Solving Systems of Linear Equations

A "system" of equations is a set or collection of equations that you deal with all together at once.

Linear equations (ones that graph as straight lines) are simpler than non-linear equations, and the simplest linear system is one with two equations and two variables.

Think back to linear equations. For instance, consider the linear equation $y = 3x - 5$.

A "solution" to this equation was any x, y-point that "worked" in the equation. So $(2, 1)$ was a solution because, plugging in 2 for x:

LS RS

$$\begin{array}{c|l} y & = 3x - 5 \\ \hline 1 & 3(2) - 5 \\ & 6-5 \\ & 1 \end{array}$$

LS=RS ∴ (2,1) is a point of the equation

therefore

On the other hand, $(1, 2)$ was not a solution, because, plugging in 1 for x:

$$\begin{array}{c|l} y & = 3x - 5 \\ \hline 2 & 3(1) - 5 \\ & 3 - 5 \\ & -2 \end{array}$$

LS ≠ RS ∴ (1,2) is not a solution

...which did not equal y (which was 2, for this point).

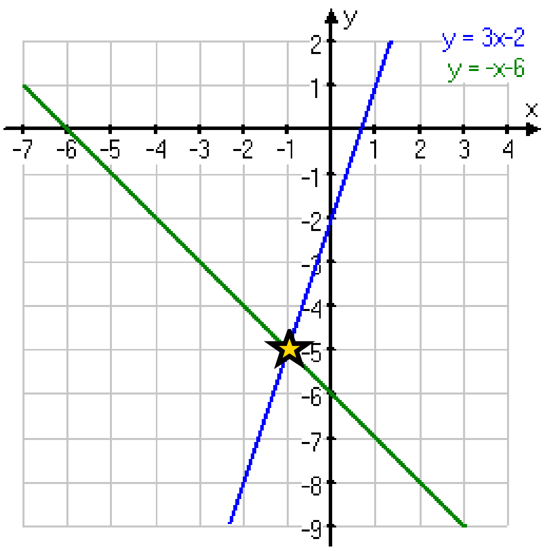
Solving by Graphing:

Now consider the following two-variable system of linear equations:

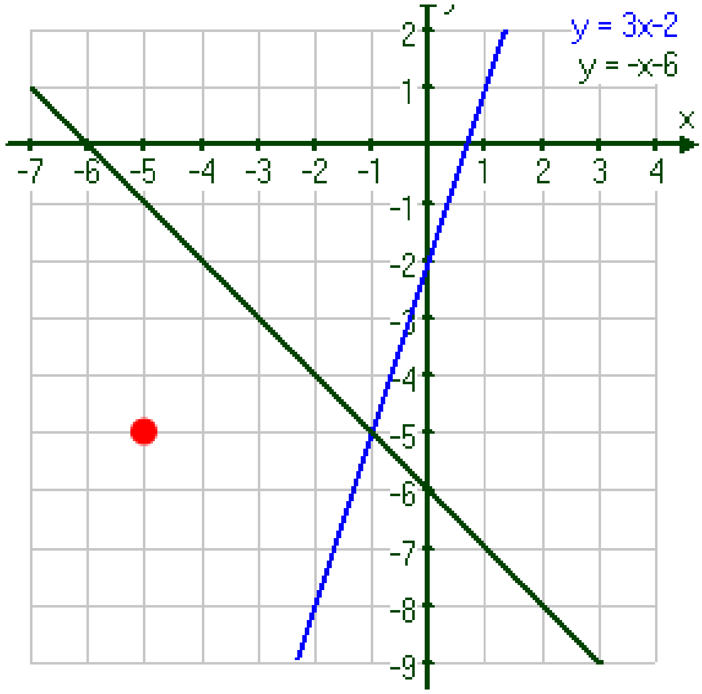
$y = 3x - 2$
$y = -x - 6$

CHECK...

