Warm Up

Differentiate the following...

$$\left(x^3 + y^5\right)^6 + 3xy = 2x^4y^5$$

Higher Order Derivatives

We can continue to find the derivatives of a derivative. We find the

- second derivative by taking the derivative of the first,
- third derivative by taking the derivative of the second ... etc

Examples:

1. Determine the higher order derivatives for *f(x)*...

$$f(x) = x^4 - 2x^3 + 3x - 5$$

2. Determine $f'''(x)$ given that $f(x) = \dfrac{5}{\sqrt{2 - 3x}}$

Nov 2-10:30 PM

3. Find the second derivative of the implicit function $xy + y^2 = 4$.

4. *Determine the fourth derivative of* $y = \cos(5x)$

Practice...

Page 111
(Higher Order Derivatives)

#2, 3, 4, 5, 7 (a)

Given $f(x) = \sqrt[4]{3x+1}$, find $f'''(5)$ ← Sub. $x = 5$

3rd derivative

$$f'(x) = \frac{1}{4}(3x+1)^{-\frac{3}{4}}(3)$$

$$f''(x) = -\frac{3}{16}(3x+1)^{-\frac{7}{4}}(9)$$

$$f'''(x) = +\frac{21}{64}(3x+1)^{-\frac{11}{4}}(27)$$

$$f'''(5) = \frac{21}{64}(16)^{-\frac{11}{4}}(27)$$

$$= \frac{21}{64}\left(\frac{1}{2^{11}}\right)(27) = \frac{(21)(27)}{(64)(2048)}$$

$$= \frac{567}{131072}$$

$$\doteq 0.004$$

Rectilinear Motion and Derivatives

Any motion along a straight line is called rectilinear motion.

$s(t) \rightarrow s'(t) \rightarrow s''(t)$

✱ **Displacement - Velocity - Acceleration**

If s represents a function that measures displacement, then $\frac{ds}{dt}$ would represent ???

Check Here

The rate of change of the velocity...ie. $\frac{\Delta v}{\Delta t}$

would represent??

Check Here

So it follows that the second derivative of displacement will give us acceleration:

$a = \frac{d^2s}{dt^2}$ ←-------- Notice the notation

Example

If the displacement (in metres) at time t (in seconds) of an object is given by

$s = 4t^3 + 7t^2 - 2t$,

find the acceleration at time $t = 10$.

$S' = 12t^2 + 14t$ (velocity)

$S'' = 24t + 14$ (Acceleration)

at 10 sec...

$S''(10) = 24(10) + 14$

$= 254 \text{ m/s}^2$

Nov 2-11:50 PM

Example:

- The position of a particle is given by the equation $s = f(t) = t^3 - 6t^2 + 9t$, where t is measured in seconds and s in meters.

a) Find the velocity at time t .

b) What is the velocity after 2 s? After 4 s?

c) When is the particle at rest?

d) When is the particle moving forward (that is, in the positive direction)?

e) Draw a diagram to represent the motion of the particle.

f) Find the total distance traveled by the particle during the first five seconds.

g) Find the acceleration at time t and after 4 s.

h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.

i) When is the particle speeding up? When is it slowing down?

$s = f(t) = t^3 - 6t^2 + 9t$

a) Find the velocity at time t.

s' or $f'(t)$

$s' = 3t^2 - 12t + 9$

b) What is the velocity after 2 s? After 4 s?

$s'(2) = 3(2)^2 - 12(2) + 9$
$= 12 - 24 + 9$
$= -3$ m/s

$s'(4) = 3(4)^2 - 12(4) + 9$
$= 48 - 48 + 9$
$= 9$ m/s

Nov 12-9:52 AM

$s = f(t) = t^3 - 6t^2 + 9t$

c) When is the particle at rest?

$s' = 3t^2 - 12t + 9$ ($V = 0$ m/s)

$\frac{0}{3} = \frac{3t^2}{3} - \frac{12t}{3} + \frac{9}{3}$

$0 = t^2 - 4t + 3$

$0 = (t-3)(t-1)$

At $t = 1$ sec & 3 sec

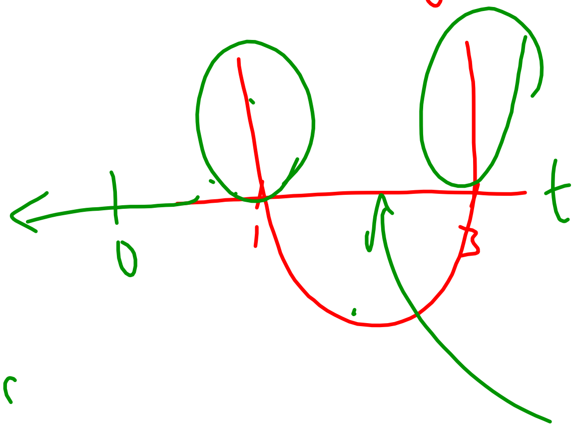
d) When is the particle moving forward (that is, in the positive direction)?

