HOMEWORK QUESTIONS?

1b

p.221 5d, 5a

for each linear inequality.

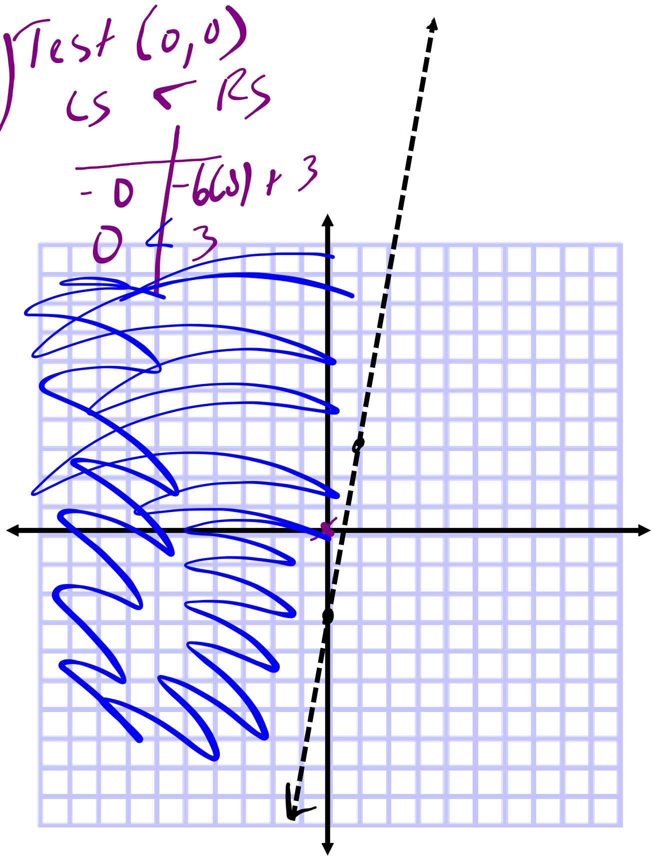
b) $-y < -6x + 3$

$$\frac{-1y}{-1} = \frac{-6x}{-1} + \frac{3}{-1}$$

$$y = 6x - 3$$

Test (0,0)

