

Homework (from yesterday)...

p. 72: #4-6

p. 78: #2, 8, 10, 12, 20

Symbols

// - parallel
⊥ - perpendicular (90°)

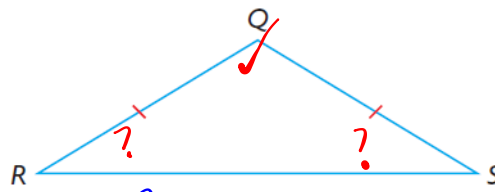
p. 90: #3, 5, 7, 9, 13 [from today's lesson]

p. 90 HW... Section 2.3: #1 - 13

Hw... 7, 8, 5, ~~4~~, 9, 12

PRACTISING

4. If $\angle Q$ is known, write an expression for the measure of one of the other two angles.



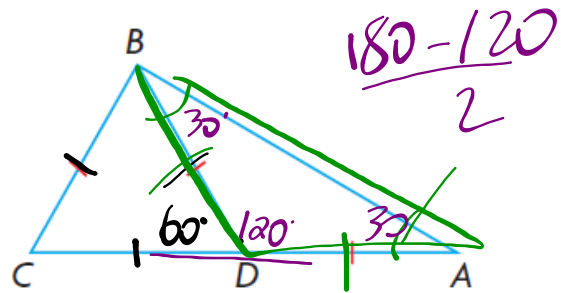
$$\angle R = \angle S$$

$$\frac{1}{2}(180 - \angle Q) \quad \text{OR}$$

$$\frac{180 - \angle Q}{2}$$

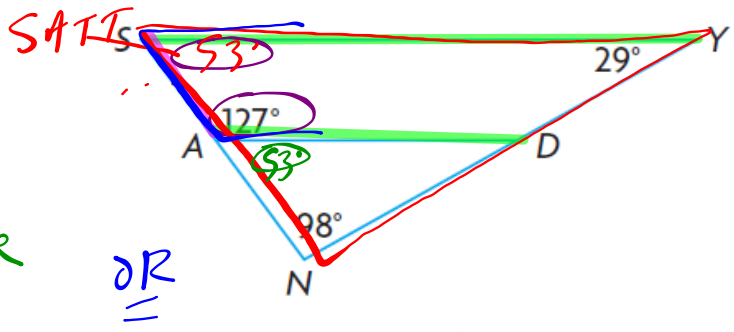
5. Prove: $\angle A = 30^\circ$

S	T
$\angle BDC = 60^\circ$	Equilateral
$\angle BDA = 120^\circ$	SAT
$\angle A = 30^\circ$	ITT \therefore SATT



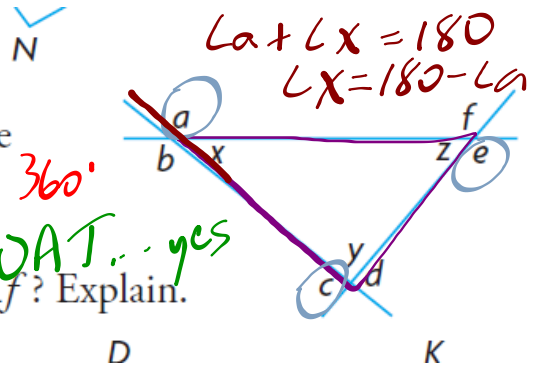
7. Prove: $SY \parallel AD$

<u>S</u>	<u>J</u>
$\angle ASY = 53^\circ$	SATT
$\angle NAD = 53^\circ$	SAT
$\angle ASY = \angle NAD$	Transitive
$\therefore SY \parallel AD$	CA



Statement	Justification
$\angle SAD = 127^\circ$	Given
$\angle ASY = 53^\circ$	SATT
$\angle SAD + \angle ASY = 180^\circ$	Addition
$\therefore SY \parallel AD$	CA

8. Each vertex of a triangle has two exterior angles, as shown.

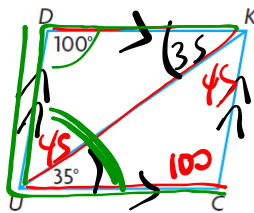


- a) Make a conjecture about the sum of the measures of $\angle a$, $\angle c$, and $\angle e$. *Add to 360°*
- b) Does your conjecture also apply to the sum of the measures of $\angle b$, $\angle d$, and $\angle f$? Explain. *OAT... yes*
- c) Prove or disprove your conjecture.

S	T
$\angle x + \angle y + \angle z = 180^\circ$	SATT
$(180 - \angle a) + (180 - \angle b) + (180 - \angle c) = 180^\circ$	Rearranging
$360 = \angle a + \angle b + \angle c$	

c) Prove or disprove your conjecture.