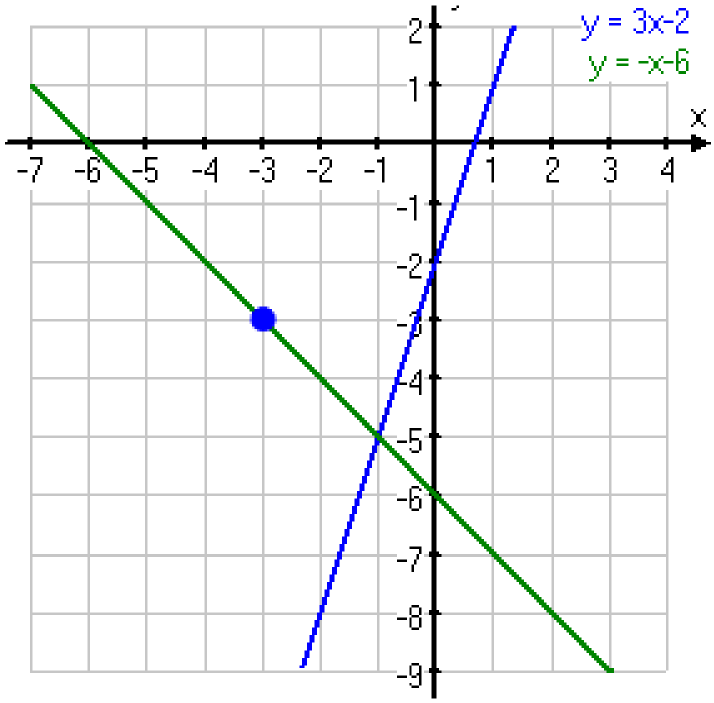
Since the two equations above are in a system, we deal with them together at the same time. In particular, we can graph them together on the same axis system, like this:



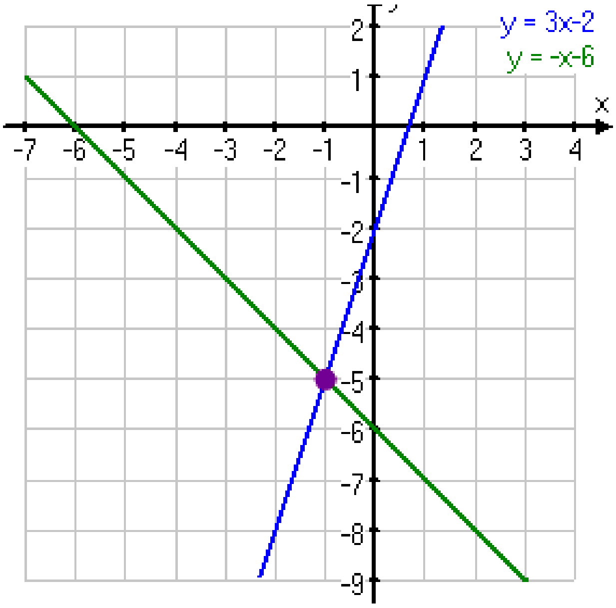
A solution for a *single* equation is any point that lies on the line for that equation. A solution for a *system* of equations is any point that lies on each line in the system. For example, the red point at right is not a solution to the system, because it is not on either line:



The blue point at right is not a solution to the system, because it lies on only one of the lines, not on *both* of them:



The purple point at right is a solution to the system, because it lies on both of the lines:

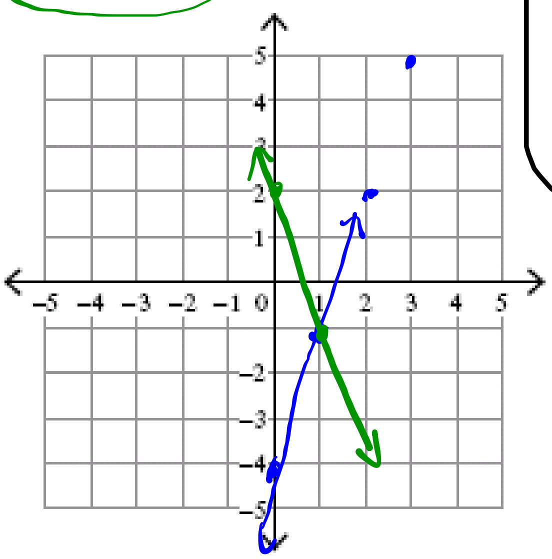


In particular, this purple point marks the intersection of the two lines. Since this point is on both lines, it thus solves both equations, so it solves the entire system of equation. And this relationship is always true: For systems of equations, "solutions" are "intersections". You can confirm the solution by plugging it into the system of equations, and confirming that the solution works in each equation.

EXAMPLES...

Solve each system by graphing.

1) $y = 3x - 4$
$y = -3x + 2$



Review...Graphing a line

Slope-intercept method...

