APRIL 10, 2018

UNIT 7: SIMILARITY AND TRANSFORMATIONS

7.4: SIMILAR TRIANGLES

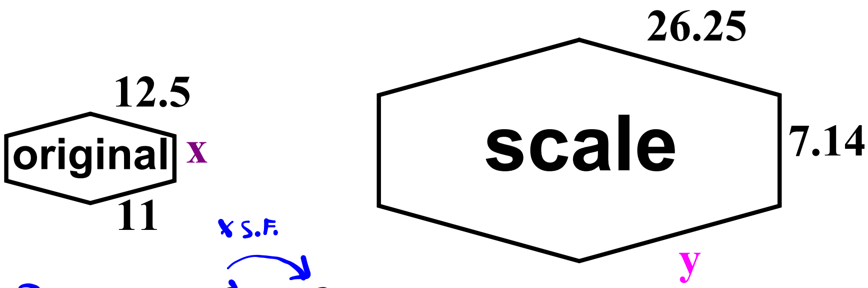
K. Sears
MATH 9

WHAT'S THE POINT OF TODAY'S LESSON?

We will begin working on the Math 9 Specific Curriculum Outcome (SCO) "Shape and Space 3" OR "SS3" which states:

"Demonstrate an understanding of similarity of polygons."

WARM UP: Find the lengths of the missing sides ("x" and "y") in the <u>proportional</u> diagrams below. Show all work.



$S.F = \frac{S}{O}$

$= \frac{26.25}{12.5}$

$= 2.1$

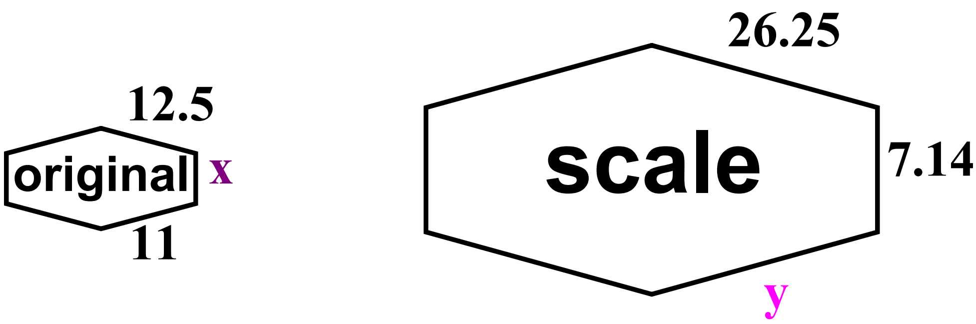
$O \underset{\div S.F.}{\overset{\times S.F.}{\rightleftarrows}} S$

$x = \frac{7.14}{2.1}$

$= 3.4$

$y = (11)(2.1)$

$= 23.1$



A SOLUTION:

Scale Factor = scale/original

= 26.25/12.5

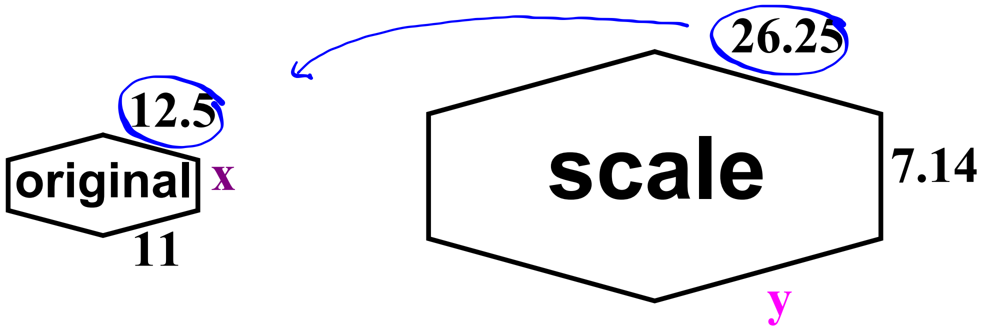
= 2.1

x = 7.14 / 2.1

x = 3.4

y = 11 x 2.1

y = 23.1



ANOTHER SOLUTION:

$$\frac{26.25}{12.5} = \frac{7.14}{x} = \frac{y}{11}$$

$$\frac{26.25}{12.5} = \frac{7.14}{x}$$
$$\frac{26.25x}{26.25} = \frac{89.25}{26.25}$$
$$x = 3.4$$

$$\frac{26.25}{12.5} = \frac{y}{11}$$
$$\frac{288.75}{12.5} = \frac{12.5y}{12.5}$$
$$23.1 = y$$

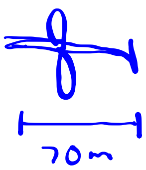
HOMEWORK QUESTIONS?

