Simple and Compound Interest

MATH *ON THE JOB*

One day you might find Andrea Thiveos decorating a home for a wedding reception, while the next day, you could find her choosing a new colour scheme for a real estate office. Andrea is an interior decorator and a certified home stager, staging professional, and colour consultant. She puts her education and skills to work operating her own business, Roomscaping by Andrea. Andrea grew up in Happy Adventure, Newfoundland.

Andrea's job skills include completing mathematical calculations. When estimating the size of a room or piece of furniture, Andrea calculates its square footage or surface area. She also measures fabric, estimates quotes for different jobs, and does the accounting for her business.

A self-employed entrepreneur might take out a loan to help start up a new business and cover the costs of materials and equipment. If Andrea took out a loan to help cover her costs she would need to calculate the total cost of the loan with interest. This would help her keep track of her expenses. If Andrea took out a loan of $20 000.00 and repaid it by making 12 monthly payments of $1698.43, what would be the total cost of her loan?

Andrea's job involves artfully and tastefully combining colours.

SOLUTION

Multiply the amount of the monthly payment by 12.

$1698.43 × 12 = $20 381.16

The total cost of the loan would be $20 381.16.

INTEREST???

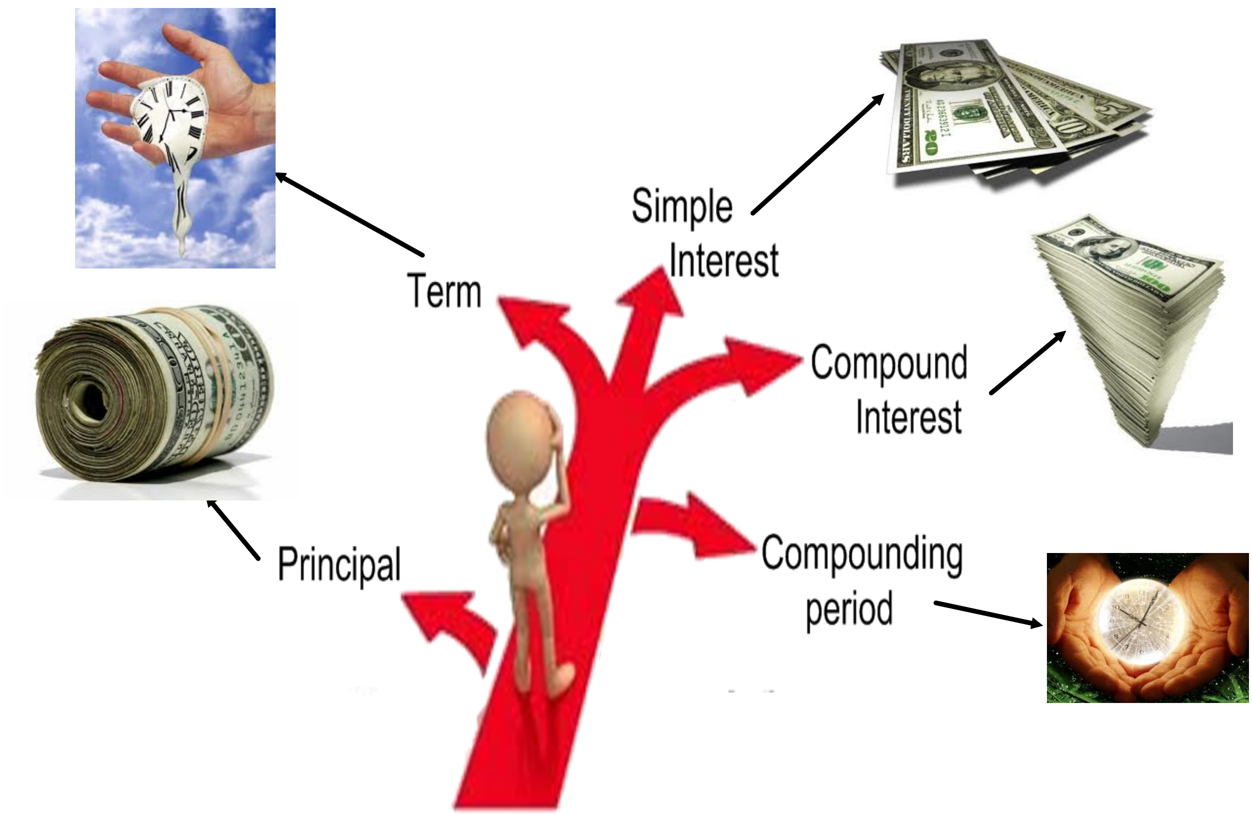
- What is Interest? money → RRSP
  tax ↓ tax
  Money that is added to an investment/loan.

