrt{9 \times 2}$

$9^{\frac{1}{2}} 2^{\frac{1}{2}}$

$\sqrt{9} \cdot \sqrt{2}$

$3\sqrt{2}$

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2 \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$9^{\frac{1}{2}} = \sqrt{9}$

$= \sqrt{9}$

$= 3$

EX.:

$$8^{\frac{1}{3}}$$

$$= \sqrt[3]{8}$$

$$= 2$$

9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

INDEX

AND

EXPONENT

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

INDEX

EX.: Evaluate $16^{\frac{3}{2}}$.

$16^{\frac{3}{2}}$ (EXPONENT) (INDEX)

$= \left(\sqrt[2]{16}\right)^3$

$= 4^3$

$= 64$

OR

$16^{\frac{3}{2}}$ (EXP.) (INDEX)

$= \sqrt[2]{16^3}$

$= \sqrt{4096}$

$= 64$

$$(-125)^{\frac{2}{3}} = \left(\sqrt[3]{-125}\right)^2$$

$$= (-5)^2$$

$$= 25$$

10. POWERS WITH NEGATIVE EXPONENTS:

$$x^{-n} = \frac{1}{x^n} \quad AND \quad \frac{1}{x^{-n}} = x^n$$

EX.: 4^{-2}

$= \frac{1}{4^2}$ $\left(\frac{1}{4}\right)^2$

$= \frac{1}{16}$

EX.: $\frac{1}{5^{-2}}$

$= 5^2$

$= 25$

$\left(\frac{3}{2}\right)^{-3} \rightarrow \left(\frac{2}{3}\right)^3$

$= \frac{8}{27}$

$\frac{3^{-3}}{2^{-3}}$

$\frac{2^3}{3^3}$

Basically, remember to take the reciprocal of the ENTIRE base and change the negative exponent to a positive exponent.

EX.: $\left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^{3}$

$= -\frac{64}{27}$

LAST MINUTE QUESTIONS???
(page 236, #1 to #8)

2. iv) $400^{1.5} = 400^{\frac{3}{2}}$

$= \left(\sqrt{400}\right)^{3}$

$= 20^{3}$

$= 8000$

v) $(-125)^{\frac{1}{3}}$

$= \sqrt[3]{-125}$

$= -5$

vi) $\left(\frac{8}{125}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{8}{125}}\right)^{2}$

$= \left(\frac{2}{5}\right)^{2}$

$= \frac{4}{25}$

8. $P = 5000(1.029)^{-3}$

$= \$4589.06$

4. $T = 17.5(85)^{\frac{1}{4}}$

$= 371.88$

4.6 - APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$(2^2)(2^6)$
$= 2^8$
$= 256$

2^{2+6}
$= 2^8$
$= 256$

$(2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2)$
2^8
$= 256$

APPLYING EXPONENT LAWS:

Let's build gradually on what we knew in grade 9...

For example:

$(2^2)(2^{-6})$
$= 2^{-4}$
$= \frac{1}{2^4}$
$= \frac{1}{16}$

$2^{2+(-6)}$
2^{-4}
$\frac{1}{2^4}$
$\frac{1}{16}$

APPLYING EXPONENT LAWS: $(a^m)^n = a^{mn}$

Let's build gradually on what we knew in grade 9...

For example:

$$\left[\left(2^2\right)\left(2^{-6}\right)\right]^{\frac{-3}{2}}$$

$$= \left(2^{-4}\right)^{\frac{-3}{2}}$$

$$= \left(\frac{1}{2^4}\right)^{\frac{-3}{2}}$$

$$= \left(\frac{1}{16}\right)^{\frac{-3}{2}}$$

$$= 16^{\frac{3}{2}}$$

$$= \left(\sqrt{16}\right)^3$$

$$= 4^3$$

$$= 64$$

$(2^{-4})^{-\frac{3}{2}}$
$= \left(\frac{1}{2^4}\right)^{-\frac{3}{2}}$
$= (2^4)^{\frac{3}{2}}$
$= 2^6$
$= 2$
$= 64$