LS	< RS
-0	$-6(0) + 3$
0	< 3



5. Graph the solution set for each linear inequality.

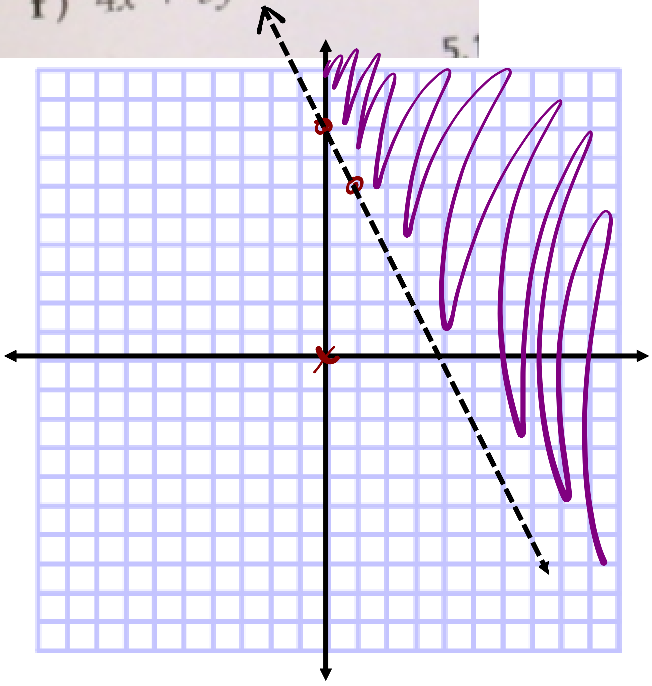
a) $y > -2x + 8$ d) $-4x - 8 > 4$

b) $-3y \leq 9x + 12$ e) $10x - 12 < -y$

c) $y < 6$ f) $4x + 3y \geq -12$

a) $y = -2x + 8$

LS	> RS
0	$-2(0) + 8$
	$\ngtr 8$ no



d) $-4x - 8 > 4$

$-4x - 8 = 4$

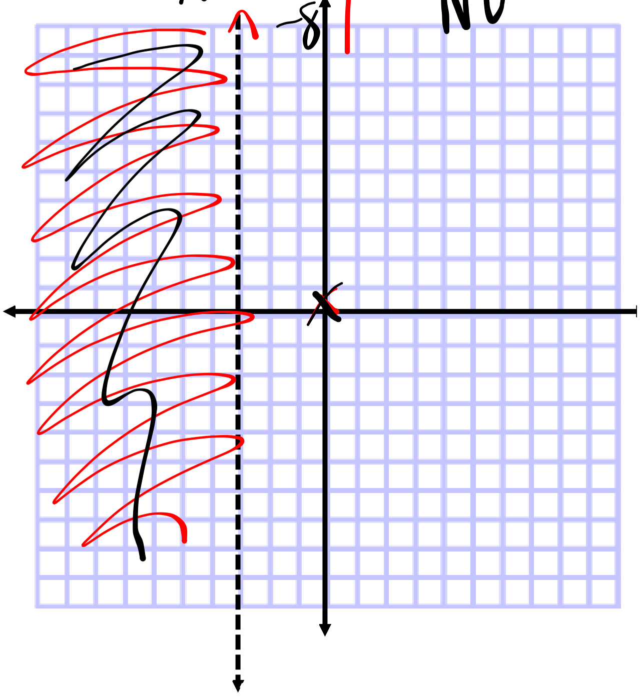
$-4x = 4 + 8$

$\frac{-4x}{-4} = \frac{12}{-4}$

$x = -3$

LS	> RS
$-4(0) - 8$	4
-8	

NO



5.1 Graphing Linear Inequalities in Two Variables

GOAL

Solve problems by modelling linear inequalities in two variables.

EXPLORE...

VERIFY

LS RS

- For which inequalities is (3, 1) a possible solution? How do you know?

x y

a) $13 - 3x > 4y$
b) $2y - 5 \leq x$
c) $y + x < 10$
d) $y \geq 9$

Let's VERIFY...

a) LS > RS

$13-3(3)$ | $4(1)$
$13-9 \not> 4$
4
NO

b) LS ≤ RS

$2(1)-5$ | 3
$-3 \leq$
yes

c) LS < RS

$1+3$ | 10
$4 <$
yes

d) LS > RS

$1 \not> 9$
NO

APPLY the Math EXAMPLE FROM TEXT P. 213

EXAMPLE 1 p. 213 — **Solving a linear inequality graphically when it has a continuous solution set in two variables**

Graph the solution set for this linear inequality.
$-2x + 5y \geq 10$

(0,0) LS $\geq$ RS
$-2(0)+5(0)$ 10
0 NO

$\in$ "belongs to" of
"is a set of"

Domain - x values
Range - y values

Robert's Solution: Using graph paper

Linear equation that represents the boundary:
$-2x + 5y = 10$

I knew that the graph of the linear equation $-2x + 5y = 10$ would form the boundary of the linear inequality $-2x + 5y \geq 10$.

$\frac{5y}{5} = \frac{2x}{5} + \frac{10}{5}$
$y = 2/5x + 2$

The variables represent numbers from the set of real numbers.
$x \in \mathbb{R}$ and $y \in \mathbb{R}$

The domain and range are not stated and no context is given, so I assumed that the domain and range are the set of real numbers. This means that the solution set is **continuous**.

continuous
A connected set of numbers. In a continuous set, there is always another number between any two given numbers. Continuous variables represent things that can be measured, such as time.