- Investments (money is earned)
  "Good interest"
  - savings account (very, very small interest)
  - RRSP (registered retirement savings plan)
  - RESP (registered educational savings plan)
  - Canada Savings Bonds
  - GIC's (guaranteed investment certificate)
  - Tax Free Savings Accounts
  - Mutual Funds
  - Stock Market (no interest, shares)

- Loans (money owed)
  "Bad Interest"
  - banks (line of credit, personal loans, mortgage)
  - business/stores
  - credit cards

INTEREST - What is a good # ?

- bank: 7-10 %
- business: 14 - 20%
- credit card (9 - 25 %)



SIMPLE Interest...

SIMPLE Interest	Based on the **principal** (original amount) that is invested/borrowed. Interest is a certain percentage per **annum** (year). Often used for personal loans and short-term investments. The length of time for the investment/loan is called the **term**.

Interest = Principal x rate x time

$\boxed{I = Prt}$

&

$\boxed{A = P + I}$

- I - interest earned
- P - principal (original investment/loan)
- r - interest rate as a percent **(change to a decimal)**
- t - is ALWAYS time in **years**
  (how long the money is invested/borrowed)
- **A - amount of money including interest**

EXAMPLE #1:

You just won 2.5 million from Saturday's 649 lottery. The bank has offered you a simple interest rate of 1.75 %/a. How much interest will you earn in one year?

$I = Prt$

$I = (2\ 500\ 000)(0.0175)(1)$

$I = \$43\ 750.00$

EXAMPLE #2:

You borrowed \$500 from your older brother who charges 4.5 % per annum. How much will you owe him after 2 years?

$I = Prt$

$I = (500)(0.045)(2)$

$I = \$45$

$A = P + I$

$A = 500 + 45$

$A =$ **\$545**

EXAMPLE #3:
Betty-Ann's bank offers a simple interst rate of 4% per annum. How much interest would Betty-Ann earn on her investment of $4000 after 8 months.

$I = Prt$

$I = 4000\,(0.04)\,(8/12)$

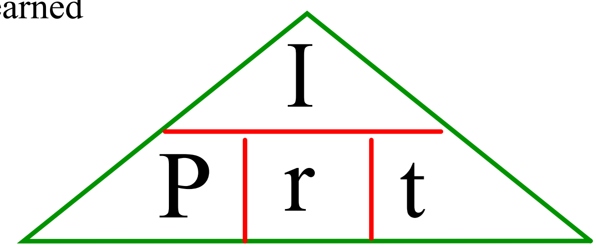
$I = \$106.67$

Time

must be in years

$I = Prt$ $\qquad$ $A = P + I$

Interest earned $\qquad$ Amount



Rearranging???

$I = Prt$

$P = \dfrac{I}{rt}$

$r = \dfrac{I}{Pt}$

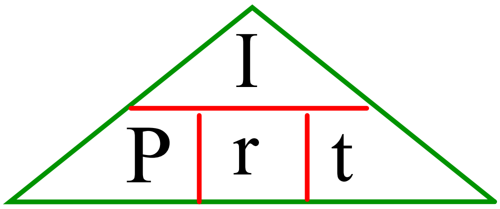
$t = \dfrac{I}{Pr}$

EXAMPLE #4:

The interest earned on a deposit is \$25 with an interest rate is 6% per annum. If the money was invested for 2 years, what is the principal?

