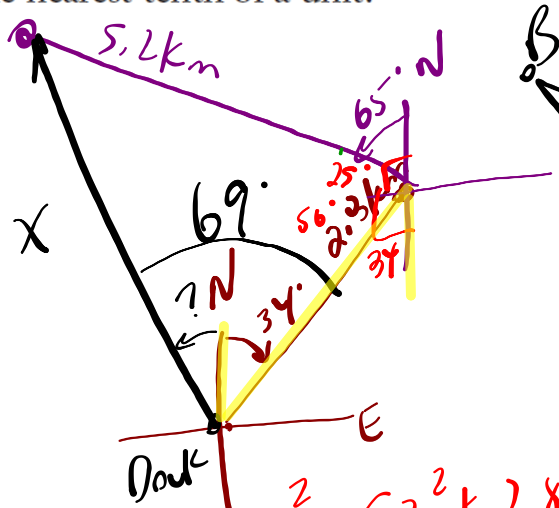
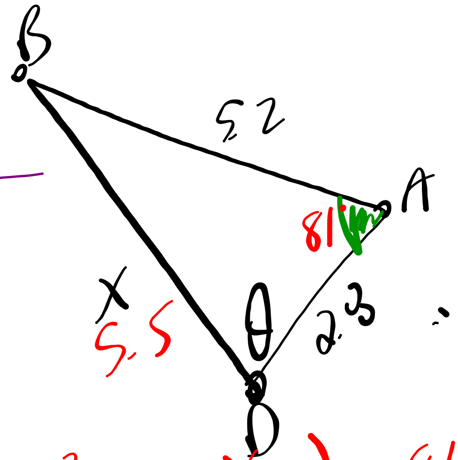
12. A canoeist starts from a dock and paddles 2.8 km N34°E. Then she paddles 5.2 km N65°W. What distance, and in which direction, should a second canoeist paddle to reach the same location directly, starting from the same dock? Round all answers to the nearest tenth of a unit.

N — W





$$x^2 = 5.2^2 + 2.8^2 - 2(5.2)(2.8)\cos 81°$$

x^2

$x =$

```
5.2²+2.8²-2*5.2*
2.8cos(81)
      30.32462838
√(Ans
      5.506780219
```

$\boxed{x = 5.5\text{ km}}$

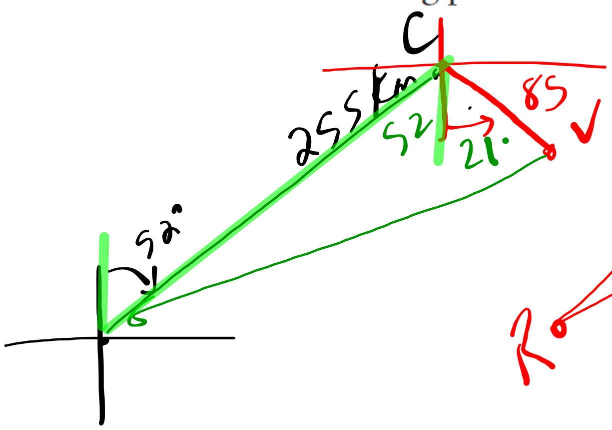
$\boxed{N\,35°\,W}$

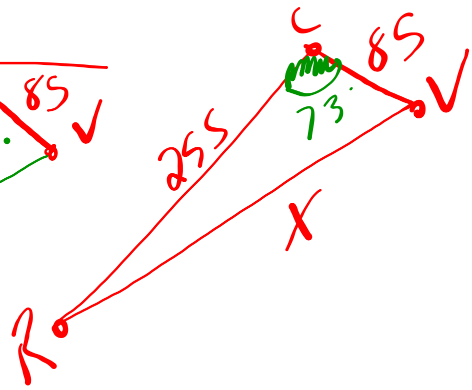
$$5.2 \cdot \frac{\sin\theta}{5.2} = \frac{\sin 81°}{5.5} \cdot 5.2$$

$$\sin^{-1}\sin\theta = \sin^{-1}(0.9327)$$

$$\theta = 69°$$

11. A bush pilot delivers supplies to a remote camp by flying 255 km in the direction N52°E. While at the camp, the pilot receives a radio message to pick up a passenger at a village. The village is 85 km S21°E from the camp. What is the total distance, to the nearest kilometre, that the pilot will have flown by the time he returns to his starting point?

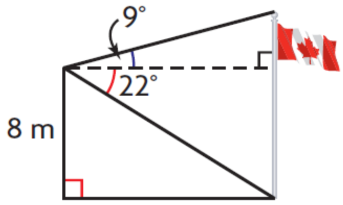




$x^2 =$ 255²+85²−2*255*85*cos(73)
59575.6866
√(Ans
$X =$ 244.0813115

TOTAL → 255
+ 85
+ 244
584 km

10. From a window in an apartment building, the angle of elevation to the top of a flagpole across the street is 9°. The angle of depression is 22° to the base of the flagpole. How tall is the flagpole, to the nearest tenth of a metre?



