

MAY 7, 2018

UNIT 8: CIRCLE GEOMETRY

**8.3: PROPERTIES OF
ANGLES IN A
CIRCLE**

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MATH 9



WHAT'S THE POINT OF TODAY'S LESSON?

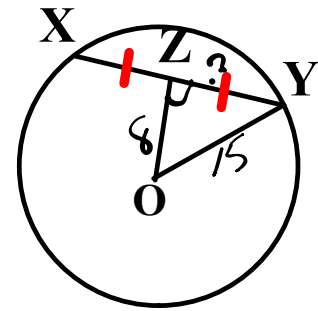
We will continue working on the Math 9 Specific Curriculum Outcome (SCO) "Shape and Space 1" OR "SS1" which states:

"Solve problems and justify the solution strategy using circle properties, including:

- * the perpendicular from the centre of a circle to a chord bisects the chord;**
- * the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc;**
- * the inscribed angles subtended by the same arc are congruent;**
- * a tangent to a circle is perpendicular to the radius at the point of tangency."**

WARM UP QUIZ:

(sketch and use the diagram to the right)



COPY AND ANSWER:

1. The chord is line XY.

2. The centre of the circle is labeled O.

3. The perpendicular bisector is line OZ.

4. The radius is line OY.

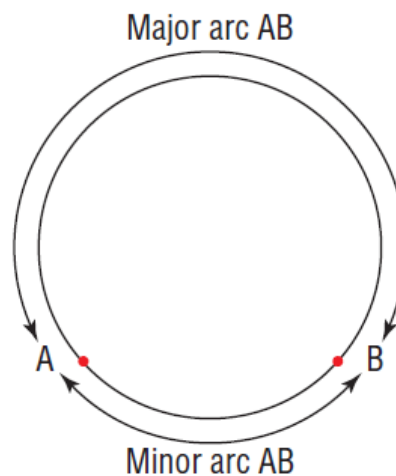
5. If **OZ** is **8 cm** and **OY** is **15 cm**, what is the length of the XY to the nearest tenth?

$$\begin{aligned}
 a^2 &= c^2 - b^2 \\
 &= 15^2 - 8^2 \\
 &= 225 - 64 \\
 &= \sqrt{161} \\
 &= 12.7 \times 2
 \end{aligned}$$

25.4 cm

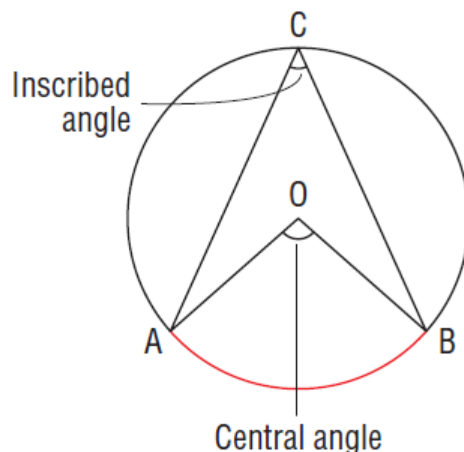
VOCABULARY:

1. **ARC:** A section of the circumference of a circle. In the diagram below, the **shorter arc AB** is the **MINOR ARC**, and the **longer arc AB** is the **MAJOR ARC**.



VOCABULARY:

2. **CENTRAL ANGLE:** The angle formed by joining the endpoints of an arc to the centre of the circle. (This is done using 2 radii.)

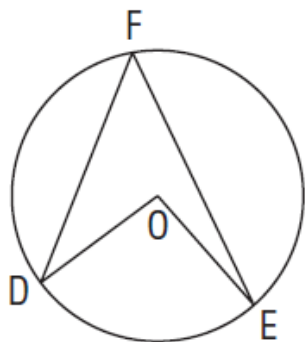


The inscribed and central angles in this circle are **SUBTENDED** by the minor arc AB.

3. **INSCRIBED ANGLE:** The angle formed by joining the endpoints of an arc to a point on the circle.

VOCABULARY:

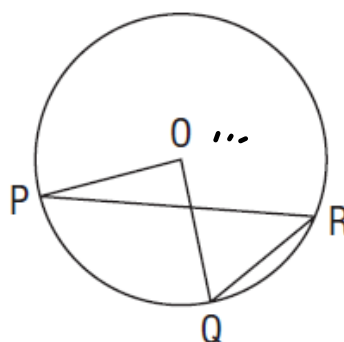
4. **CENTRAL ANGLE AND INSCRIBED ANGLE PROPERTY (CIAP):** In a circle, the measure of a central angle subtended by an arc is **TWICE** the measure of an inscribed angle subtended by the **SAME** arc.