I

t

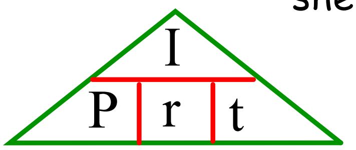


$$P = \frac{I}{rt}$$

$$= \frac{25}{(0.06)2}$$

$$= \$208.33$$

EXAMPLE #5: Liberty wants to earn \$150 simple interest from a \$1200 investment over 5 1/2 years. What rate does she need from the bank?



$$r = \frac{I}{Pt}$$

$$= \frac{150}{1200(5.5)}$$

$$= 2.\overline{27}\%$$

THE ROOTS OF MATH

THE CANADIAN DOLLAR

The loonie. The buck. The dollar. These are terms we use every day to talk about our currency. But where do they come from and how did the Canadian dollar come to be what we use today?

The dollar was a coin issued in the late 1600s by the Hudson's Bay Company. Its value was equal to a beaver pelt, also called a "buck" by fur traders. That is why today we sometimes call the dollar a "buck."

Before 1763, Canada was one of five North American colonies that made up New France. The first paper money appeared in New France in the form of playing cards. Eventually the card system collapsed, and for a long time people mistrusted paper money.

In 1812, the United States of America and Great Britain went to war in North America. Army bills for amounts between $1.00 and $400.00 were issued to help finance the war. After the war, the bills were easily exchanged for coins, so people no longer doubted the value of paper money.

For a time, both the British sterling monetary system of pounds, shillings, and pence, and the US system of dollars and cents were in use in what are the present-day Canadian provinces. In 1858, the dollar was chosen over the pound, and a single Canadian dollar was finally established.

At that time, the Bank of Canada and the Royal Canadian Mint did not yet exist, and banks were allowed to issue their own money. The first bank notes based on the dollar were issued by the Bank of Montreal. The British North America Act in 1867 gave government control over coins and currency, and it began producing all coins and bills.

In 1987, $1.00 bills were replaced by the $1.00 coin, the "loonie," so-called because of the image of a loon on one side. The $2.00 bill was replaced in 1996 by a $2.00 coin commonly called the "toonie."

Since 1976, Winnipeg's Royal Canadian Mint has produced every one of Canada's coins.

1. A $2.00 bill used to cost six cents to make, but only lasted about a year. The "toonie" costs 16 cents to make but lasts 20 years. Explain why it made good sense for the Royal Canadian Mint to switch from $2.00 bills to $2.00 coins.
2. Canadian bills have gone through many changes since the 1800s. They now include many security features to prevent counterfeiting, and accessibility features to help blind and visually impaired people recognize denominations.

   Research some security features that the Bank of Canada has put into bills to prevent counterfeiting and to help visually impaired Canadians.

HW: Answer both of these AND examine a Canadian bill and list all of it's features.

SOLUTIONS

1. In the long term, the cost of the toonie is much lower than the cost of the $2.00 bill. The $2.00 bill had only a 1-year life, so the cost to supply the currency for 20 years was $1.20 (20 times 6 cents). The cost of a toonie, which lasts 20 years, is only $0.16.
2. Security features of Canadian bills include:
   - a metallic (holographic) stripe
   - a ghost image (watermark)
   - metallic dashes printed on the bill shift from gold to green when tilted
   - a see-through number indicating the value of the bill appears when held up to light
   - raised ink on some elements of the bill
   - under UV (fluorescent) light, text appears

   Accessibility features of Canadian bills include:
   - a tactile feature of raised dots on one corner of the bill
   - large, high-contrast numerals identify the denomination (dark numeral on pale background and white number on dark background)
   - the various denominations are printed in contrasting colours

   The Bank of Canada also provides a hand-held bank note reader that informs the user of the value of the bill.