Notes - Ambiguous Case.pdf

In Summary

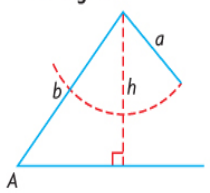
Key Idea

- The ambiguous case of the sine law may occur when you are given two side lengths and the measure of an angle that is opposite one of these sides. Depending on the measure of the given angle and the lengths of the given sides, you may need to construct and solve zero, one, or two triangles.

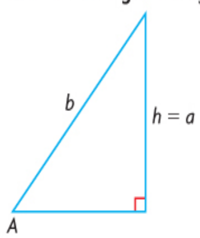
Need to Know

- In $\triangle ABC$ below, where h is the height of the triangle, $\angle A$ and the lengths of sides a and b are given, and $\angle A$ is acute, there are four possibilities to consider:

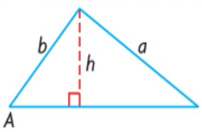
If $\angle A$ is acute and $a < h$, there is **no triangle.**



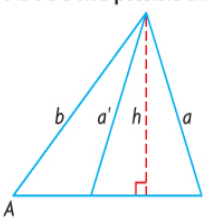
If $\angle A$ is acute and $a = h$, there is **one right triangle.**



If $\angle A$ is acute and $a > b$ or $a = b$, there is **one triangle.**

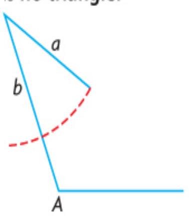


If $\angle A$ is acute and $h < a < b$, there are **two possible triangles.**

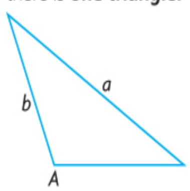


- If $\angle A$, a, and b are given and $\angle A$ is obtuse, there are two possibilities to consider:

If $\angle A$ is obtuse and $a < b$ or $a = b$, there is **no triangle.**



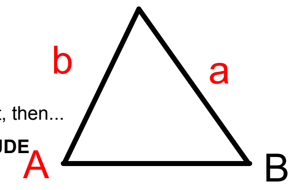
If $\angle A$ is obtuse and $a > b$, there is **one triangle.**



Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b

*** If ALL 3 criteria are met, then...



CALCULATE THE ALTITUDE

$\boxed{\text{alt} = b \sin A}$

CASE 1: a < altitude; there is NO SOLUTION

CASE 2: a = altitude; there is ONE SOLUTION [Right Triangle]

CASE 3: a >altitude; this is the 'AMBIGUOUS CASE'...TWO SOLUTIONS

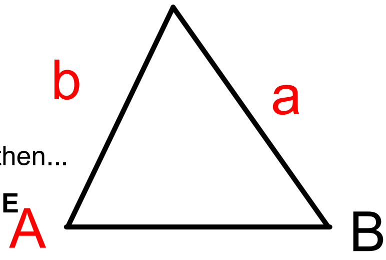
1) Acute Triangle (angle, θ, is found with Law of Sines)
2) Obtuse Triangle (angle is 180º - θ)

MUST MEMORIZE THESE NOTES IN ORDER TO KNOW AMBIGUOUS CASE

3 C's

Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b



*** If ALL 3 criteria are met, then...

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1) Acute Triangle (angle, θ, is found with Law of Sines)

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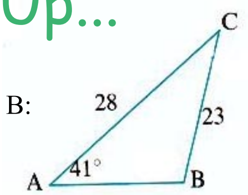
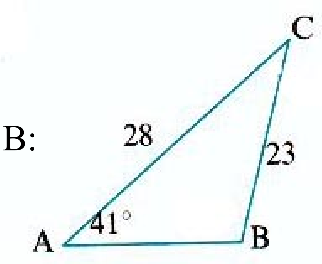
Back to the Warm-Up...

Determine the measure of the obtuse angle B:



Warm Up

Determine the measure of the obtuse angle B:



* SSA ✓
* acute ✓
* a < b ∴ ✓

$$\frac{28 \sin B}{28} = \frac{28 \sin 41}{23}$$

$\sin^{-1}(\sin B) = \sin^{-1}(0.7987)$

$\angle B = 53$???

$h = b \sin A$

$h = 28 \sin 41$

$h = 18.4$

1) $a < h$ 2) $a = h$ 3) $a > h$ *

$a > h$

23 vs 18.4

Ambiguous

OR

$\angle B = 180 - 53°$

$\angle B = 127°$ *

EXAMPLE 1 Connecting the *SSA* situation to the number of possible triangles

p. 177

4.3 Ambiguous Case of the Sine Law

Given each *SSA* situation for $\triangle ABC$, determine how many triangles are possible. ✓SSA ✓ acute ✓ a < b

a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m

c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m

d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m

d) 1 solution

$h = 12 \sin 30°$

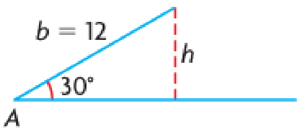
$h = 6$

a) ① $a < h$ → No solution

b) ② $a = h$ → 1 right Δ

c) ③ $a > h$ → * 2 solutions

Saskia's Solution



I drew the beginning of a triangle with a 30° angle and a 12 m side.