(pages 329 / 330 / 331, #4 to #6, #8 to #11 & #20)

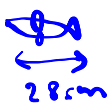
Pg 331

\# 20.

O



S



a) $SF = \frac{S}{O}$
$= \frac{28}{7000}$
$= 0.004$

b) $O = \frac{24}{0.004}$
$= 6000\text{cm}$
$= 60\text{m}$

c) $\text{Tail} = \frac{7.6\text{cm}}{0.004}$
$= 1900\text{cm}$
$= 19\text{m}$

PLEASE TURN TO PAGE 316 IN *MMS9*.

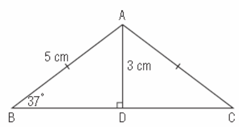
This is a review of several properties you should already know about triangles.

Start Where You Are

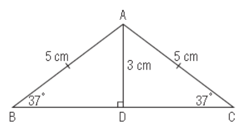
What Should I Recall?

Suppose I have to solve this problem:
Determine the unknown measures of the angles and sides in ΔABC.
The given measures are rounded to the nearest whole number.

I think of what I already know about triangles.
I see that AB and AC have the same hatch marks; this means the sides are equal.
AC = AB
So, AC = 5 cm

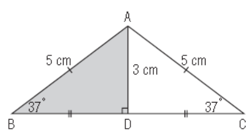


I know that a triangle with 2 equal sides is an isosceles triangle.
So, ΔABC is isosceles.
An isosceles triangle has 2 equal angles that are formed where the equal sides intersect the third side.
I use 3 letters to describe an angle.



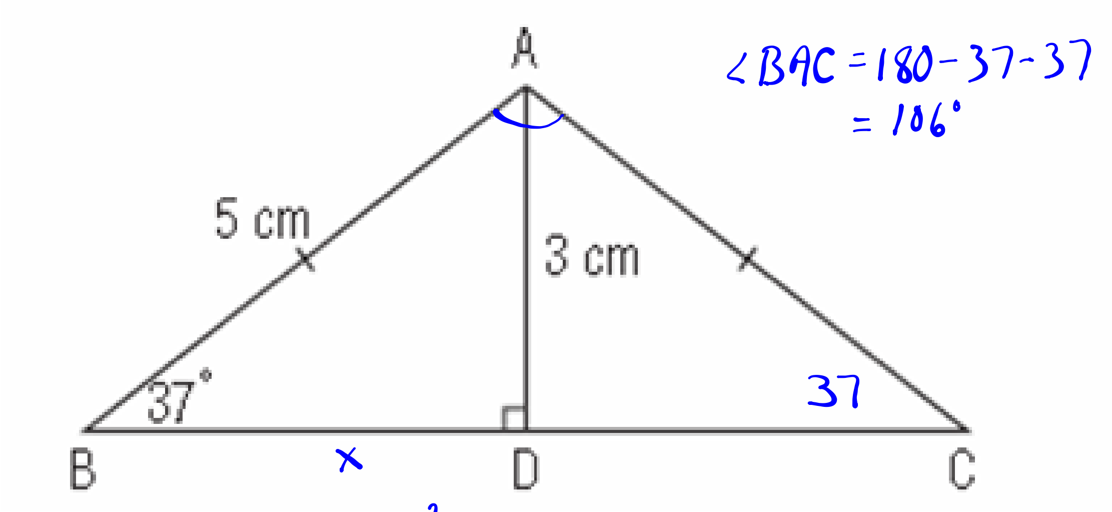
So, $\angle ACD = \angle ABD$
$= 37°$

Since ΔABC is isosceles, the height AD is the perpendicular bisector of the base BC.
So, BD = DC and $\angle ADB = 90°$
I can use the Pythagorean Theorem in ΔABD to calculate the length of BD.



$$AD^2 + BD^2 = AB^2$$
$$3^2 + BD^2 = 5^2$$
$$9 + BD^2 = 25$$
$$9 - 9 + BD^2 = 25 - 9$$
$$BD^2 = 16$$
$$BD = \sqrt{16}$$
$$BD = 4$$

