MAY 7, 2018

UNIT 8: CIRCLE GEOMETRY

8.3: PROPERTIES OF ANGLES IN A CIRCLE

K. Sears
MATH 9

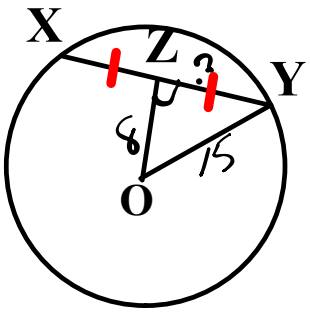
WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the Math 9 Specific Curriculum Outcome (SCO) "Shape and Space 1" OR "SS1" which states:

"Solve problems and justify the solution strategy using circle properties, including:

* **the perpendicular from the centre of a circle to a chord bisects the chord;**
* **the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc;**
* **the inscribed angles subtended by the same arc are congruent;**
* **a tangent to a circle is perpendicular to the radius at the point of tangency."**

WARM UP QUIZ:
(sketch and use the diagram to the right)



$a^2 = c^2 - b^2$
$= 15^2 - 8^2$
$= 225 - 64$
$= \sqrt{161}$
$= 12.7 \times 2$

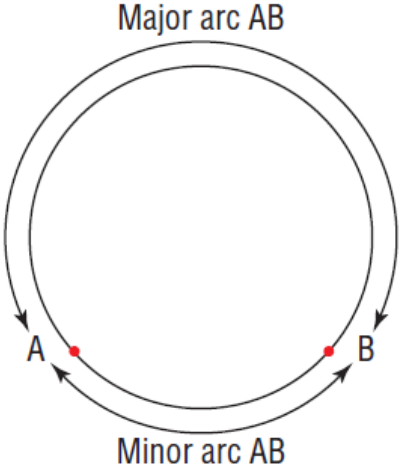
25.4 cm

COPY AND ANSWER:

1. The chord is line XY.
2. The centre of the circle is labeled O.
3. The perpendicular bisector is line OZ.
4. The radius is line OY.
5. If OZ is 8 cm and OY is 15 cm, what is the length of the XY to the nearest tenth?

VOCABULARY:

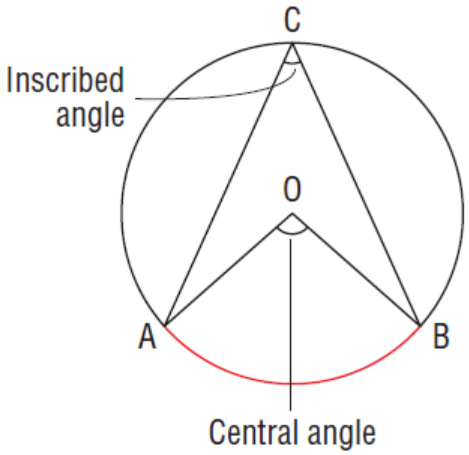
1. ARC: A section of the circumference of a circle. In the diagram below, the shorter arc AB is the MINOR ARC, and the longer arc AB is the MAJOR ARC.



VOCABULARY:

2. **CENTRAL ANGLE:** The angle formed by joining the endpoints of an arc to the centre of the circle. (This is done using 2 radii.)

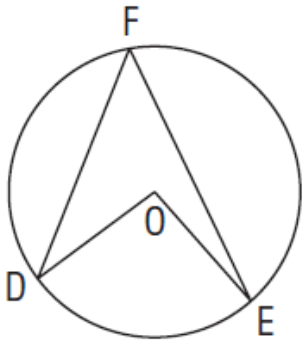
3. **INSCRIBED ANGLE:** The angle formed by joining the endpoints of an arc to a point on the circle.



The inscribed and central angles in this circle are SUBTENDED by the minor arc AB.

VOCABULARY:

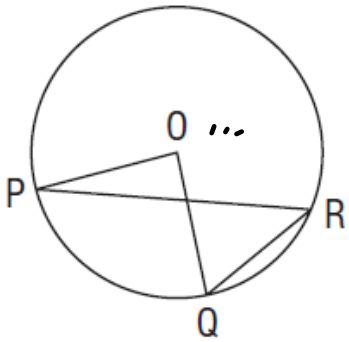
4. **CENTRAL ANGLE AND INSCRIBED ANGLE PROPERTY (CIAP):** In a circle, the measure of a central angle subtended by an arc is TWICE the measure of an inscribed angle subtended by the SAME arc.