9. *DUCK* is a parallelogram. Benji determined the measures of the unknown angles in *DUCK*. Paula says he has made an error.



Benji's Solution

Statement

$\angle DKU = \angle KUC$

$\angle DKU = 35^\circ$ ✓

$\angle UDK + \angle DUC = 180^\circ$

$\angle DUK + \angle KUC = 100^\circ$

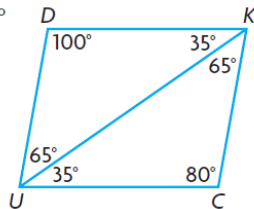
$\angle DUK = 65^\circ$

$\angle UKC = 65^\circ$

$\angle UCK = 180^\circ - (\angle KUC + \angle UKC)$

$\angle UCK = 180^\circ - (35^\circ + 65^\circ)$

$\angle UCK = 80^\circ$



Justification

$\angle DKU$ and $\angle KUC$ are alternate interior angles. ✓

$\angle UDK$ and $\angle DUC$ are ~~corresponding~~ ^{supplementary} angles. ✗

$\angle DUK$ and $\angle UKC$ are alternate interior angles.

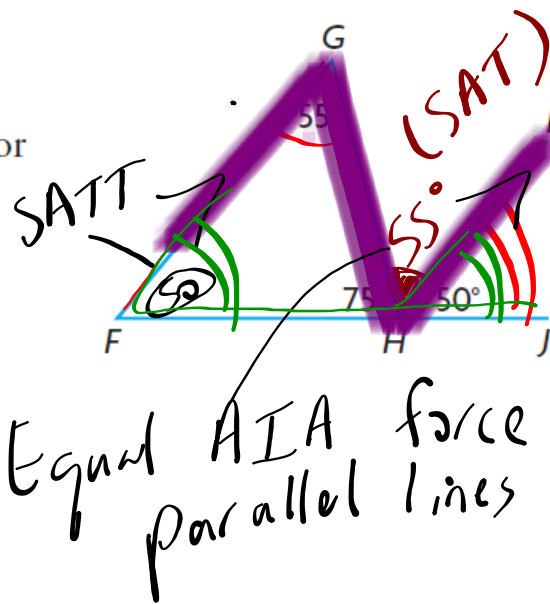
The sum of the measures of the angles in a triangle is 180° .

I redrew the diagram, including the angle measures I determined.

ERROR

- a) Explain how you know that Benji made an error.
- b) Correct Benji's solution.

12. a) Tim claims that FG is not parallel to HI because $\angle FGH \neq \angle IHJ$. Do you agree or disagree? Justify your decision.
- b) How else could you justify your decision? Explain.



2.4

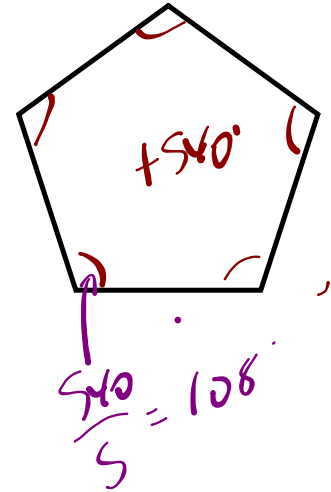
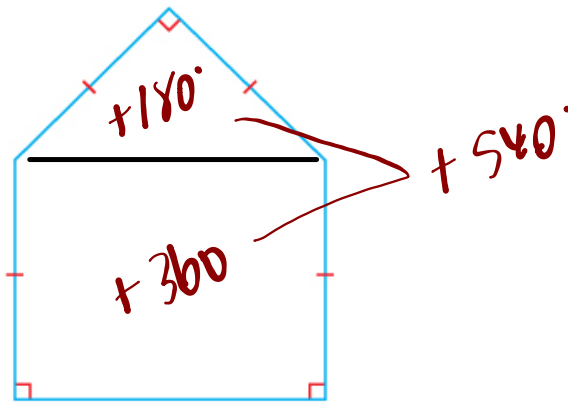
Angle Properties in Polygons

GOAL

Determine properties of angles in polygons, and use these properties to solve problems.

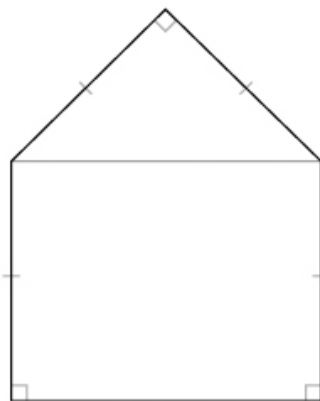
EXPLORE...