316 UNIT 7: Similarity and Transformations

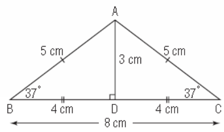


$\angle BAC = 180 - 37 - 37$
$= 106°$

$b^2 = c^2 - a^2$
$x^2 = 5^2 - 3^2$
$= 25 - 9$
$= 16$
$x = \sqrt{16}$
$= 4$

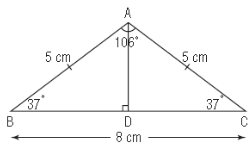
BD = 4 cm
So, BC = 2 × 4 cm
= 8 cm

I know that the sum of the angles in a triangle is 180°.
So, I can calculate the measure of ∠BAC.
∠BAC + ∠ACD + ∠ABD = 180°
∠BAC + 37° + 37° = 180°
∠BAC + 74° = 180°
∠BAC + 74° − 74° = 180° − 74°
∠BAC = 106°



My friend Janelle showed me a different way to calculate.
She recalled that the line AD is a line of symmetry for an isosceles triangle.
So, ΔABD is congruent to ΔACD.

This means that ∠BAD = ∠CAD
Janelle calculated the measure of ∠BAD in ΔABD.
∠BAD + 37° + 90° = 180°
∠BAD + 127° = 180°
∠BAD + 127° − 127° = 180° − 127°
∠BAD = 53°
Then, ∠BAC = 2 × 53°
= 106°

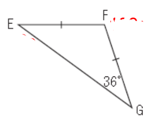


Check

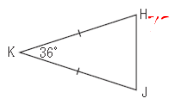
1. Calculate the measure of each angle.

a) ∠ACB

b) ∠GEF and ∠GFE



c) ∠HJK and ∠KHJ



SIMILAR TRIANGLES

TO IDENTIFY SIMILAR TRIANGLES:

* the measures of the 3 pairs of corresponding angles must be EQUAL

OR

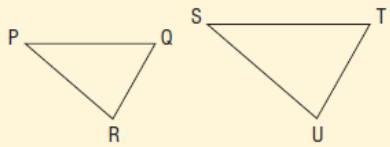
* the ratios of the lengths of the 3 pairs of corresponding sides must be EQUAL; in other words, corresponding sides are proportional

MMS9, Page 344:

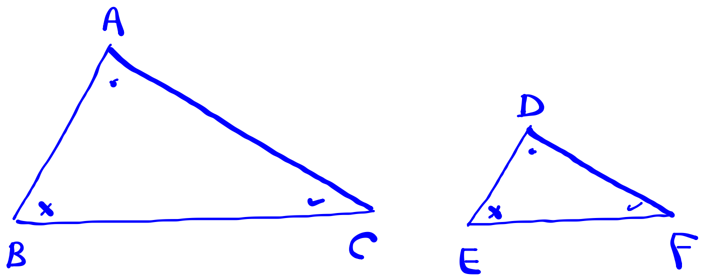
Properties of Similar Triangles

To identify that ΔPQR and ΔSTU are similar, we only need to know that:

- ∠P = ∠S and ∠Q = ∠T and ∠R = ∠U; *or*
- $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$



Similar Triangles



I. Angles are congruent

∠A = ∠D Given

∠B = ∠E Given

∠C = ∠F Given

ΔABC ~ ΔDEF by AAA

II Sides are proportional

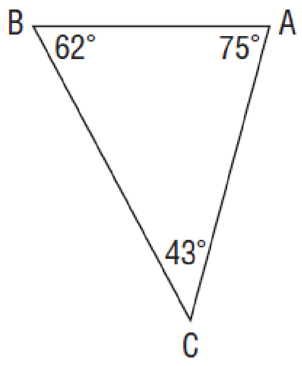
$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

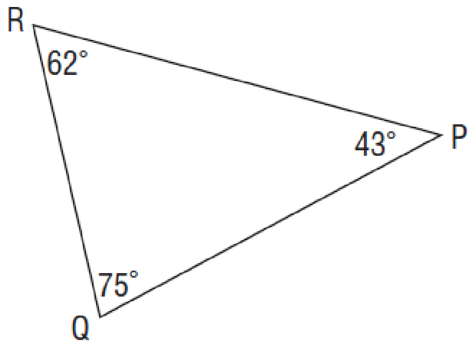
or

$\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$

ΔABC ~ ΔDEF by SSS

ARE THESE TWO TRIANGLES SIMILAR?





EXAMPLE -*MMS9*, PAGE 344:

These triangles are similar because:

$\angle A = \angle Q = 75°$

$\angle B = \angle R = 62°$

$\angle C = \angle P = 43°$

When we name two similar triangles, we order the letters to match the corresponding angles.