$\angle O = 2\angle F$

OR

$\angle F = \frac{1}{2}\angle O$



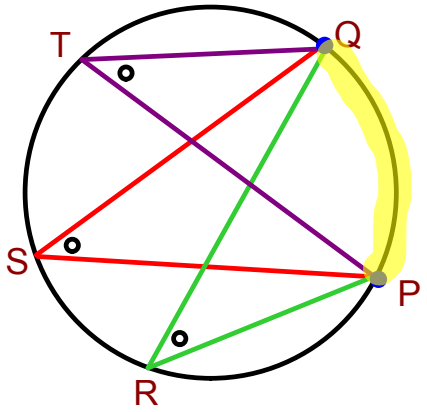
$\angle O = 2\angle R$

OR

$\angle R = \frac{1}{2}\angle O$

VOCABULARY:

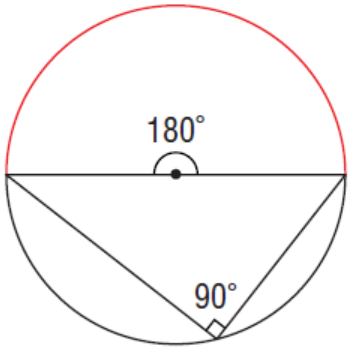
5. INSCRIBED ANGLES PROPERTY (IAP): In a circle, ALL inscribed angles subtended by the SAME arc are congruent (equal).

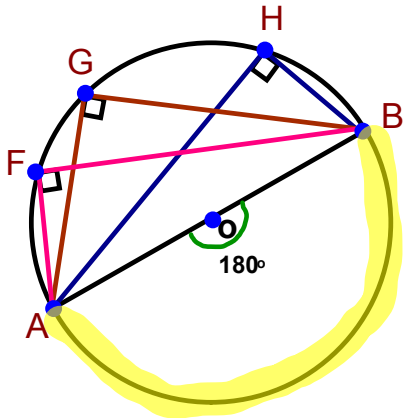


$\angle R = \angle S = \angle T$

VOCABULARY:

6. ANGLES IN A SEMICIRCLE PROPERTY (ASP): All inscribed angles subtended by a semicircle are RIGHT angles.





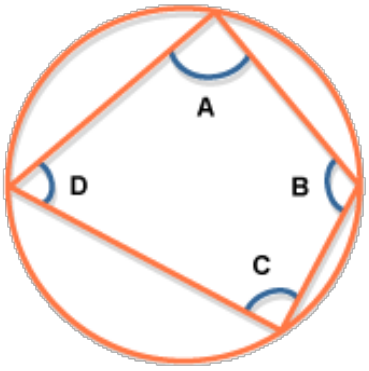
This makes sense - think of CIAP; an inscribed angle is half the central angle when the are subtended by the same arc.

$\angle F = \angle G = \angle H = 90^\circ$

VOCABULARY:

7. OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL PROPERTY (CQP):

The opposite angles in a cyclic quadrilateral (a quadrilateral whose vertices all touch the circumference of a circle) add up to 180^o.



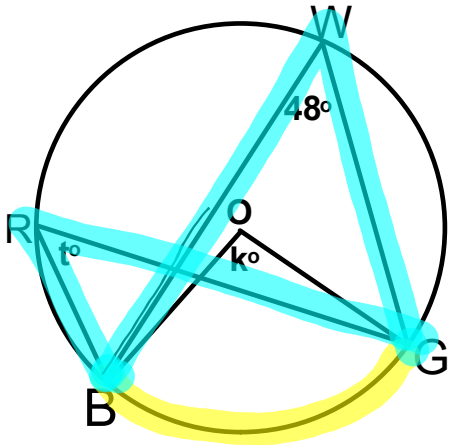
$\angle A + \angle C = 180^o$

$\angle B + \angle D = 180^o$

EXAMPLE: USING INSCRIBED AND CENTRAL AN

Point O is the center of a circle.
Determine the values of k and t.

$\angle t = 48^o$ (IAP)
$\angle k = 96^o$ (CIAP)



EXAMPLE: APPLYING THE ANGLES IN A SEMICIRCLE P

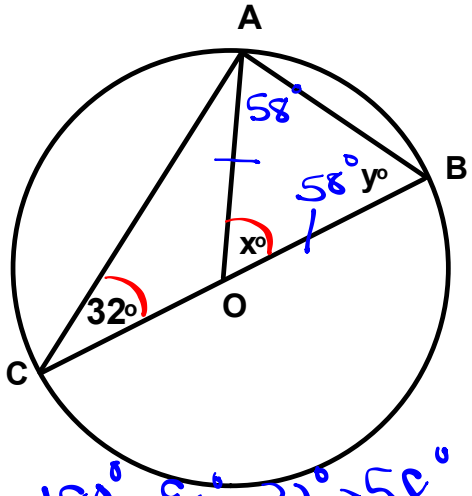
Point O is the center of the circle.
Determine the values of x° and y°.