EXAMPLE:

a) $0.3^{-3} \cdot 0.3^5$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^2 \cdot \left[\left(-\frac{3}{2}\right)^2\right]^3$

c) $\frac{(1.4^3)(1.4^4)}{1.4^{-2}}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^6$

SOLUTIONS:

a) $0.3^{-3} \cdot 0.3^{5}$ $= 0.3^{-3+5}$

$= 0.3^{2}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3}$

$= \left(-\frac{3}{2}\right)^{-8} \left(-\frac{3}{2}\right)^{6}$

$= \left(-\frac{3}{2}\right)^{-8+6}$

$= \left(-\frac{3}{2}\right)^{-2}$

$= \left(-\frac{2}{3}\right)^{2}$

$= \frac{4}{9}$

c) $\dfrac{(1.4^3)(1.4^4)}{1.4^{-2}}$

$= \dfrac{1.4^7}{1.4^{-2}}$

$= 1.4^{7-(-2)}$

$= 1.4^9$

$(1.4^7)(1.4^2)$

1.4^{7+2}

1.49

d) $\left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^6$

$\left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{1}{3}+\frac{5}{3}}}\right)^6$

$= \left(\dfrac{7^{\frac{2}{3}}}{7^{\frac{6}{3}}}\right)^6$

$= \left(7^{\frac{2}{3}-\frac{6}{3}}\right)^6$

$= \left(7^{-\frac{4}{3}}\right)^{\frac{6}{1}}$

$= 7^{-\frac{24}{3}}$

$= 7^{-8}$

$= \dfrac{1}{7^8}$

SOLUTIONS:

a) $0.3^{-3} \cdot 0.3^{5} = 0.3^{(-3)+5}$

$= 0.3^{2}$

b) $\left[\left(-\frac{3}{2}\right)^{-4}\right]^{2} \cdot \left[\left(-\frac{3}{2}\right)^{2}\right]^{3} = \left(-\frac{3}{2}\right)^{-8} \cdot \left(-\frac{3}{2}\right)^{6}$

$= \left(-\frac{3}{2}\right)^{-2}$

$= \left(-\frac{2}{3}\right)^{2}$

c) $\frac{(1.4^{3})(1.4^{4})}{1.4^{-2}}$

$= \frac{1.4^{3+4}}{1.4^{-2}}$

$= \frac{1.4^{7}}{1.4^{-2}}$

$= 1.4^{7-(-2)}$

$= 1.4^{9}$

d) $\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}}\right)^{6}$

$= \left(\frac{7^{\frac{2}{3}}}{7^{\frac{6}{3}}}\right)^{6}$

$= \left(7^{\frac{2}{3}-\frac{6}{3}}\right)^{6}$

$= \left(7^{-\frac{4}{3}}\right)^{6}$

$= 7^{-\frac{24}{3}}$

$= 7^{-8}$

$= \frac{1}{7^{8}}$

YOU TRY!

a) $0.8^{2} \cdot 0.8^{-7}$

b) $\left[\left(-\frac{4}{5}\right)^{2}\right]^{-3} \div \left[\left(-\frac{4}{5}\right)^{4}\right]^{-5}$

c) $\frac{(1.5^{-3})^{-5}}{1.5^{5}}$

d) $\frac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$

[Answers: a) $\frac{1}{0.8^{5}}$ b) $\left(-\frac{4}{5}\right)^{14}$

c) 1.5^{10} d) $9^{\frac{1}{4}}$]

SOLUTIONS:

a) $0.8^2 \cdot 0.8^{-7}$ $= 0.8^{-5}$

$= \frac{1}{0.8^5}$

or $\left(\frac{1}{0.8}\right)^5$

b) $\left[\left(-\frac{4}{5}\right)^2\right]^{-3} \div \left[\left(-\frac{4}{5}\right)^4\right]^{-5}$

