NOTES - Chapter 1 Definitions.docx

1.1 Making Conjectures: Inductive Reasoning

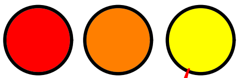
GOAL

Use reasoning to make predictions.

1) G B P
2) G B I V
3) R O Y
4) O R
5) Y O R

EXPLORE...

- If the first three colours in a sequence are red, orange, and yellow, what colours might be found in the rest of the sequence? Explain.

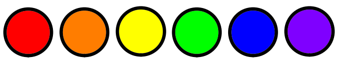


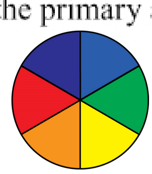
R O Y

SAMPLE ANSWER

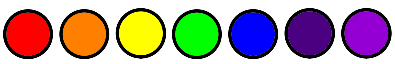
Here are three possible answers:

- If the colour sequence is red, orange, and yellow, the rest of the sequence may be green, blue, and purple. These colours are the primary and secondary colours seen on a colour wheel.

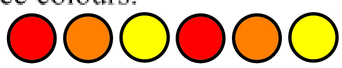




- If the colour sequence is red, orange, and yellow, the rest of the sequence may be green, blue, indigo, and violet. These colours are those of a rainbow.



- If the colour sequence is red, orange, and yellow, the rest of the sequence may repeat these three colours.



INVESTIGATE the Math

Georgia, a fabric artist, has been patterning with equilateral triangles. Consider Georgia's **conjecture** about the following pattern.

> **conjecture**
> A testable expression that is based on available evidence but is not yet proved.

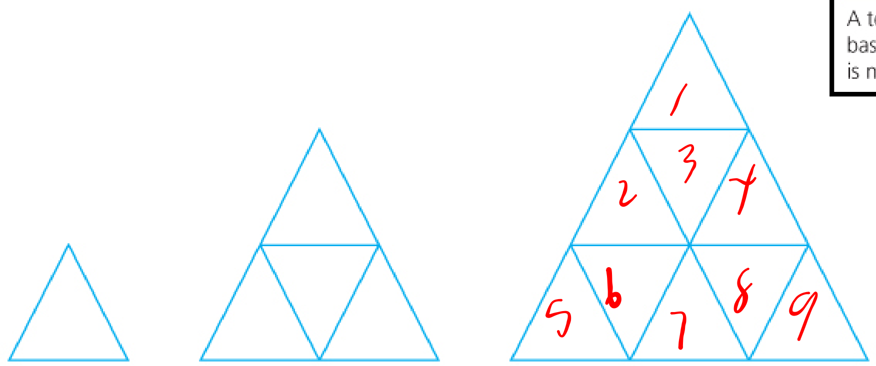


Figure 1 Figure 2 Figure 3

I think Figure 10 in this pattern will have 100 triangles, and all these triangles will be congruent to the triangle in Figure 1.

How did Georgia arrive at this conjecture?

A. Organize the information about the pattern in a table.

Figure	1	2	3	4	5	6	7	8	9	10
Number of Triangles	1	4	9	16	25	36	49	64	81	100

+3 +5 +7

B. With a partner, discuss what you notice about the data in the table.

C. Extend the pattern for two more figures.

D. What numeric pattern do you see in the table?

Answers

A.

Figure	1	2	3	4	5	6	7	8	9	10
Number of Triangles	1	4	9	16	25	36	49	64	81	100

B. The pattern in the table shows that the number of triangles equals the square of the figure number.

C.

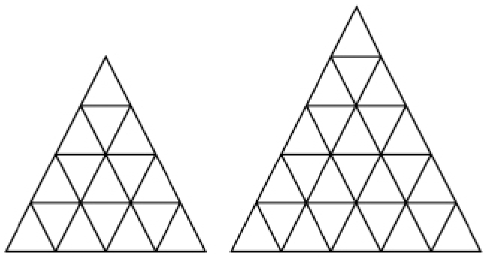


Figure 4 Figure 5

D. Figure 11 has 11^2 or 121 triangles.
Figure 12 has 12^2 or 144 triangles.

The numeric pattern in the table shows that each figure will have a perfect square of congruent triangles. The number of congruent triangles in each figure is the square of the figure number.