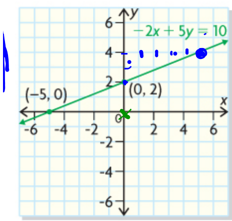
y-intercept:
$$-2x + 5y = 10$$
$$-2(0) + 5y = 10$$
$$\frac{5y}{5} = \frac{10}{5}$$
$$y = 2$$
The *y*-intercept is at (0, 2).

I knew that I needed to plot and join only two points to graph the linear equation. I decided to plot the two intercepts.

To determine the *y*-intercept, I substituted 0 for *x*.

x-intercept:
$$-2x + 5y = 10$$
$$-2x + 5(0) = 10$$
$$\frac{-2x}{-2} = \frac{10}{-2}$$
$$x = -5$$
The *x*-intercept is at $(-5, 0)$.

To determine the *x*-intercept, I substituted 0 for *y*.



Since the linear inequality has the possibility of equality ($\geq$), and the variables represent real numbers, I knew that the **solution region** includes all the points on its boundary. That's why I drew a solid green line through the intercepts.

solution region
The part of the graph of a linear inequality that represents the solution set; the solution region includes points on its boundary if the inequality has the possibility of equality.

Test (0, 0) in $-2x + 5y \geq 10$.

LS	RS
$-2(0) + 5(0)$	10
0	

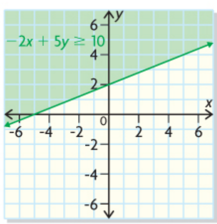
Since 0 is not greater than or equal to 10, (0, 0) is not in the solution region.

I needed to know which **half plane**, above or below the boundary, represents the solution region for the linear inequality.

To find out, I substituted the coordinates of a point in the half plane below the line. I used (0, 0) because it made the calculations simple.

I already knew that the solution region includes points on the boundary, so I didn't need to check a point on the line.

half plane
The region on one side of the graph of a linear relation on a Cartesian plane.



Since my test point below the boundary was not a solution, I shaded the half plane that did not include my test point. This was the region above the boundary.

I used green shading to show that the solution set belongs to the set of real numbers.

Since the domain and range are in the set of real numbers, I knew that the solution set is continuous. Therefore, the solution region includes all points in the shaded area and on the solid boundary.

Communication *Tip*
If the solution set to a linear inequality is continuous and the sign includes equality ($\leq$ or $\geq$), a solid green line is used for the boundary, and the solution region is shaded green.

Graphs of Linear In-Equalities

Sometimes the domain and range are stated as being in the set of integers. This means that the solution set is **discrete** and consists of separate or distinct parts. Discrete variables represent things that can be counted, such as people in a room. This means that the solution region is not shaded but rather stippled with points.

So when interpreting the solution region for a linear inequality, consider the restriction on the domain and range of the variables.

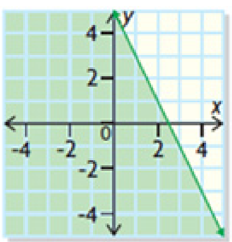
If the solution set is **continuous**, all the points in the solution region are in the solution set. (Shaded) R

If the solution set is **discrete**, only specific point in the solution region are in the solution set. This is represented graphically by stippling. (DOTS) → N, W, I

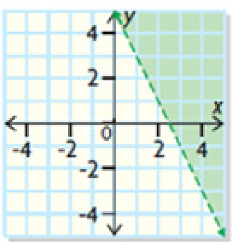
Some solution sets may be restricted to specific quadrants. For example, most linear inequalities representing real-world problem situations have graphs that are restricted to the first quadrant.

Here are some examples:

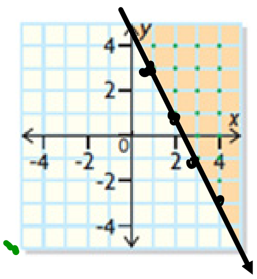
$\{(x, y) \mid y \leq -2x + 5,\ x \in R,\ y \in R\}$



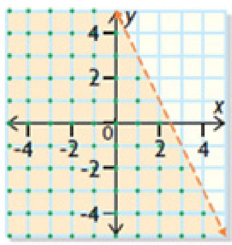
$\{(x, y) \mid y > -2x + 5,\ x \in R,\ y \in R\}$



$\{(x, y) \mid y \geq -2x + 5,\ x \in I,\ y \in I\}$



$\{(x, y) \mid y < -2x + 5,\ x \in I,\ y \in I\}$



Let's do a couple more...