- A pentagon has three right angles and four sides of equal length, as shown. What is the sum of the measures of the angles in the pentagon?



SAMPLE ANSWER

I drew a diagonal joining the two angles that are not right angles. This cut the pentagon into a rectangle and a triangle. I knew that the quadrilateral was a rectangle, not a trapezoid, because the two right angles share an arm, so their other arms must be parallel. As well, the other arms are equal length. I knew that the sum of the measures of the angles in a rectangle is 360° and the sum of the measures of the angles in a triangle is 180° , so the sum of the measures of the angles in the pentagon must be 540° .



Function Notation

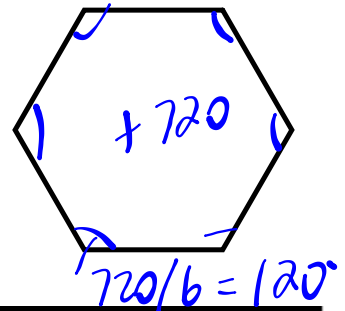
$$S(6) = 180(6-2) = 720^\circ$$

convex polygon ← focus

A polygon in which each interior angle measures less than 180° .

convex non-convex (concave)

$f(x)$



This is my conjecture: The sum of the measures of the interior angles in a polygon, $S(n)$, is:

$$S(n) = 180^\circ(n - 2)$$

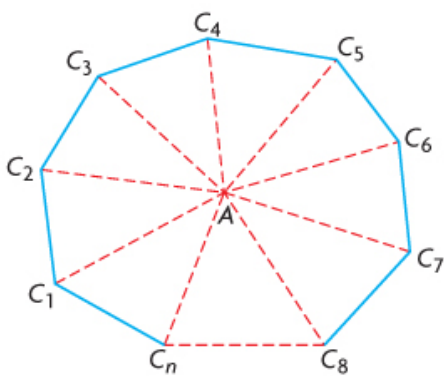
$$S = 180(n - 2)$$

APPLY the Math Deriving the formula...

EXAMPLE 1 Reasoning about the sum of the interior angles of a polygon

Prove that the sum of the measures of the interior angles of any n -sided **convex polygon** can be expressed as $180^\circ(n - 2)$.

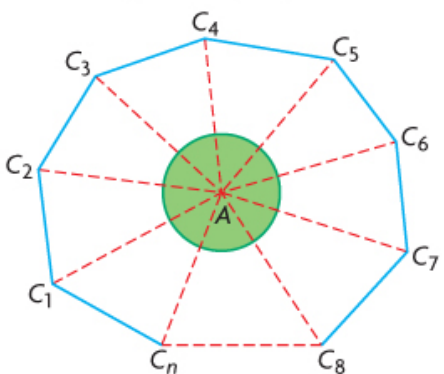
Viktor's Solution



I drew an n -sided polygon. I represented the n th side using a broken line. I selected a point in the interior of the polygon and then drew line segments from this point to each vertex of the polygon. The polygon is now separated into n triangles.

The sum of the measures of the angles in each triangle is 180° .

The sum of the measures of the angles in n triangles is $n(180^\circ)$.



Two angles in each triangle combine with angles in the adjacent triangles to form two interior angles of the polygon.

Each triangle also has an angle at vertex A . The sum of the measures of the angles at A is 360° because these angles make up a complete rotation. These angles do not contribute to the sum of the interior angles of the polygon.

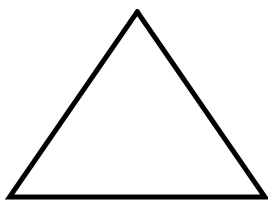
The sum of the measures of the interior angles of the polygon, $S(n)$, where n is the number of sides of the polygon, can be expressed as:

$$S(n) = 180^\circ n - 360^\circ$$

$$S(n) = 180^\circ(n - 2)$$

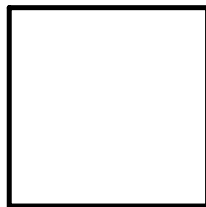
The sum of the measures of the interior angles of a convex polygon can be expressed as $180^\circ(n - 2)$.