Reflecting

E. Is Georgia's conjecture reasonable? Explain.

F. How did Georgia use **inductive reasoning** to develop her conjecture?

G. Is there a different conjecture you could make based upon the pattern you see? Explain.

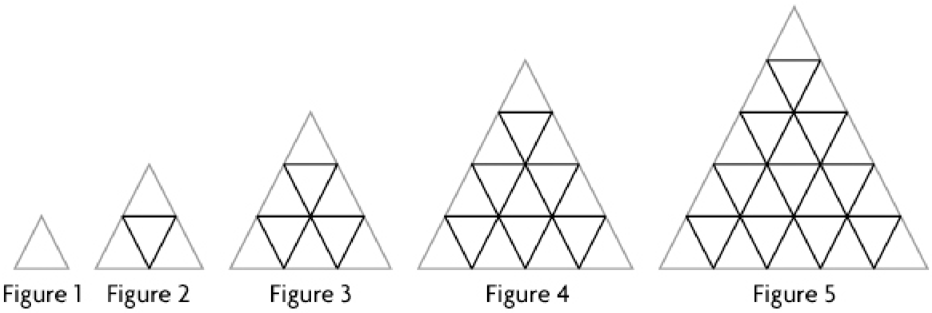
inductive reasoning
Drawing a general conclusion by observing patterns and identifying properties in specific examples.

Answers

E. Georgia's conjecture is reasonable because, when the table is extended to the 10th figure, the pattern of values is the same as Georgia's prediction.

F. Georgia used inductive reasoning by gathering evidence about more cases. This evidence established a pattern. Based on this pattern, Georgia made a prediction about what the values would be for a figure not shown in the evidence.

G. A different conjecture could be made because a different pattern could have been seen. If the focus had been only on the congruent triangles with their vertices at the bottom and their horizontal sides at the top, then the following conjecture could have been made: The 5th figure will have 10 congruent triangles.



EXAMPLE 2 Using inductive reasoning to develop a conjecture about integers

Make a conjecture about the product of two odd integers.

NOTE: Must do 3 Examples

Jay's Solution

$(+3)(+7) = (+21)$

Odd integers can be negative or positive. I tried two positive odd integers first. The product was positive and odd.

$(-5)(-3) = (+15)$

Next, I tried two negative odd integers. The product was again positive and odd.

$(+3)(-3) = (-9)$

Then I tried the other possible combination: one positive odd integer and one negative odd integer. This product was negative and odd.

My conjecture is that the product of two odd integers is an odd integer.

I noticed that each pair of integers I tried resulted in an odd product.

$(-211)(-17) = (+3587)$

I tried other integers to test my conjecture. The product was again odd.

EXAMPLE 2 Using inductive reasoning to develop a conjecture about integers

Make a conjecture about the product of two odd integers.

NOTE: Must do 3 Examples

Jay's Solution

1) $3 \times 3 = 9$
2) $-11 \times 9 = -99$
3) $101 \times -73 = -7373$

Always be ODD

EXAMPLE 2 Using inductive reasoning to develop a conjecture about integers

Make a conjecture about the product of two odd integers.

Your Turn

Do you find Jay's conjecture convincing? Why or why not?

Answer

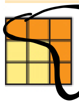
Here are two possible answers:

- Yes. Jay's conjecture is convincing because all the different combinations with positive and negative odd integers were used as samples. These three samples showed a pattern in their products, which Jay then tested with different integers. Jay's conjecture was supported by this last sample.
- No. Jay looked at only three cases before he made his conjecture, then tested it with only one more example. This is not a lot of evidence to base a conjecture on.

EXAMPLE 3 Using inductive reasoning to develop a conjecture about perfect squares

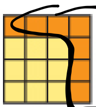
Make a conjecture about the difference between consecutive perfect squares.

Steffan's Solution: Comparing the squares geometrically

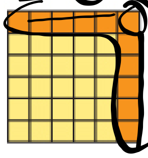


9-4=5

I represented the difference using unit tiles for each perfect square. First, I made a 3×3 square in orange and placed a yellow 2×2 square on top. When I subtracted the 2×2 square, I had 5 orange unit tiles left.