$$\angle O = 2\angle F$$

OR

$$\angle F = \frac{1}{2}\angle O$$



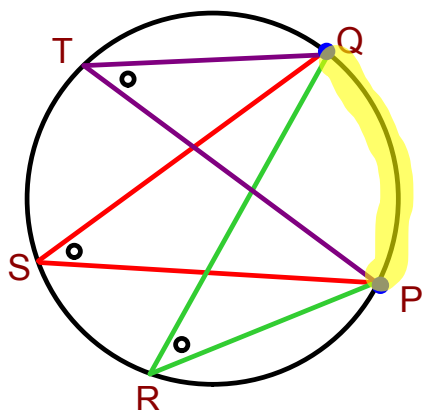
$$\angle O = 2\angle R$$

OR

$$\angle R = \frac{1}{2}\angle O$$

VOCABULARY:

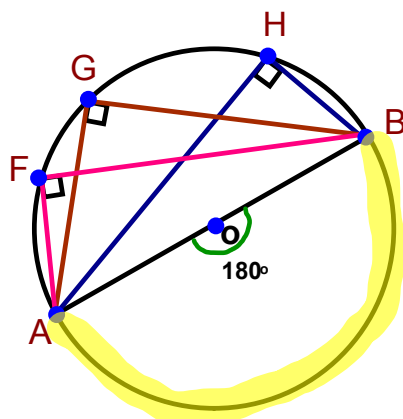
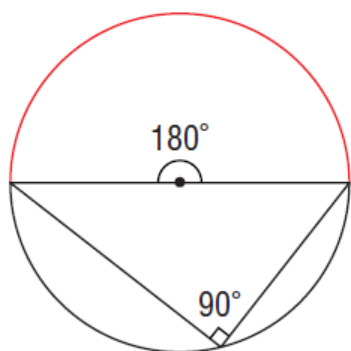
5. INSCRIBED ANGLES PROPERTY (IAP): In a circle, ALL inscribed angles subtended by the SAME arc are congruent (equal).



$$\angle R = \angle S = \angle T$$

VOCABULARY:

6. ANGLES IN A SEMICIRCLE PROPERTY (ASP): All inscribed angles subtended by a semicircle are RIGHT angles.



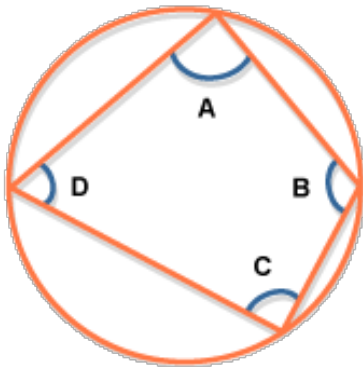
This makes sense - think of CIAP; an inscribed angle is half the central angle when the arc subtended by the same arc.

$$\angle F = \angle G = \angle H = 90^\circ$$

VOCABULARY:

7. OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL PROPERTY (CQP):

The **opposite angles** in a cyclic quadrilateral (a quadrilateral whose vertices all touch the circumference of a circle) **add up to 180°**.



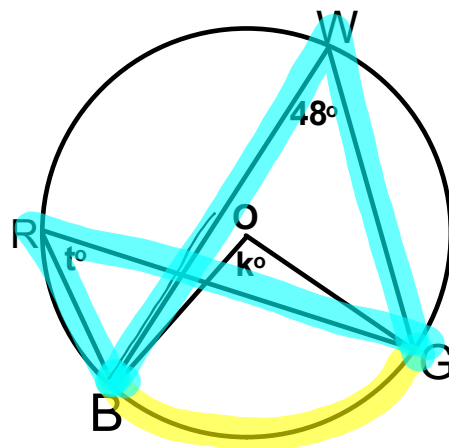
$$\angle A + \angle C = 180^\circ$$

$$\angle B + \angle D = 180^\circ$$

EXAMPLE: USING INSCRIBED AND CENTRAL AN

Point O is the center of a circle.
Determine the values of k and t°.

$\angle t = 48$ (IAP)
 $\angle k = 96$ (CIAP)



EXAMPLE: APPLYING THE ANGLES IN A SEMICIRCLE P

Point O is the center of the circle.
Determine the values of x° and y° .

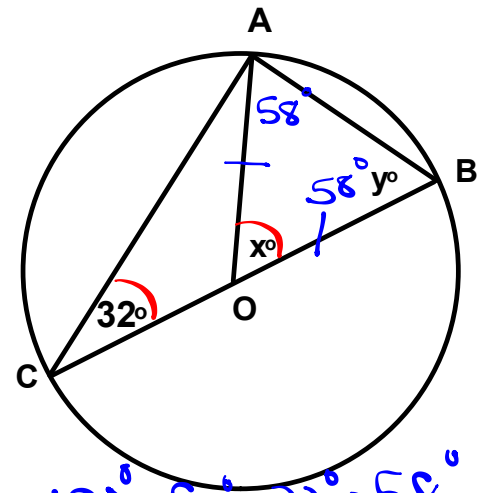
$\angle x = 64^\circ$ (CIAP)

$\angle A = 90^\circ$ (ASP)

$\angle y = 58^\circ$ (SATT)

OR

$\angle y = 58^\circ$ [ITT/SATT; $(180^\circ - 64^\circ)/2$]