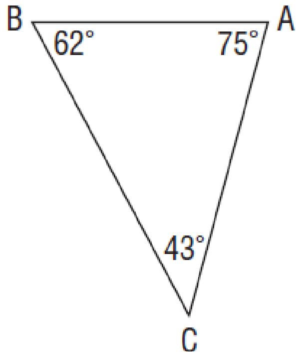
We write: $\Delta ABC \sim \Delta QRP$

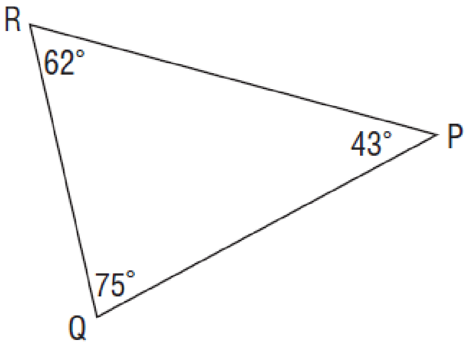
Then we can identify corresponding sides:

AB corresponds to QR.

BC corresponds to RP.

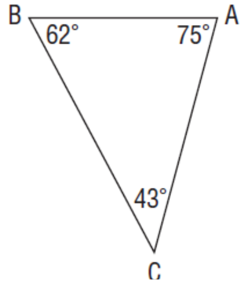
AC corresponds to QP.





EXAMPLE - How you show PROOF OF SIMILARITY (AAA) in your work:

(NOTE: "AAA" = angle; angle; angle)

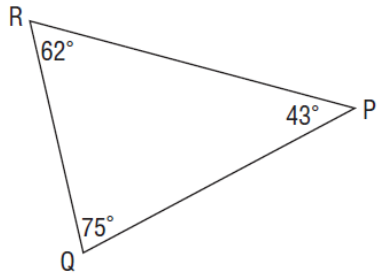


$\angle A = \angle Q$ (GIVEN)

$\angle B = \angle R$ (GIVEN)

$\angle C = \angle P$ (GIVEN)

$\therefore \triangle ABC \sim \triangle QRP$ (AAA)

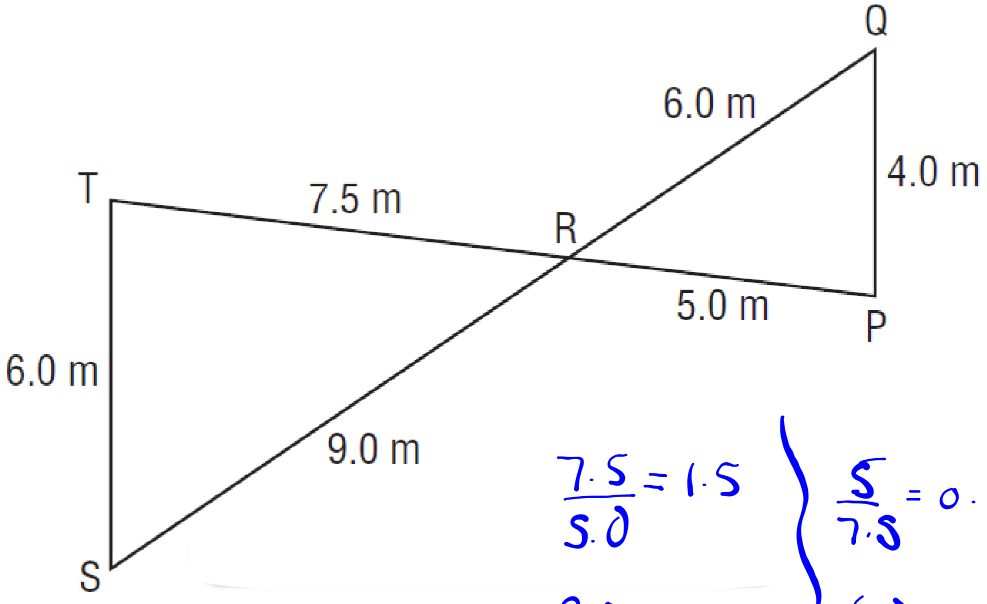


$$\Delta ABC \sim \Delta DEF$$

SYMBOL FOR "SIMILAR TO"

THIS IS CALLED A "SIMILARITY STATEMENT".

ARE THESE TWO TRIANGLES SIMILAR?