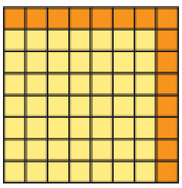
16-9=7



Next, I made 3×3 and 4×4 squares. When I subtracted the 3×3 square, I was left with 7 orange unit tiles. I decided to try greater squares.

My conjecture is that the difference between consecutive squares is always an odd number.

I saw the same pattern in all my examples: an even number of orange unit tiles bordering the yellow square, with one orange unit tile in the top right corner. So, there would always be an odd number of orange unit tiles left, since an even number plus one is always an odd number.



I tested my conjecture with the perfect squares 7×7 and 8×8. The difference was an odd number.

The example supports my conjecture.

EXAMPLE 3 Using inductive reasoning to develop a conjecture about perfect squares

Make a conjecture about the difference between consecutive perfect squares.

1) $100 - 81 = 19$
2) $25 - 16 = 9$
3) $49 - 36 = 13$

Always be ODD

EXAMPLE 3 Using inductive reasoning to develop a conjecture about perfect squares

Make a conjecture about the difference between consecutive perfect squares.

Francesca's Solution: Describing the difference numerically

$2^2 - 1^2 = 4 - 1$
$2^2 - 1^2 = 3$

I started with the smallest possible perfect square and the next greater perfect square: 1^2 and 2^2. The difference was 3.

$4^2 - 3^2 = 7$
$9^2 - 8^2 = 17$

Then I used the perfect squares 3^2 and 4^2. The difference was 7. So, I decided to try even greater squares.

My conjecture is that the difference between consecutive perfect squares is always a prime number.

I thought about what all three differences—3, 7, and 17—had in common. They were all prime numbers.

$12^2 - 11^2 = 23$

To test my conjecture, I tried the perfect squares 11^2 and 12^2. The difference was a prime number.

The example supports my conjecture.

EXAMPLE 3 Using inductive reasoning to develop a conjecture about perfect squares

Make a conjecture about the difference between consecutive perfect squares.

Your Turn

How is it possible to have two different conjectures about the same situation? Explain.

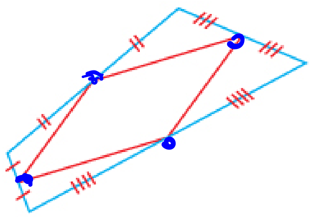
Answer

It is possible to have two different conjectures about the same situation because different samples were used to develop the conjecture. Francesca used different values for the sizes of consecutive squares. When she examined her evidence, the common feature from her examples was different from the common feature that Steffan found from the evidence he had developed.

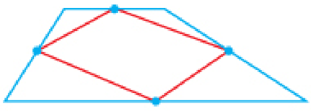
EXAMPLE 4 Using inductive reasoning to develop a conjecture about quadrilaterals

Make a conjecture about the shape that is created by joining the midpoints of adjacent sides in any quadrilateral.

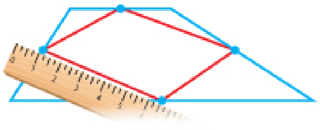
Marc's Solution: Using a protractor and ruler



I drew an irregular quadrilateral on tracing paper. I used my ruler to determine the midpoints of each side. I joined the midpoints of adjacent sides to form a new quadrilateral. This quadrilateral looked like a parallelogram.



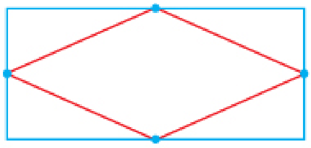
Next, I drew a trapezoid with sides that were four different lengths. I determined the midpoints of the sides. When the midpoints were joined, the new quadrilateral looked like a parallelogram.



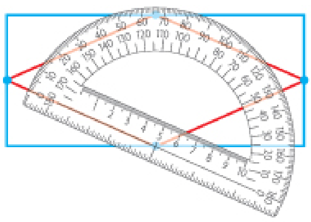
I used my ruler to confirm that the opposite sides were equal.

My conjecture is that joining the adjacent midpoints of any quadrilateral will create a parallelogram.

