Linear Relations

5.6 Properties of Linear Relations

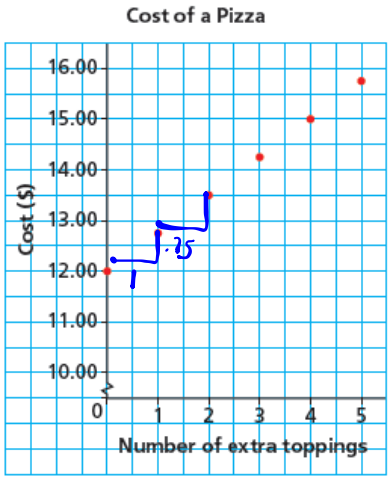
LESSON FOCUS Identify and represent linear relations in different ways.

Make Connections

The table of values and graph show the cost of a pizza with up to 5 extra toppings.

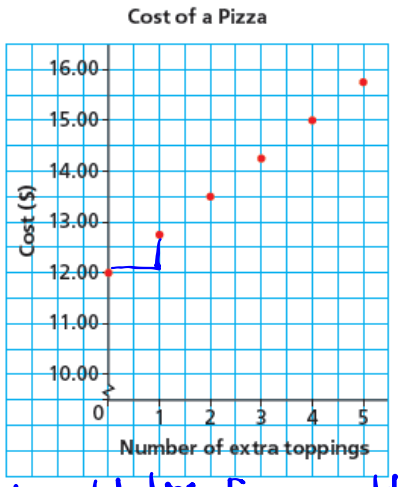
Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

+0.75 +0.75 +0.75 +0.75 +0.75



Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

0.75 0.75 0.75 0.75



What patterns do you see in the table? straight line. For every 1 topping cost ↑ 0.75

Write a rule for the pattern that relates the cost of a pizza to the number of its toppings. $C = 12 + 0.75t$

How are the patterns in the table shown in the graph? topping on horizontal axis + height cost/topping

How can you tell from the table that the graph represents a linear relation? The ratio of the height (0.75) divided by the length is the same between points

5.6 Properties of Linear Relations

EXERCISE...

Which table of values represents a linear relation? Justify your answer.

a) The relation between the number of bacteria in a culture, n, and time, t minutes.

t	n
0	1
20	2
40	4
60	8
80	16
100	32

20 20 20 20 20 +1 +2 +4 +8 +16

1	10
2	20
3	30
4	40
5	50

+1 +1 +1 +1 10 10 10 10

1	10
2	20
4	40
5	50

+1 +2 1 +10 +20 +10

non linear (rise/run) are not the same

b) The relation between the amount of goods and services tax charged, T dollars, and the amount of the purchase, A dollars

A	T
60	3
120	6
180	9
240	12
300	15

60 60 60 60 +3 +3 +3 +3

$m = \frac{3}{60} = \frac{1}{20}$ linear (all the same slope)

Here is another example of a linear relation...

The cost for a car rental is \$60, plus \$20 for every 100 km driven.
The independent variable is the distance driven and the dependent variable is the cost.
We can identify that this is a linear relation in different ways.

- a table of values

x (independent) y (dependent)

Independent variable → | Distance (km) | Cost (\$) | ← Dependent variable

	Distance (km)	Cost (\$)	
	0	60	
+100	100	80	+20
+100	200	100	+20
+100	300	120	+20
+100	400	140	+20

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

5.6 Properties of Linear Relations

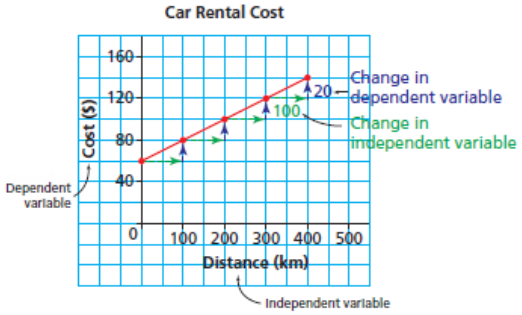
How to identify a linear relation...