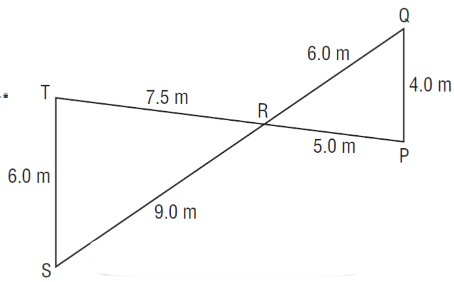
$\Delta TSR \sim \Delta PQR$

$\frac{7.5}{5.0} = 1.5$ $\qquad \frac{5}{7.5} = 0.\overline{6}$

$\frac{9.0}{6.0} = 1.5$ $\qquad \frac{6.0}{9.0} = 0.\overline{6}$

$\frac{6}{4} = 1.5$ $\qquad \frac{4}{6} = 0.\overline{6}$

EXAMPLE -*MMS9*, PAGE 345:



Find out if the corresponding sides are proportional.

$$\frac{ST}{PQ} = \frac{6.0}{4.0} = 1.5$$
$$\frac{TR}{PR} = \frac{7.5}{5.0} = 1.5$$
$$\frac{RS}{QR} = \frac{9.0}{6.0} = 1.5$$

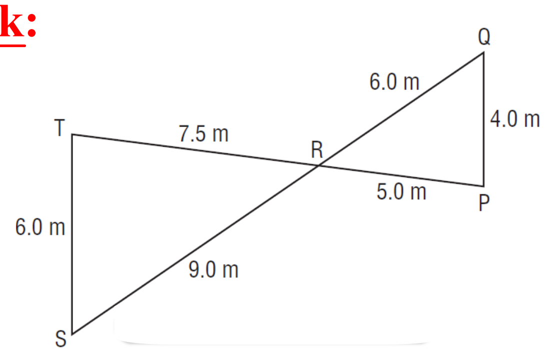
Since the corresponding sides are proportional, the triangles are similar.

P and T are the vertices where the two shorter sides in each triangle meet, so $\angle P$ corresponds to $\angle T$.

Similarly, $\angle Q$ corresponds to $\angle S$ and $\angle TRS$ corresponds to $\angle QRP$.

So, $\Delta PQR \sim \Delta TSR$

EXAMPLE - How you show PROOF OF SIMILARITY (RATIOS) in your work:



$$\frac{PQ}{TS} = \frac{QR}{SR} = \frac{PR}{TR} = \frac{2}{3}$$

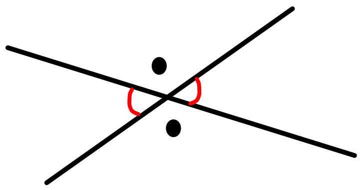
OR

$$\frac{RS}{RQ} = \frac{ST}{QP} = \frac{RT}{RP} = \frac{3}{2} = 1.5$$

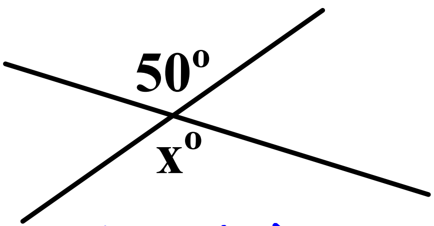
$\therefore \triangle PQR \sim \triangle TSR$ **(RATIOS)**

There are two angle theorems that you will need for your similar triangles proofs:

1. OPPOSITE ANGLES THEOREM (OAT): opposite angles are EQUAL



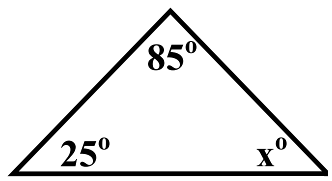
Ex.:



x = 50° OAT

2. SUM OF THE ANGLES IN A TRIANGLE THEOREM (SATT): the sum of the angles in a triangle is 180°.

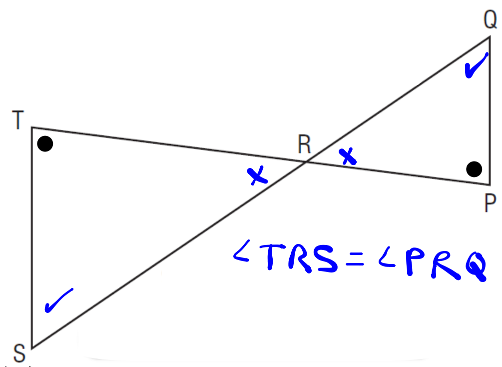
Ex.: Calculate the unknown angle measure.