$180^\circ - 90^\circ - 32^\circ = 58^\circ$
 $y^\circ = 58^\circ$

$180^\circ - 58^\circ - 58^\circ$
 $x^\circ = 64^\circ$

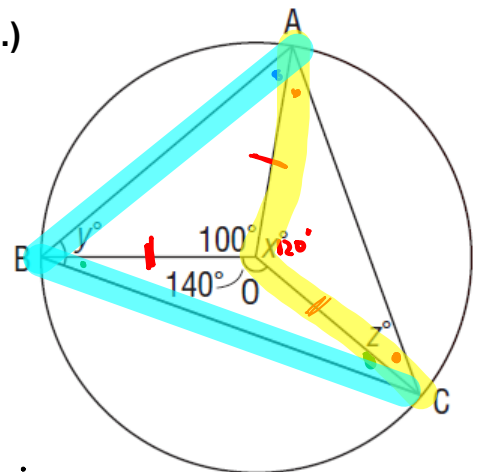
EXAMPLE: DETERMINING ANGLES IN AN INSCRIBED TRI

Determine the values of x° , y° , and z° .
(HINT: There are 360° in a circular rotation.)

$\angle x = 120^\circ$ [$360^\circ - (100^\circ + 140^\circ)$]

$\angle y = 60^\circ$ (CIAP)

$\angle z = 30^\circ$ [ITT / SATT; $(180^\circ - 120^\circ)/2$]



$x^\circ = 360^\circ - 100^\circ - 140^\circ$
 $x^\circ = 120^\circ$

$y^\circ = 60^\circ$ (CIAP)

$z^\circ = 30^\circ$ (ITT)

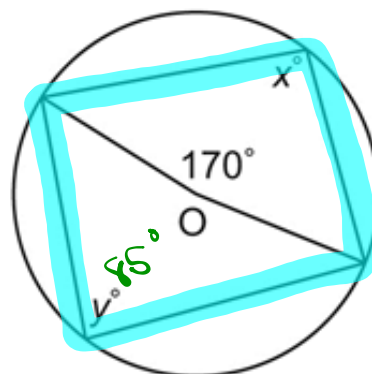
$180^\circ - 120^\circ = \frac{60^\circ}{2}$

EXAMPLE: DETERMINING ANGLES IN A CYCLIC QUAD

Point O is the center of the circle.
Determine the values of x° and y° .

$$\angle y = 85^\circ \text{ (CIAP)}$$

$$\angle x = 95^\circ \text{ (CQP)}$$



$$y^\circ = 85^\circ \text{ (CIAP)}$$

$$\begin{aligned} x^\circ &= 180^\circ - 85^\circ \\ &= 95^\circ \text{ (CQP)} \end{aligned}$$

CONCEPT REINFORCEMENT:

MMS9:

PAGE 410: #3 TO #5

PAGE 411: #6, #9, & #11

PAGE 419: #9 & #10

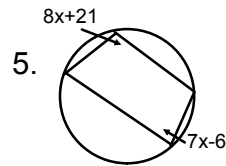
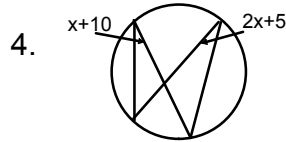
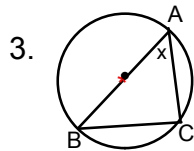
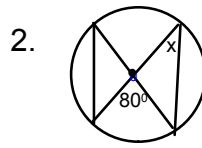
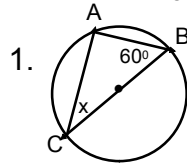
PAGE 420: Practice Test (#1 TO #6)

WORKSHEETS:

"Extra Practice 1, 2 and 3"

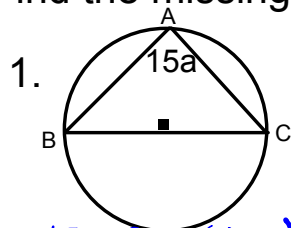
Additional questions:

Find the missing values.

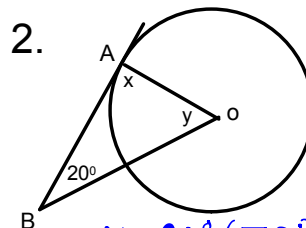


Additional questions (page 2)

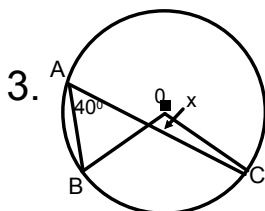
Find the missing value.



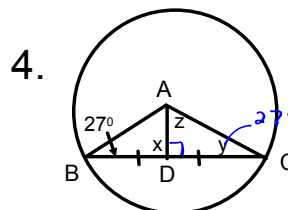
$15a = 90^\circ$ (ASP)
 $a = \frac{90}{15}$
 $= 6$



$x = 90^\circ$ (TRP)
 $y = 180 - 20 - 90$ SATT
 $= 70^\circ$



$x = 80^\circ$ (CIA P)



$x = 90^\circ$ (PCP)
 $y = 27^\circ$ (ITT)
 $z = 180 - 90 - 27$ SATT
 $= 63^\circ$

Attachments

Worksheet - Angles in a Circle.doc