<x = 64° (CIAP)

<A= 90° (ASP)
<y = 58° (SATT)

OR

<y = 58° [ITT/SATT; (180°-64°)/2]



$180^\circ - 90^\circ - 32^\circ = 58^\circ$

$y^\circ = 58^\circ$

$180^\circ - 58^\circ - 58^\circ$

$x^\circ = 64^\circ$

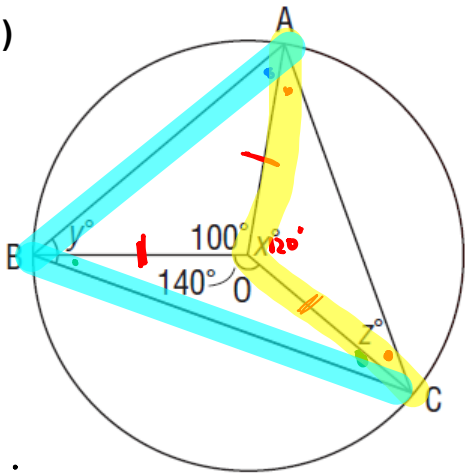
EXAMPLE: DETERMINING ANGLES IN AN INSCRIBED TI

Determine the values of x°, y°, and z°.
(HINT: There are 360° in a circular rotation.)

<x = 120° [360° - (100° + 140°)]

<y = 60° (CIAP)

<z = 30° [ITT / SATT; (180° - 120°)/2]



$z^\circ = 30^\circ$ (ITT)

$\frac{180^\circ - 120^\circ}{2} = 60^\circ$

$x^\circ = 360^\circ - 100^\circ - 140^\circ$

$x^\circ = 120^\circ$

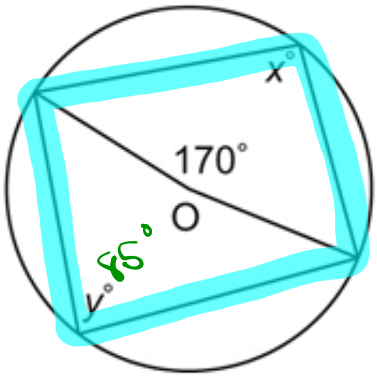
$y^\circ = 60^\circ$ (CIAP)

EXAMPLE: DETERMINING ANGLES IN A CYCLIC QU

Point O is the center of the circle.
Determine the values of x° and y°.

<y = 85° (CIAP)

<x = 95° (CQP)



$y^\circ = 85^\circ$ (CIAP)

$x^\circ = 180^\circ - 85^\circ$
$= 95^\circ$ (CQP)

CONCEPT REINFORCEMENT:

MMS9:

PAGE 410: #3 TO #5

PAGE 411: #6, #9, & #11

PAGE 419: #9 & #10

PAGE 420: Practice Test (#1 TO #6)

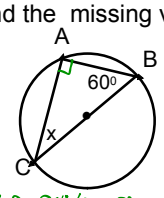
WORKSHEETS:

"Extra Practice 1, 2 and 3"

Additional questions:

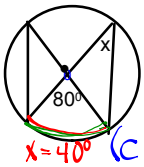
Find the missing values.

1.



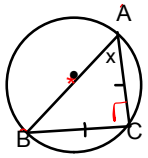
∠A = 90° (ASP)
x = 180 - 90 - 60
= 30° SATT

2.



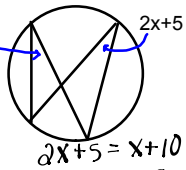
x = 40° (CIAP)

3.



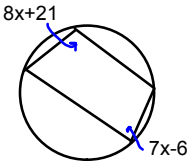
C = 90° (ASP)
x = 180-90 SATT
2 ITT
= 45°

4.



2x+5 = x+10 (IAP)
2x - x = 10 - 5
x = 5

5.



8x + 21 + 7x - 6 = 180 (CAP)
15x + 15 = 180
15x = 180 - 15
15x = 165
15 15
x = 11

Attachments

Worksheet - Angles in a Circle.doc