$x = 180 - 25 - 85$

$= 70°$ SATT

EXAMPLE: **PROVE** that the triangles in the diagram below are **SIMILAR**.



<T = <P (given)

<R = <R (OAT)

<S = <Q (SATT)

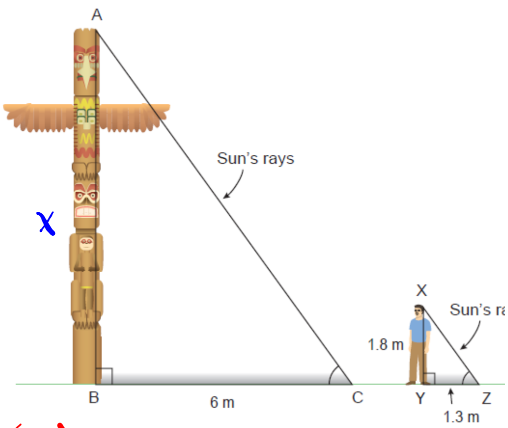
∴ △RST ∼ △RQP (AAA)

EXAMPLE -*MMS9*, PAGE 345 (continued):

In *Example 1*, we can say that ΔTSR is an enlargement of ΔPQR with a scale factor of 1.5.
Or, since $1.5 = \frac{3}{2}$, we can also say that ΔPQR is a reduction of ΔTSR with a scale factor of $\frac{2}{3}$.
We can use the properties of similar triangles to solve problems that involve scale diagrams.
These problems involve lengths that cannot be measured directly.

EXAMPLE -*MMS9*, PAGE 346:

At a certain time of day, a person who is 1.8 m tall has a shadow 1.3 m long. At the same time, the shadow of a totem pole is 6 m long. The sun's rays intersect the ground at equal angles. How tall is the totem pole, to the nearest tenth of a metre?



$(\cancel{1.8})\ \frac{x}{\cancel{1.8}} = \frac{6}{1.3}(1.8)$

$x = \frac{6(1.8)}{1.3}$

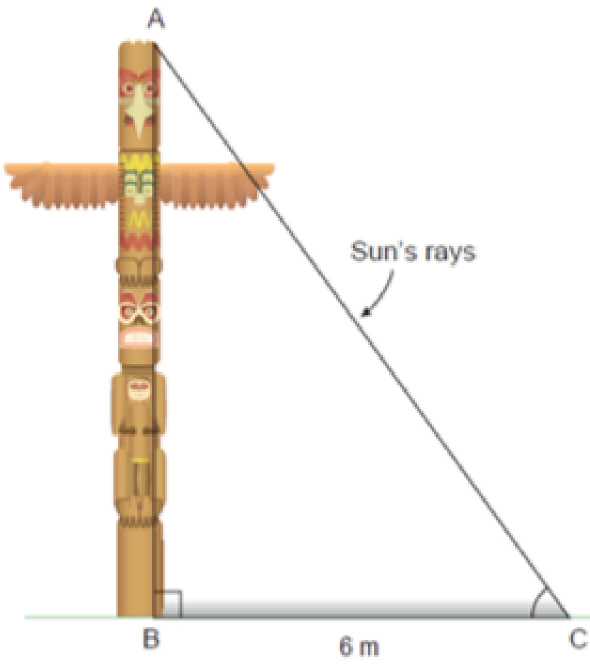
$= 8.31\text{m}$

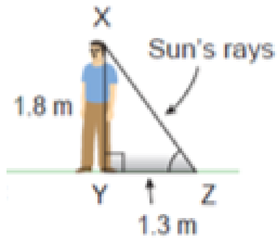
$\frac{x}{1.8} = \frac{6}{1.3}$

$x = \frac{6(1.8)}{1.3}$

$= 8.31\text{m}$

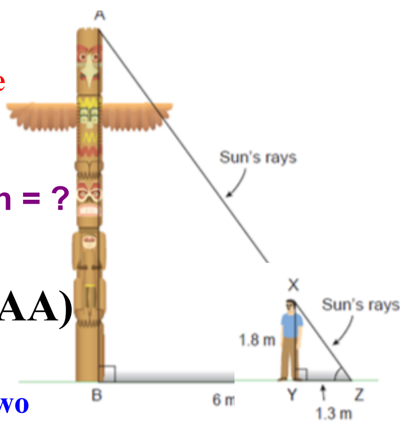
Merge these 2 diagrams together to see how the sun's rays work:





Now, you can prove that the 2 triangles are similar to determine the height of the totem pole:

h = ?