Each time I joined the midpoints, a parallelogram was formed.



To check my conjecture one more time, I drew a rectangle. I determined its midpoints and joined them. This quadrilateral also looked like a parallelogram.



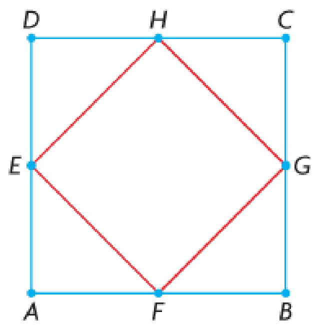
I checked the measures of the angles in the new quadrilateral. The opposite angles were equal. The new quadrilateral was a parallelogram, just like the others were.

The rectangle example supports my conjecture.

EXAMPLE 4 Using inductive reasoning to develop a conjecture about quadrilaterals

Make a conjecture about the shape that is created by joining the midpoints of adjacent sides in any quadrilateral.

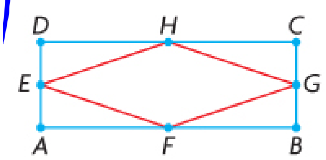
Tracey's Solution: Using dynamic geometry software



$\overline{HE} = 2.4$ cm
$\overline{EF} = 2.4$ cm
$\overline{FG} = 2.4$ cm
$\overline{GH} = 2.4$ cm

$\angle EFG = 90°$
$\angle FGH = 90°$
$\angle GHE = 90°$
$\angle HEF = 90°$

I constructed a square and the midpoints of the sides. Then I joined the adjacent midpoints. *EFGH* looked like a square. I checked its side lengths and angle measures to confirm that it was a square.



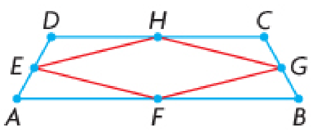
$\overline{HE} = 1.6$ cm
$\overline{EF} = 1.6$ cm
$\overline{FG} = 1.6$ cm
$\overline{GH} = 1.6$ cm

$\angle EFG = 143°$
$\angle FGH = 37°$
$\angle GHE = 143°$
$\angle HEF = 37°$

Next, I constructed a rectangle and joined the adjacent midpoints to create a new quadrilateral, *EFGH*. The side lengths and angle measures of *EFGH* showed that *EFGH* was a rhombus but not a square.

My conjecture is that the quadrilateral formed by joining the adjacent midpoints of any quadrilateral is a rhombus.

Since a square is a rhombus with right angles, both of my examples resulted in a rhombus.



$\overline{HE} = 1.6$ cm
$\overline{EF} = 1.6$ cm
$\overline{FG} = 1.6$ cm
$\overline{GH} = 1.6$ cm

$\angle EFG = 152°$
$\angle FGH = 28°$
$\angle GHE = 152°$
$\angle HEF = 28°$

To check my conjecture, I tried an isosceles trapezoid. The new quadrilateral, *EFGH*, was a rhombus.

The isosceles trapezoid example supports my conjecture.

EXAMPLE 4 Using inductive reasoning to develop a conjecture about quadrilaterals

Make a conjecture about the shape that is created by joining the midpoints of adjacent sides in any quadrilateral.

Your Turn

a) Why did the students draw different conjectures?

b) Do you think that both conjectures are valid? Explain.

Answers

a) The quadrilaterals that Marc and Tracey used were different. The quadrilaterals that Marc used were more varied than those that Tracey used.

b) Based on the evidence used, both conjectures seem valid. The conjecture that Marc developed would hold true for all of Tracey's quadrilaterals, since a rhombus is a special type of parallelogram. But Tracey's conjecture would not hold true for all of Marc's quadrilaterals, since not all parallelograms are rhombuses.

In Summary

Key Idea

- Inductive reasoning involves looking at specific examples. By observing patterns and identifying properties in these examples, you may be able to make a general conclusion, which you can state as a conjecture.

Need to Know

- A conjecture is based on evidence you have gathered.
- More support for a conjecture strengthens the conjecture, but does not prove it.

HW... Trig Test

p. 12: #1 - 3; #6 - 11; 13; 15; 16

Attachments

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