- a set of ordered pairs

+100 +100 +100 +100
{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)}
+20 +20 +20 +20

Why is it important that the ordered pairs are listed so their first elements are in numerical order?

- a graph



The graph of a linear relation is a straight line.

We can use each representation to calculate the **rate of change**.

The rate of change can be expressed as a fraction:

$$\frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{\$20}{100 \text{ km}}$$

$$= \$0.20/\text{km}$$

5.6 Properties of Linear Relations

Rate of Change = $\frac{\text{change in dependent variable}}{\text{change in independent variable}}$
(ROC)

$ROC = \frac{\Delta y}{\Delta x}$

$y = \_\_x + \_\_$

slope (ROC) y-intercept

$y = mx + b$

Example:

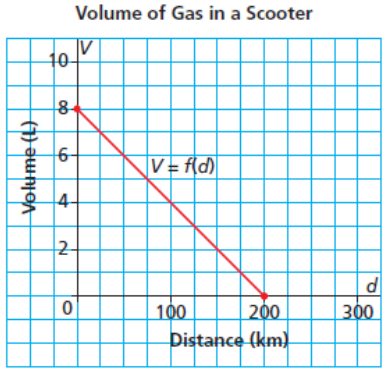
$y = 3x - 2$

$m = 3$

$b = -2$

EXERCISE...

This graph shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.



a) Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

(0, 8) tank full + not moved

(200, 0) travelled 200km + no gas.

b) What are the domain and range of this function?

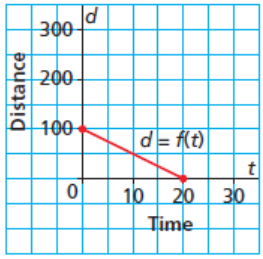
$\{x \mid 0 \le x \le 200, x \in R\}$

$\{y \mid 0 \le y \le 8, y \in R\}$

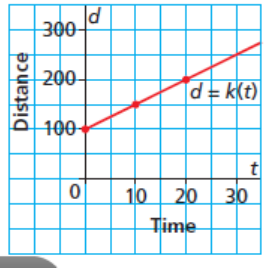
ONE MORE...

Which graph has a rate of change of −5 and a vertical intercept of 100? Justify your answer.

a)



b)



a) $ROC = \frac{\Delta y}{\Delta x} = \frac{-100}{20} = -5$

b) $ROC = \frac{\Delta y}{\Delta x} = \frac{-50}{-10} = 5$

The rate of change is \$0.20/km; that is, for each additional 1 km driven, the rental cost increases by 20¢. The rate of change is constant for a linear relation.

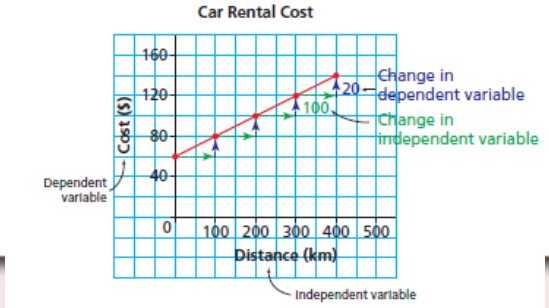
We can determine the rate of change from the equation that represents the linear function.

Let the cost be C dollars and the distance driven be d kilometres.

An equation for this linear function is:

$C = 0.20d + 60$

- initial amount (y intercept)
- independent variable (x)
- rate of change (dependent/independent)
- dependent variable (y)



5.6 Properties of Linear Relations

- Graphing Relations

I. Using a table of values:

Using a Table of Values to Graph a Linear Relation Worksheet

Find the value of "y" in the following table(s) of values.