<B = <Y (given)
<C = <Z (common)
<A = <X (SATT)

$\therefore \triangle ABC \sim \triangle XYZ$ **(AAA)**

Now that we have proven these two triangles are similar, we know that their corresponding sides are proportional. We use two pairs of corresponding sides to determine the height of the totem pole:

$$\frac{AB}{XY} = \frac{BC}{YZ}$$

$$\frac{h}{1.8} = \frac{6}{1.3}$$

$$\frac{1.3h}{1.3} = \frac{10.8}{1.3}$$

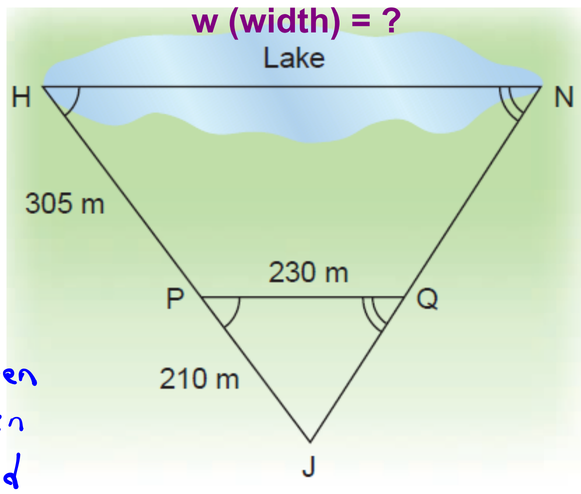
$$h \doteq 8.3076...$$

$$h \doteq 8.3 \text{ m}$$

EXAMPLE:

a) Prove that these 2 triangles are similar.

b) Find the width of the lake to the nearest whole metre.



a) $\angle N = \angle PQJ$ given

$\angle H = \angle JPQ$ given

$\angle J = \angle J$ shared

$\Delta JPQ \sim \Delta JHN$

b) $\frac{HN}{PQ} = \frac{HJ}{PJ}$

$\frac{x}{230} = \frac{515}{210}$

$x = \frac{515(230)}{210}$

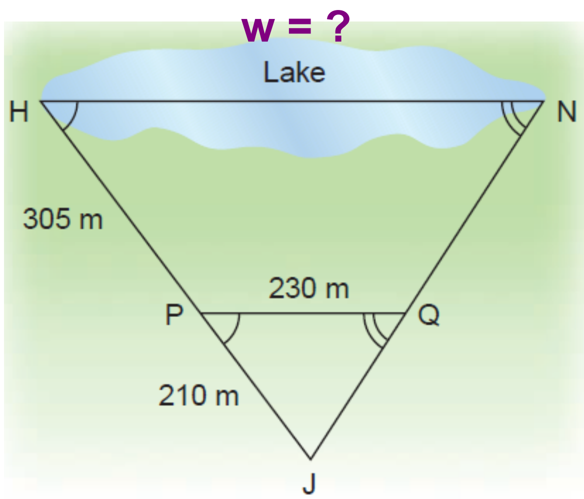
$= 564.05\text{m}$

a) $\angle H = \angle P$ (given)

$\angle N = \angle Q$ (given)

$\angle J = \angle J$ (common)

$\therefore \Delta HJN \sim \Delta PJQ$ (AAA)



b) $\frac{HN}{PQ} = \frac{HJ}{PJ}$

$\frac{w}{230} = \frac{515}{210}$

$\frac{210w}{210} = \frac{118\,450}{210}$

$w \doteq 564.0476...$

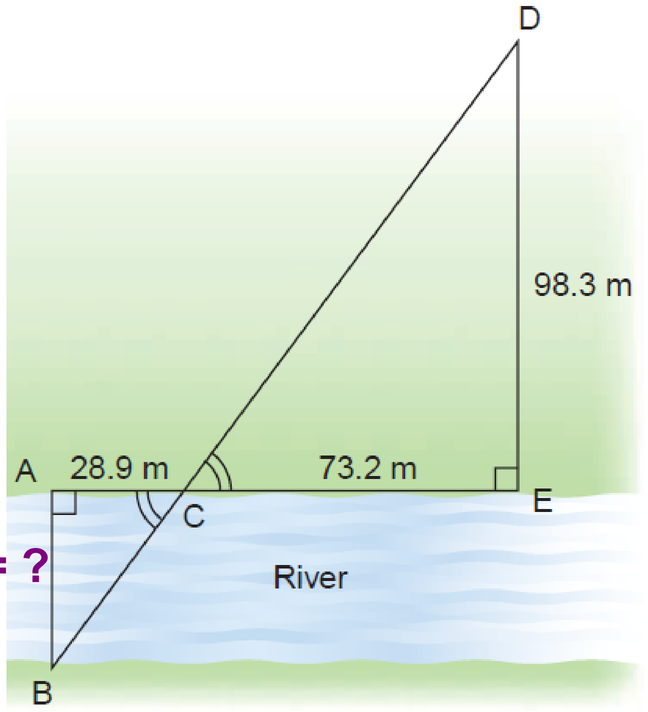
$w \doteq 564\text{ m}$

EXAMPLE:

a) Prove that these 2 triangles are similar.