1) $\{(x,y) \mid 2x+5y \le -20, x \varepsilon I, y \varepsilon I\}$

SET

such that

belongs to

Integers

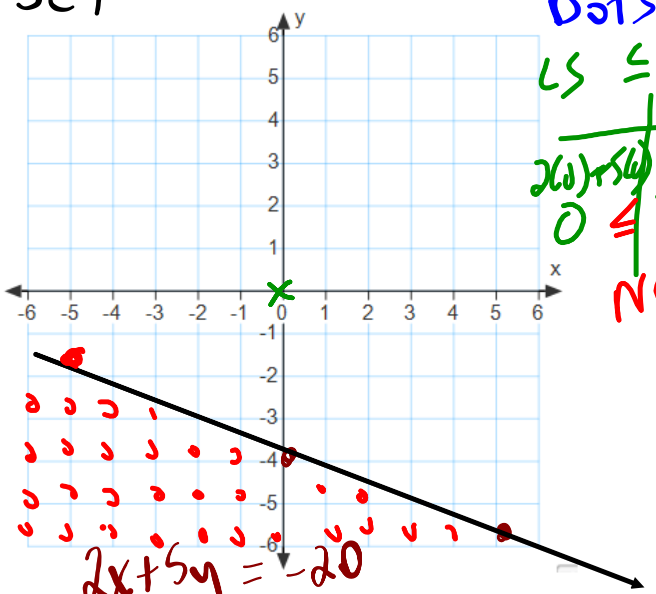
DOTS

LS ≤ RS

$2(0)+5(0) \mid -20$

$0 \le -20$

NO



$2x+5y=-20$

$\frac{5y}{5} = \frac{-2x}{5} - \frac{20}{5}$

$y = -\frac{2}{5}x - 4$

2) $\{(x,y) \mid 3x+4y>4, x \varepsilon W, y \varepsilon W\}$

LS > RS

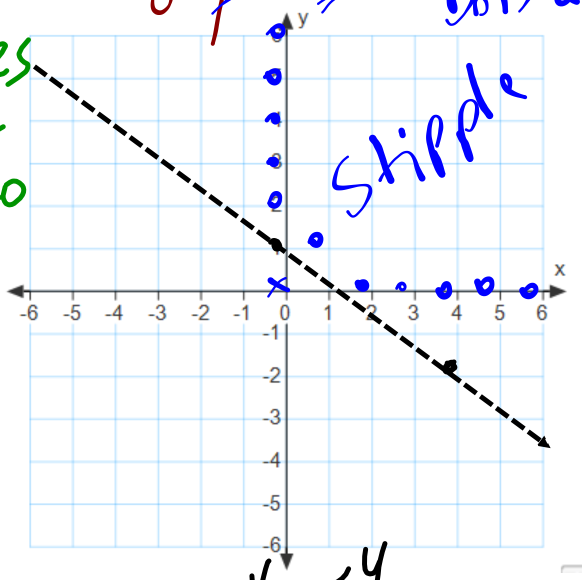
$3(0)+4(0) \mid 4$

$0 > 4$ NO

Whole

DOTS Q1

stipple



$3x+4y=4$

$\frac{4y}{4} = \frac{-3x}{4} + \frac{4}{4}$

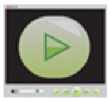
$y = -\frac{3}{4}x + 1$

EXAMPLE 1 Solving a linear inequality graphically when it has a continuous solution set in two variables

Graph the solution set for this linear inequality:
$-2x + 5y \geq 10$

Your Turn

Compare the graphs of the following relations. What do you notice?
$-2x + 5y \geq 10$ $\quad$ $-2x + 5y = 10$ $\quad$ $-2x + 5y < 10$



Answer

They all have the same line or boundary, but the inequality $-2x + 5y \geq 10$ has a solution region that includes the boundary and the half plane above it. The equation $-2x + 5y = 10$ has a solution that includes only the values on the line. The inequality $-2x + 5y < 10$ has a solution region that is the half plane below the boundary and does not include values on the boundary.