$= \left(-\frac{4}{5}\right)^{-6} \div \left(-\frac{4}{5}\right)^{-20}$

$= \left(-\frac{4}{5}\right)^{-6-(-20)}$

$= \left(-\frac{4}{5}\right)^{14}$

c) $\dfrac{(1.5^{-3})^{-5}}{1.5^{5}}$

$= \dfrac{1.5^{15}}{1.5^{5}}$

$= 1.5^{15-5}$

$= 1.5^{10}$

d) $\dfrac{9^{\frac{5}{4}} \cdot 9^{-\frac{1}{4}}}{9^{\frac{3}{4}}}$

$= \dfrac{9^{\frac{5}{4}+(-\frac{1}{4})}}{9^{\frac{3}{4}}}$

$= \dfrac{9^{\frac{4}{4}}}{9^{\frac{3}{4}}}$

$= 9^{\frac{4}{4}-\frac{3}{4}}$

$= 9^{\frac{1}{4}}$

$\sqrt[4]{9}$

EXAMPLE:

a) $(x^3y^2)(x^2y^{-4})$

$= x^{3+2}\ y^{2+(-4)}$

$= x^5\ y^{-2}$

b) $\dfrac{10a^5b^3}{2a^2b^{-2}}$

$5a^{5-2}\ b^{3-(-2)}$

$\frac{10}{2}\ \frac{a^5}{a^2}\ b^3b^2$

$5a^3b^5$

SOLUTIONS:

a) $(x^3y^2)(x^2y^{-4}) = x^3 \cdot y^2 \cdot x^2 \cdot y^{-4}$

$= x^3 \cdot x^2 \cdot y^2 \cdot y^{-4}$

$= x^{3+2} \cdot y^{2+(-4)}$

$= x^5 \cdot y^{-2}$

$= x^5 \cdot \dfrac{1}{y^2}$

$= \dfrac{x^5}{y^2}$

b) $$\frac{10a^5b^3}{2a^2b^{-2}} = \frac{10}{2} \cdot \frac{a^5}{a^2} \cdot \frac{b^3}{b^{-2}}$$

$$= 5 \cdot a^{5-2} \cdot b^{3-(-2)}$$

$$= 5 \cdot a^3 \cdot b^5$$

$$= 5a^3b^5$$

YOU TRY!

a) $m^4n^{-2} \cdot m^2n^3$

b) $\frac{6x^4y^{-3}}{14xy^2}$

SOLUTIONS:

a) $m^4n^{-2} \cdot m^2n^3$

$= m^{4+2}\, n^{-2+3}$

$= m^6 n^1$

b) $\dfrac{6x^4y^{-3}}{14xy^2}$ $= \dfrac{6}{14}\, \dfrac{x^4}{x^1}\, \dfrac{y^{-3}}{y^2}$

$= \dfrac{3}{7} x^3 y^{-5}$

$= \dfrac{3x^3}{7y^5}$

[Answers: a) m^6n b) $\frac{3x^3}{7y^5}$]

EXAMPLE:

a) $(8a^3b^6)^{\frac{1}{3}}$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1})$

c) $\frac{4a^{-2}b^{\frac{2}{3}}}{2a^2b^{\frac{1}{3}}}$

d) $\left(\frac{100a}{25a^5b^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

SOLUTIONS:

a) $(8a^3b^6)^{\frac{1}{3}} = 8^{\frac{1}{3}}(a^3)^{\frac{1}{3}}(b^6)^{\frac{1}{3}}$

$= 2a^{\frac{3}{3}}b^{\frac{6}{3}}$

$= 2ab^2$

b) $(x^{\frac{3}{2}}y^2)(x^{\frac{1}{2}}y^{-1}) = x^{\frac{3}{2}}\cdot x^{\frac{1}{2}}y^2y^{-1}$