b) Find the width of the river to the nearest tenth of a metre.

w (width) = ?



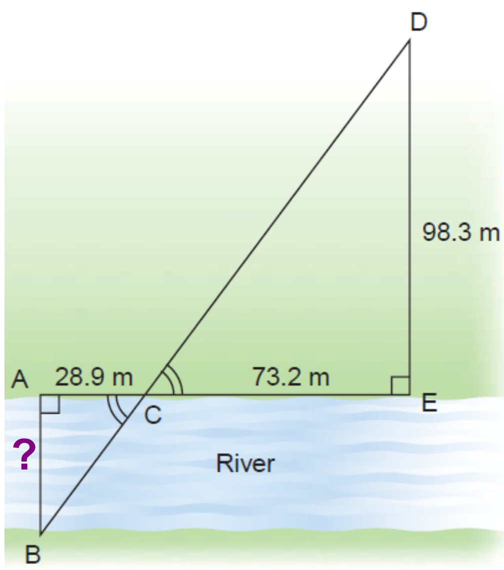
a) <A = <E (given)

<C = <C (given)

<B = <D (SATT)

$\therefore \Delta ABC \sim \Delta EDC$ (AAA)

w = ?



b) $\frac{AB}{ED} = \frac{AC}{EC}$

$\frac{w}{98.3} = \frac{28.9}{73.2}$

$\frac{73.2w}{73.2} = \frac{2\,840.87}{73.2}$

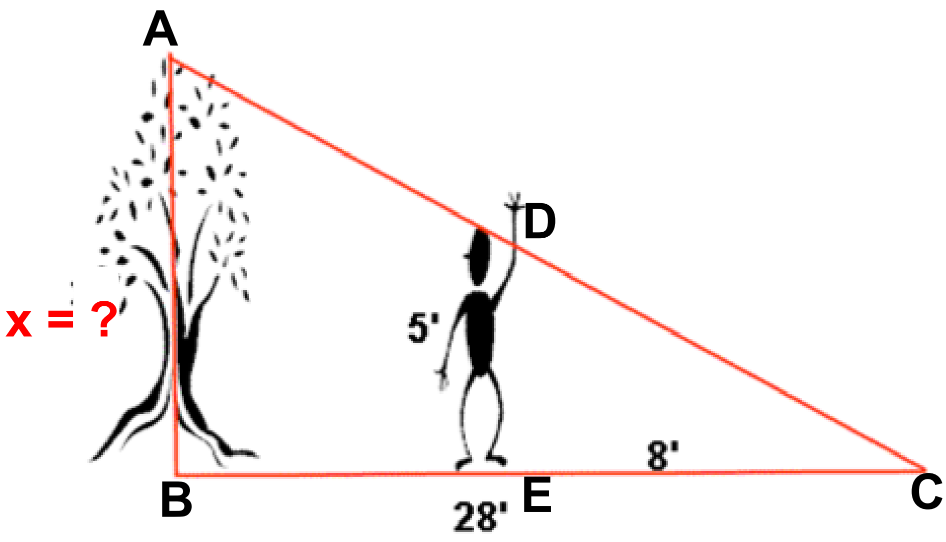
$w \doteq 38.8096...$

$w \doteq 38.8$ m

WARM UP QUIZ:

a) Prove that these 2 triangles are similar.

b) Find the height of the tree to the nearest tenth of a metre.



At a certain time of the day, the shadow of a 5' boy is 8' long. The shadow of a tree at this same time is 28' long. How tall is the tree?

CONCEPT REINFORCEMENT:

MMS9:

Page 341: 4, 5, 6, 9, 11

PAGE 349: #4 (should say, "Are the triangles in each pair..."), 5, 6, 7

PAGE 350: 9 - 11

PAGE 351: #12 TO #15

PAGE 352: #7

PAGE 378: #9 TO #11