Regular Polygon → all angles / sides are equal

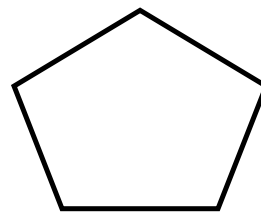


Equilateral

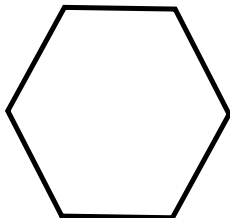
Triangle



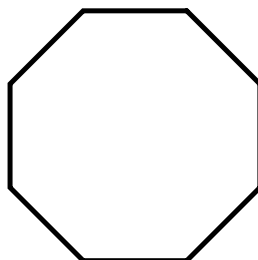
Square



Pentagon



Hexagon



Octagon



EXAMPLE 2 Reasoning about angles in a regular polygon

Outdoor furniture and structures like gazebos sometimes use a regular hexagon in their building plan. Determine the measure of each interior angle of a regular hexagon.



Nazra's Solution

Let $S(n)$ represent the sum of the measures of the interior angles of the polygon, where n is the number of sides of the polygon.

$S(n) = 180^\circ(n - 2)$ ----- A hexagon has six sides, so $n = 6$.

$S(6) = 180^\circ[(6) - 2]$

$S(6) = 720^\circ$

$\frac{720^\circ}{6} = 120^\circ$ -----

The measure of each interior angle of a regular hexagon is 120° .

Since the measures of the angles in a regular hexagon are equal, each angle must measure $\frac{1}{6}$ of the sum of the angles.

Tiling Using Regular Polygons...

Regular Polygon	Measure of Interior Angle (degrees)
Equilateral Triangle	60
Square	90
Pentagon	108
Hexagon	120
Heptagon (7 sided)	128.3
Octagon	135
Nonagon (9 sided)	140
Decagon (10 sided)	144

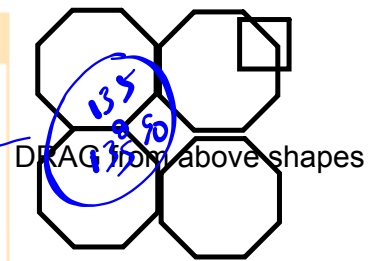
$$\frac{180(10-2)}{10}$$

←

EXAMPLE 3 Visualizing tessellations

A floor tiler designs custom floors using tiles in the shape of regular polygons. Can the tiler use congruent regular octagons and congruent squares to tile a floor, if they have the same side length?

360°



Vanessa's Solution

$$S(n) = 180^\circ(n - 2)$$

$$S(8) = 180^\circ[(8) - 2]$$

$$S(8) = 1080^\circ$$

$$\frac{1080^\circ}{8} = 135^\circ$$

Since an octagon has eight sides, $n = 8$.

First, I determined the sum of the measures of the interior angles of an octagon. Then I determined the measure of each interior angle in a regular octagon.

The measure of each interior angle in a regular octagon is 135° .
The measure of each internal angle in a square is 90° .

Two octagons fit together, forming an angle that measures:

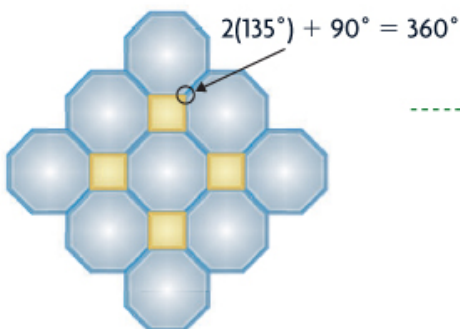
$$2(135^\circ) = 270^\circ$$

$$\text{This leaves a gap of } 90^\circ.$$

$$2(135^\circ) + 90^\circ = 360^\circ$$

I knew that three octagons would not fit together, as the sum of the angles would be greater than 360° .

A square can fit in this gap if the sides of the square are the same length as the sides of the octagon.

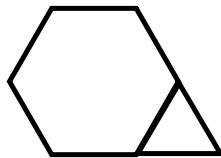


I drew what I had visualized using dynamic geometry software.

The tiler can tile a floor using regular octagons and squares when the polygons have the same side length.

Your Turn

Can a tiling pattern be created using regular hexagons and equilateral triangles that have the same side length? Explain.

Answer

In Summary

Key Idea

- You can prove properties of angles in polygons using other angle properties that have already been proved.

Need to Know

- The sum of the measures of the interior angles of a convex polygon with n sides can be expressed as $180^\circ(n - 2)$.
- The measure of each interior angle of a regular polygon is $\frac{180^\circ(n - 2)}{n}$.
- The sum of the measures of the exterior angles of any convex polygon is 360° .

HOMEWORK...

$$S = 180(n-2)$$

Page 99: 1, 3, 4, 5, 10, 11, 16

HISTORY on Buckyball Do A, B and C

Attachments

2s4e3 finalt.mp4