y = mx + b

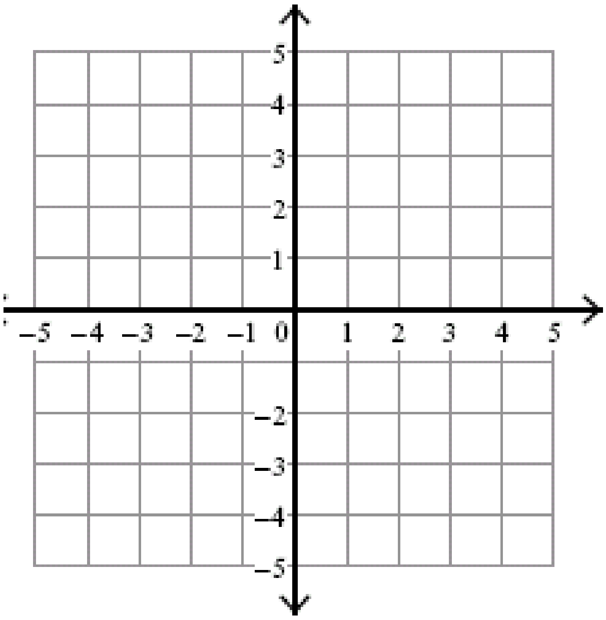
STEPS:

1) Plot the y-intercept
2) Use slope to get 2nd point

(1, -1)

2) $y = x - 4$

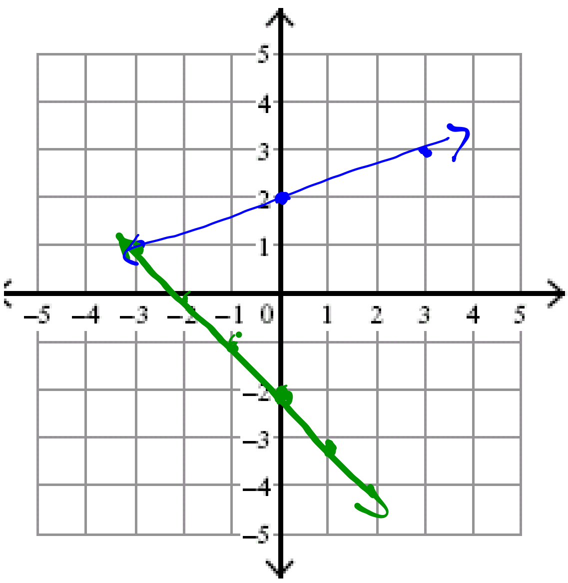
$y = -x + 2$



YOUR TURN...

$y = \frac{1}{3}x + 2$

$y = -x - 2$



(-3,1)

Solving Systems of Equations with 2 Unknowns

By Graphing... Ability to plot both lines on the same grid to give a common intersection point.

Example #3: FIRST...Put each equation in Slope-Intercept Form!!!

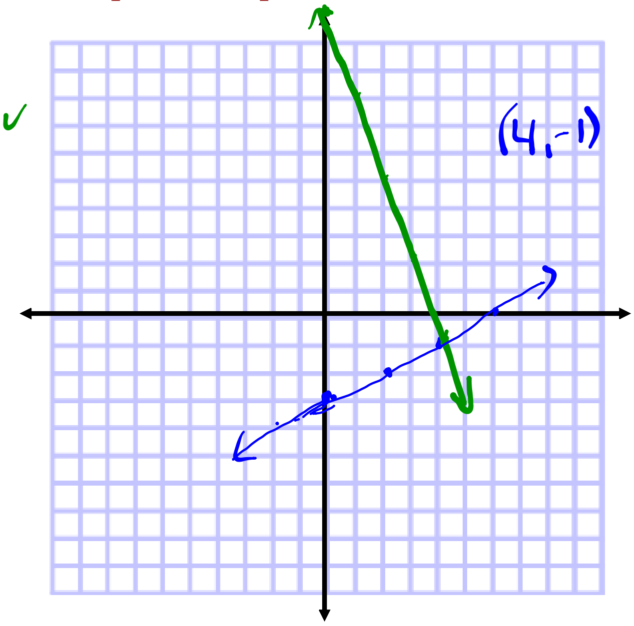
(1) $x - 2y = 6$ & (2) $3x + y = 11$

$-2y = -x + 6$

$\frac{-2y}{-2} = \frac{-x}{-2} + \frac{6}{-2}$

$y = \frac{1}{2}x - 3$

$y = -3x + 11$ ✓



Verifying A Solution...

- Determine whether either of the points $(-1, -5)$ and $(0, -2)$ is a solution to the given system of equations.

$y = 3x - 2$
$y = -x - 6$

$(-1, -5)$: $-5 \mid 3(-1) - 2$, $-3 - 2$, -5 ✓

$(-1, -5)$: $-5 \mid -(-1) - 6$, $1 - 6$, -5 ✓

Yes solution

$(0, -2)$: $-2 \mid 3(0) - 2$, -2 ✓

$(0, -2)$: $-2 \mid -(-0) - 6$, $0 - 6$, -6 ✗

No - not a solution

Homework...worksheet

1 - 13 odd numbers

17, 21 - 29 odd numbers

26

Attachments

Worksheet - Solve by Graphing.pdf

u4l2_-_systems_by_graphing_.pdf