HOMEWORK...

Worksheet - Simple Interest.doc

1. a) $I = Prt$

$= 500(0.09)(90/365)$

COMPOUND Interest...

Terminology Tango

Click on the picture to verify the match.

daily
bi-weekly
semi-annually
weekly
monthly
quarterly
annually

once a year
twice a year
four times a year
twelve times a year
26 times a year
52 times a year
365 times a year

COMPOUND Interest	Interest is added to the principal periodically throughout the year. New interest may be paid on the principal plus the interest. The interest rate is stated per annum and is divided by the number of **compounding periods**.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$I = A - P$$

A = final value of the investment ...(principal + interest)
P = principal
r = annual interest rate
n = number of compounding periods in a year
t = term of the investment or loan in number of years

EXAMPLE #1: If \$1000 is invested at 8 %/a compounded semi-annually for 2 years, how much will the investment be worth?

Using the simple interest formula...

I = 1000(0.08)(6/12)
= \$40 (after 1st interest period)

New principal = 1000 + 40
= \$1040

I = 1040(0.08)(6/12)
= \$41.60 (after 2nd interest period)

New Principal = 1040 + 41.60
= \$1081.60

I = 1081.60(0.08)(6/12)
= \$43.26 (after 3rd interest period)

New Principal = 1081.60 + 43.26
= \$1124.86

I = 1124.86(0.08)(6/12)
= \$44.99 (after 4th interest period)

New Principal = 1124.86 + 44.99
= \$1169.85

Using the formula... [redacted]

$$A = P(1 + i)^n$$
$$= 1000(1+0.08/2)^{2\times2}$$
$$= \$1169.86$$

EXAMPLE #2:

Calculate the final value of an initial investment of $6000.00. Interest is paid at 4% per annum, compounded semi-annually, for three years.

A = final value of the investment ...(principal + interest)
P = principal
r = annual interest rate
n = number of compounding periods in a year
t = term of the investment or loan in number of years

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = \quad .\quad \left(1 + \frac{.}{.}\right)^{(.)(.)}$$

$$A = \quad .\quad (1 + \quad . \quad)$$

$$A = \quad .\quad (\quad . \quad)$$

$$A = \quad .\quad (\quad . \quad)$$

$$A = \$ \quad .$$

EX #3: Maggie invests $30 000 at 10% /a compounded quarterly for 20 years. Determine...

a) How much will this investment be worth?

b) How much interest did you earn?

EXAMPLE #4...

A keen MVHS student wants to save some money from their summer employment. They decide to take out a Canada Savings Bond which pays 2.5 % interest per year compounded monthly. If the student invests $850 into the bond, how much interest will they earn if they don't touch the money for 3 years?

the RULE of 72

ACTIVITY 3.5
THE RULE OF 72

There is a quick way to estimate the time it takes for an investment compounded annually to double in value. This method is called the Rule of 72.

Rule of 72: a quick method of estimating the time it takes for an investment to double in value

To calculate the approximate length of time in years it takes for an investment to double, divide 72 by the annual interest rate expressed as a percentage.

If you wanted to know approximately how long it would take an investment with an interest rate of 3.00% per annum to double in value, you would divide 72 by 3.

$$72 \div 3 = 24 \text{ years}$$

Using the Rule of 72, you can estimate that it would take about 24 years for the investment to double in value.

1. Using the information above, write a formula that describes the Rule of 72. Use the formula to answer question 2.
2. If you wanted to double your money in 10 years, at what rate of interest would you need to invest your money?

SOLUTIONS

1. The Rule of 72 can be expressed with the following formula.

   Years to double investment = 72 ÷ interest rate

   $$y = 72 \div r$$

2. $y = 72 \div r$

   $10 = 72 \div r$

   $r = 72 \div 10$

   $r = 7.2\%$

   You would need to invest your money at an interest rate of 7.2%.