a) $y = x + 2$

x	y
0	
1	
2	
3	
4	

b) $y = 2x + 1$

x	y
3	
4	
5	
6	
20	

c) $y = 3x - 1$

x	y
8	
9	
10	
11	
25	

d) $y = 2x$

x	y
2	
3	
4	
5	
100	

e) $y = \frac{1}{2}x + 2$

x	y
0	
2	
4	
6	
8	

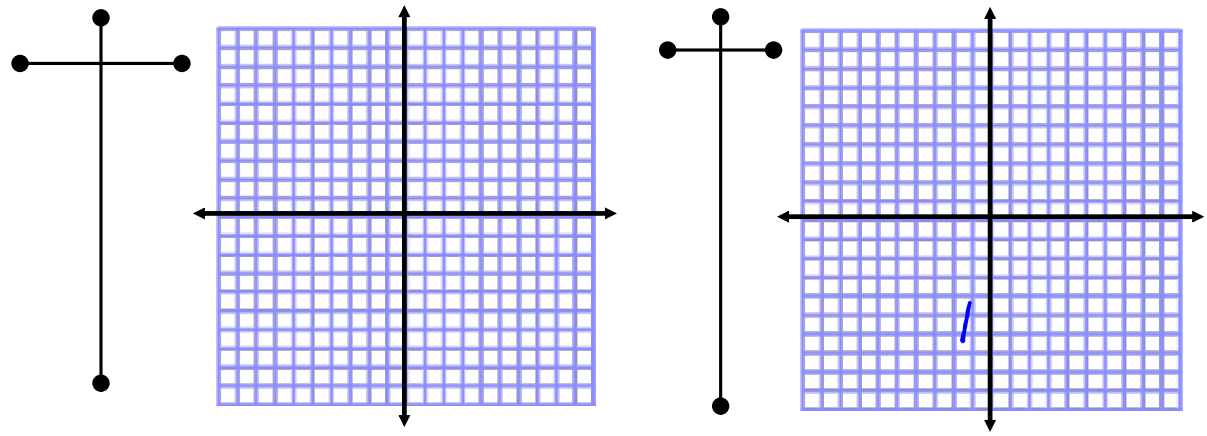
f) $y = \frac{1}{3}x - 3$

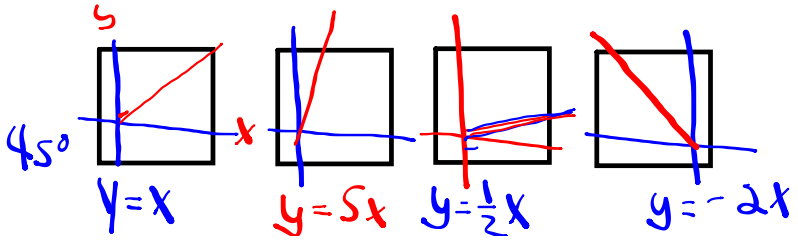
x	y
3	
6	
9	
12	

EXAMPLE: Create a table of values and graph...

- Graph $y = 2x + 3$

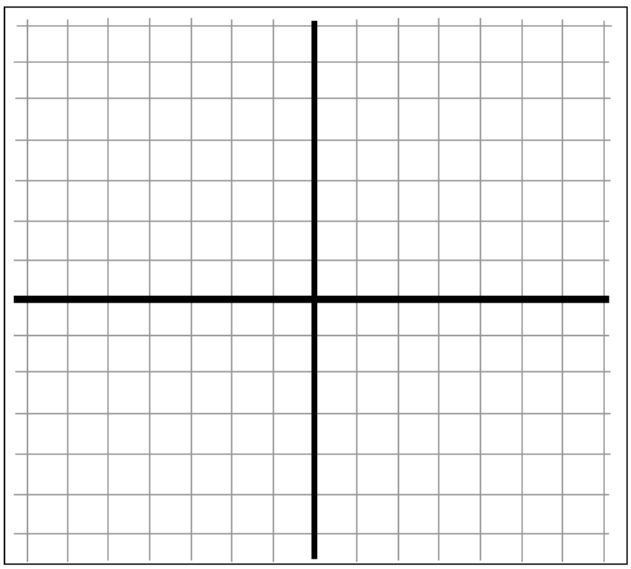
$$y = -\frac{2}{3}x + 1$$





YOUR TURN...

Graph the equation: $y = -3x + 5$



PRACTICE PROBLEMS...

p. 308: #3 - 8, 12, 14, 16