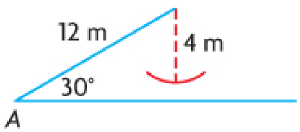
$$\sin 30° = \frac{h}{12}$$

I used the sine ratio to calculate the height of the triangle.

$$12 \sin 30° = h$$
$$6 \text{ m} = h$$

I can use this height as a benchmark to decide on side lengths opposite the 30° angle that will result in zero, one, or two triangles.

a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

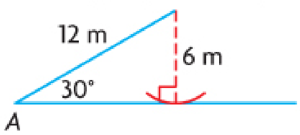


No triangles are possible.

Since $a < b$ and $a < h$, I knew that no triangles are possible.

I used a compass to be certain. I set the compass tips to represent 4 m. I placed one tip of the compass at the open end of the 12 m side and swung the pencil tip toward the other side. The pencil couldn't reach the base, so a 4 m side could not close the triangle.

b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m

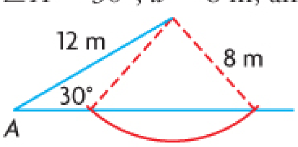


One triangle is possible.

Since $a < b$ and $a = h$, there is only one possible triangle, a right triangle.

A compass arc intersects the base at only one point.

c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m

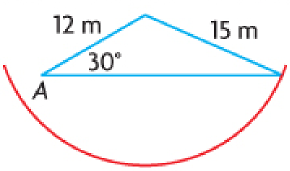


Two triangles are possible.

Since $a < b$ and $a > h$, there are two possible triangles.

A compass arc intersects the base at two points.

d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m



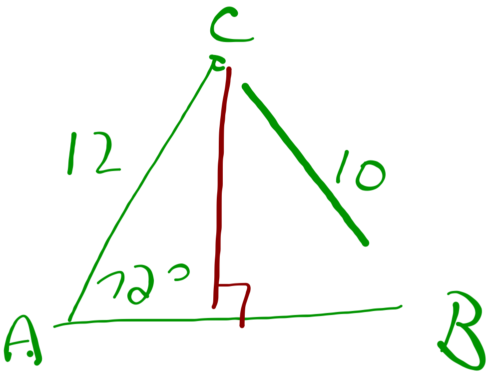
One triangle is possible.

Since $a > b$, only one triangle is possible.

A compass arc intersects the base at only one point.

Example 2:

Solve the triangle ABC if $a = 10$, $b = 12$ and angle A = 72°.



*SSA

✓ - acute

✓ - $a < b$

$h = 12 \sin 72°$

$h = 11.4$

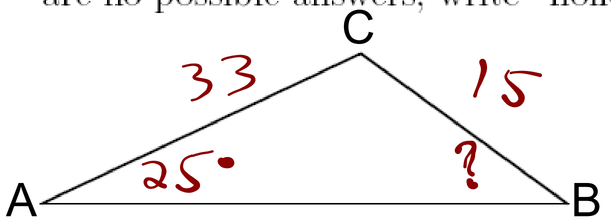
a vs h

$10 < 11.4$

No Solution

Example 3:

Given that A$= 25°$, $a = 15$, and $b = 33$, find the measure of angle B to the nearest degree. If there are two answers, give both of them. If there are no possible answers, write "none".



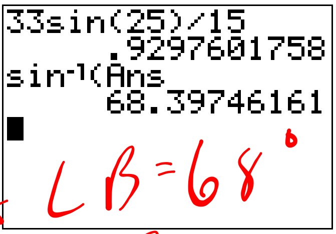
✓ * SSA
✓ – acute
✓ – a < b

alt = 33 sin 25°
alt = 13.9

1) a < alt 2) a = alt 3) a > alt *

$$\frac{\sin B}{33} = \frac{\sin 25}{15}$$

sin B =



∠B =

* ∠B = 68°

or

∠B = 180 − 68°
∠B = 112°

a vs alt
15 > 13.9
(2 solutions)
Ambiguous

HOMEWORK...

Worksheet - Ambiguous Case.pdf

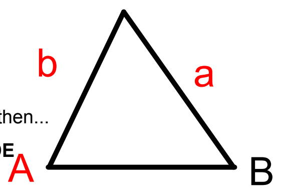
Do questions #1, 2 & 4

MEMORIZE!!!

Criteria for the Ambiguous Case...

- Must be given SSA
- Given angle is acute
- a < b

*** If ALL 3 criteria are met, then...



CALCULATE THE ALTITUDE

$\boxed{\text{alt} = b \sin A}$

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Attachments

Notes - Ambiguous Case.pdf

Worksheet - Ambiguous Case.pdf