DISCUSS THE IDEAS

GUARANTEED INVESTMENT CERTIFICATES

Vyanjana has received a special gift of $5000.00 from her grandparents, which she plans to invest for the future. She has researched investment options at her bank, and has decided to buy a Guaranteed Investment Certificate (GIC). GICs guarantee that the investor will receive his or her principal as well as a fixed amount of interest.

She has narrowed her choices down to three options:

Option 1: A GIC that offers 1.125% interest per annum, compounded monthly, with a one-year term. This GIC cannot be redeemed before the end of the term, so Vyanjana will not be able to access her money before the end of the one-year term.

Option 2: A GIC that offers 0.875% interest per annum, compounded monthly, with a one-year term. This GIC can be redeemed before the end of the term, but if Vyanjana wants to access her money before the end of the year, her investment will earn only 0.050% interest per annum.

Option 3: A GIC that offers 1.250% interest per annum, compounded annually, with a one-year term. The GIC cannot be redeemed before the end of the term.

Working in a small group, discuss Vyanjana's investment options.

1. Calculate how much interest Vyanjana would earn with each option. For option 2, calculate how much interest Vyanjana would earn after 6 months and after the full term of the investment.
2. Suggest reasons why Vyanjana might choose each of the three options.

SOLUTION
???

SOLUTIONS

1. Calculate how much interest Vyanjana would earn with each option.

Option 1:

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = \$5000.00\left(1+\frac{0.01125}{12}\right)^{12}$

$A \approx \$5056.54$

$I = A - P$

$I = \$5056.54 - \$5000.00$

$I = \$56.54$

Option 2a:

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = \$5000.00\left(1+\frac{0.00875}{12}\right)^{12}$

$A \approx \$5043.93$

$I = A - P$

$I = \$5043.93 - \$5000.00$

$I = \$43.93$

Option 2b:

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = P\left(1+\frac{0.0005}{12}\right)^{6}$

$A \approx \$5001.25$

$I = A - P$

$I = \$5001.25 - \$5000.00$

$I = \$1.25$

Option 3:

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = \$5000.00\left(1+\frac{0.0125}{1}\right)^{1}$

$A = \$5000.00\,(1.0125)^{1}$

$A \approx \$5062.50$

$I = A - P$

$I = \$5062.50 - \$5000.00$

$I = \$62.50$

ACTIVITY 3.6
THE EFFECT OF DIFFERENT COMPOUNDING PERIODS

1. Calculate the interest and the final value for an investment of $4000.00 at 3.00% per annum over 2 years for the following different compounding periods. Show your answers in a table like the one below. Use any method you wish to calculate your answers.
2. Which compounding period yields the greatest interest on the investment? Which yields the least? How would knowing this affect your choice of investment?

SOLUTIONS

1.

Interest period	*Final value of investment (A)*	*Interest (I)*
Annually	$\$4000.00\left(1+\frac{0.03}{1}\right)^{(1\times 2)} \approx \$4243.60$	$243.60
Semi-annually	$\$4000.00\left(1+\frac{0.03}{2}\right)^{(2\times 2)} \approx \$4245.45$	$245.45
Quarterly	$\$4000.00\left(1+\frac{0.03}{4}\right)^{(4\times 2)} \approx \$4246.40$	$246.40
Monthly	$\$4000.00\left(1+\frac{0.03}{12}\right)^{(12\times 2)} \approx \$4247.03$	$247.03
Daily	$\$4000.00\left(1+\frac{0.03}{365}\right)^{(365\times 2)} \approx \$4247.34$	$247.34

2. The daily compounding period yields the most interest. The annual compounding period yields the least interest. Knowing this, you would choose an investment which is compounded the most times per year to accumulate the most interest.

Hang on.... HOMEWORK!!!

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Questions:
1-7

3.2 Build Your Skills Detaild Solutions.pdf

Compound Worksheet

Attachments

Assignment - Simple Interest.doc

3.2 Build Your Skills Detaild Solutions.pdf

Compound Interest.pdf