$= x^{\frac{4}{2}}y^1$

$= x^2y$

c) $\dfrac{4a^{-2}b^{\frac{2}{3}}}{2a^{2}b^{\frac{1}{3}}} =$

$= \frac{4}{2}\,\frac{a^{-2}}{a^{2}}\,\frac{b^{\frac{2}{3}}}{b^{\frac{1}{3}}}$

$= 2\,\frac{1}{a^{4}}\,b^{\frac{2}{3}-\frac{1}{3}}$

$= \frac{2}{a^{4}}\,b^{\frac{1}{3}}$

$=$

d) $\left(\dfrac{100a}{25a^{5}b^{-\frac{1}{2}}}\right)^{\frac{1}{2}} = \left(\frac{100}{25}\right)^{\frac{1}{2}}\left(\frac{a}{a^{5}}\right)^{\frac{1}{2}}\left(b^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= 4^{\frac{1}{2}}\left(a^{-4}\right)^{\frac{1}{2}}\left(b^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= 2\left(\frac{1}{a^{4}}\right)^{\frac{1}{2}}\,b^{\frac{1}{4}}$

$= 2\,\frac{1}{a^{\frac{4}{2}}}\,b^{\frac{1}{4}}$

$= \frac{2b^{\frac{1}{4}}}{a^{2}}$

$=$

YOU TRY!

a) $(25a^4b^2)^{\frac{3}{2}}$

b) $(x^3y^{-\frac{3}{2}})(x^{-1}y^{\frac{1}{2}})$

c) $\dfrac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}}$ It's a mean one!

d) $\left(\dfrac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}}$

SOLUTIONS:

a) $(25a^4b^2)^{\frac{3}{2}} = 25^{\frac{3}{2}}(a^{\frac{4}{1}})^{\frac{3}{2}}(b^{\frac{2}{1}})^{\frac{3}{2}}$

$= 5^3 a^{\frac{12}{2}} b^{\frac{6}{2}}$

$= 125a^6b^3$

b) $(x^3y^{-\frac{3}{2}})(x^{-1}y^{\frac{1}{2}})$

$= x^{3+(-1)}\, y^{-\frac{3}{2}+\frac{1}{2}}$

$= x^2 y^{-\frac{2}{2}}$

$= x^2 y^{-1}$

$= \frac{x^2}{y}$

c) $\frac{12x^{-5}y^{\frac{5}{2}}}{3x^{\frac{1}{2}}y^{-\frac{1}{2}}}$

$= 4x^{-5-\frac{1}{2}}\, y^{\frac{5}{2}-(\frac{1}{2})}$

$= 4x^{-\frac{10}{2}-\frac{1}{2}}\, y^{\frac{4}{2}}$

$= 4x^{-\frac{11}{2}}y^2$

$= \frac{4y^2}{x^{\frac{11}{2}}}$

d) $\left(\frac{50x^2y^4}{2x^4y^7}\right)^{\frac{1}{2}} = \left(25x^{2-4}y^{4-7}\right)^{\frac{1}{2}}$

$= \left(25x^{-\frac{2}{1}}y^{-\frac{3}{1}}\right)^{\frac{1}{2}}$

$= 5x^{-\frac{2}{2}}y^{-\frac{3}{2}}$

$= 5x^{-1}y^{-\frac{3}{2}}$

$= \frac{5}{xy^{\frac{3}{2}}}$

[Answers: a) $125a^6b^3$ b) $\frac{x^2}{y}$

c) $\frac{4y^3}{x^{\frac{11}{2}}}$ d) $\frac{5}{xy^{\frac{3}{2}}}$]

CONCEPT REINFORCEMENT:

***FPCM 10*:**
Page 241: #3 to #6
Page 242: #7 to #11, #14 to #17 & #19
Page 243: #21 & #22

UNIT 1 TEST PREPARATION

***FPCM 10*:**
Page 197: Skills Summary (3.1 / 3.2)
Page 198: Review Questions (3.1 / 3.2)
Page 201: Practice Test (#1 & #3)

Page 244: Study Guide
Page 245: Skills Summary
Pages 246 to 248: Review Questions
Page 249: Practice Test