EXAMPLE 2 Graphing linear inequalities with vertical or horizontal boundaries

p. 216

Graph the solution set for each linear inequality on a Cartesian plane.

a) $\{(x, y) \mid x - 2 > 0, x \in \mathrm{R}, y \in \mathrm{R}\}$

b) $\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

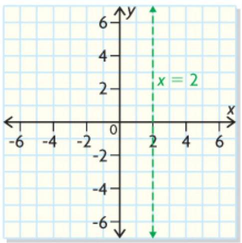
Wynn's Solution

a) $x - 2 > 0$
$x > 2$

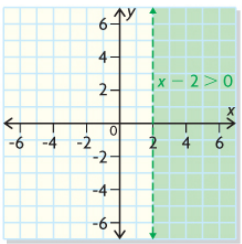
I isolated x so I could graph the inequality.

The variables represent numbers from the set of real numbers.
$x \in \mathrm{R}$ and $y \in \mathrm{R}$

The domain and range are stated as the set of real numbers. The solution set is continuous, so the solution region and its boundary will be green in my graph.



I drew the boundary of the linear inequality as a dashed green line because I knew that the linear inequality (>) does not include the possibility of x being equal to 2.



I needed to decide which half plane to shade. For x to be greater than 2, I knew that any point to the right of the boundary would work.

The solution region includes all the points in the shaded area because the solution set is continuous. The solution region does not include points on the boundary.

Communication Tip

If the solution set to a linear inequality is continuous and the sign does not include equality (< or >), a dashed green line is used for the boundary and the solution region is shaded green.

b) $-3y + 6 \geq -6 + y$
$-4y \geq -12$
$\frac{-4y}{-4} \leq \frac{-12}{-4}$
$y \leq 3$

Since the linear inequality has only one variable, y, I isolated the y.

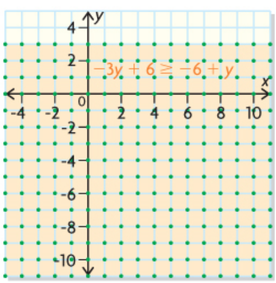
As I rearranged the linear inequality, I divided both sides by −4. That's why I reversed the sign from ≥ to ≤.

The variables represent integers.
$x \in \mathrm{I}$ and $y \in \mathrm{I}$

The domain and range are stated as being in the set of integers. I knew this means that the solution set is **discrete**.

discrete

Consisting of separate or distinct parts. Discrete variables represent things that can be counted, such as people in a room.



$\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

I knew that points with integer coordinates below the line $y = 3$ were solutions, so I shaded the half plane below it orange.

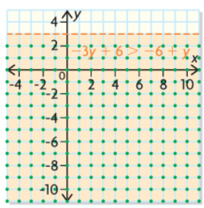
I knew the linear inequality includes 3, so points on the boundary with integer coordinates are also solutions to the linear inequality.

I stippled the boundary and the orange half plane with green points to show that the solution set is discrete.

The solution region includes only the points with integer coordinates in the shaded region and along the boundary.

Communication Tip

If the solution set to a linear inequality is discrete, the solution region is shaded orange and stippled with green points. If the sign includes equality (≥ or ≤), the boundary is also stippled. An example of this is shown to the left. If equality is not possible (< or >), the boundary is a dashed orange line. An example of this is shown below.



HOMEWORK...

p. 221: #2, #4 and #6

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