$s' > 0$

* $3t^2 - 12t + 9 > 0$ ← Above the x-axis

① $3t^2 - 12t + 9 = 0$ ⇐ (Finding zeros)

$t = 1, 3$



$t > 3$ sec

OR

$0 \leq t < 1$ sec

$1 < t < 3$

e) Draw a diagram to represent the motion of the particle.

$s = f(t) = t^3 - 6t^2 + 9t$

	t	s
start →	0	0
$s' = 0$ →	1	4
$s' = 0$ →	3	0
end →	5	20

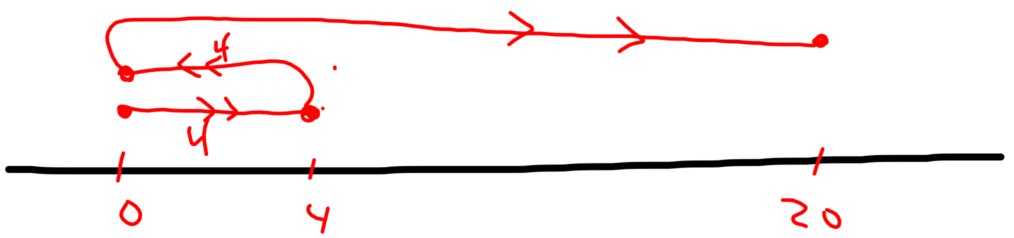
$f(0) = 0 - 0 + 0 = 0$

$f(3) = 27 - 6(9) + 27 = 0$

$f(5) = 125 - 150 + 45$

f) Find the total distance traveled by the particle during the first five seconds.

distance $= 4 + 4 + 20 = 28$ m



g) Find the acceleration at time t and after 4 s.

$s = f(t) = t^3 - 6t^2 + 9t$

$s' = 3t^2 - 12t + 9$

$s'' = 6t - 12$

$s''(4) = 6(4) - 12$

$= 12 \, m/s^2$

h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.

i) When is the particle speeding up? When is it slowing down?

$s = f(t) = t^3 - 6t^2 + 9t$

Velocity:

(+) $3t^2 - 12t + 9 > 0$

$\{0 \le t < 1 \text{ or } t > 3\}$

(−) $3t^2 - 12t + 9 < 0$

$\{1 < t < 3\}$

could happen

$-6t - 12 > 0$

$-6t > 12$

$\frac{-6t}{-6} > \frac{12}{-6}$

$t < -2$

Accel.

(+) $6t - 12 > 0$

$6t > 12$

$\frac{6t}{6} > \frac{12}{6}$

$t > 2$

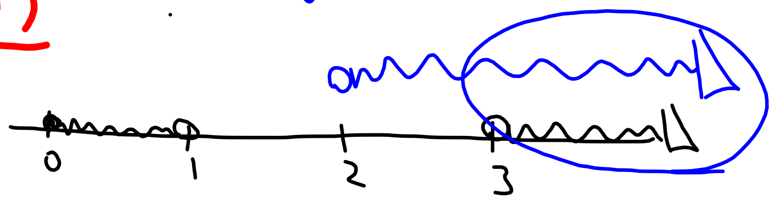
(−) $6t - 12 < 0$

$6t < 12$

$t < 2$

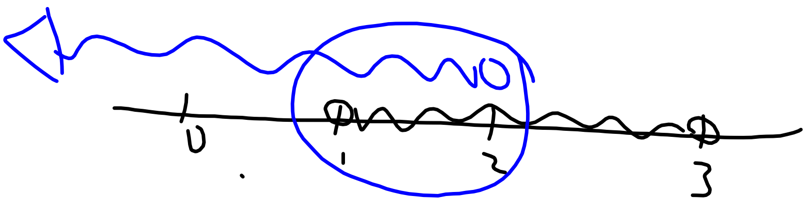
Both (+)

Speeding Up



$t > 3$

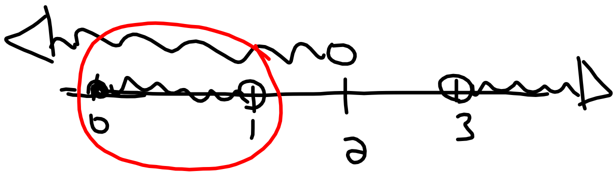
Both (−)



$1 < t < 2$

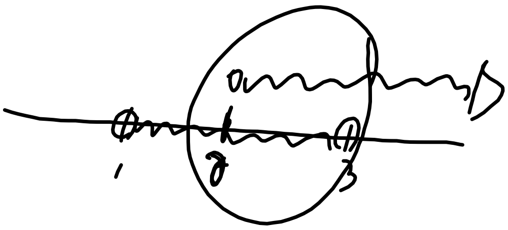
Slowing Down

vel > 0 & accel < 0



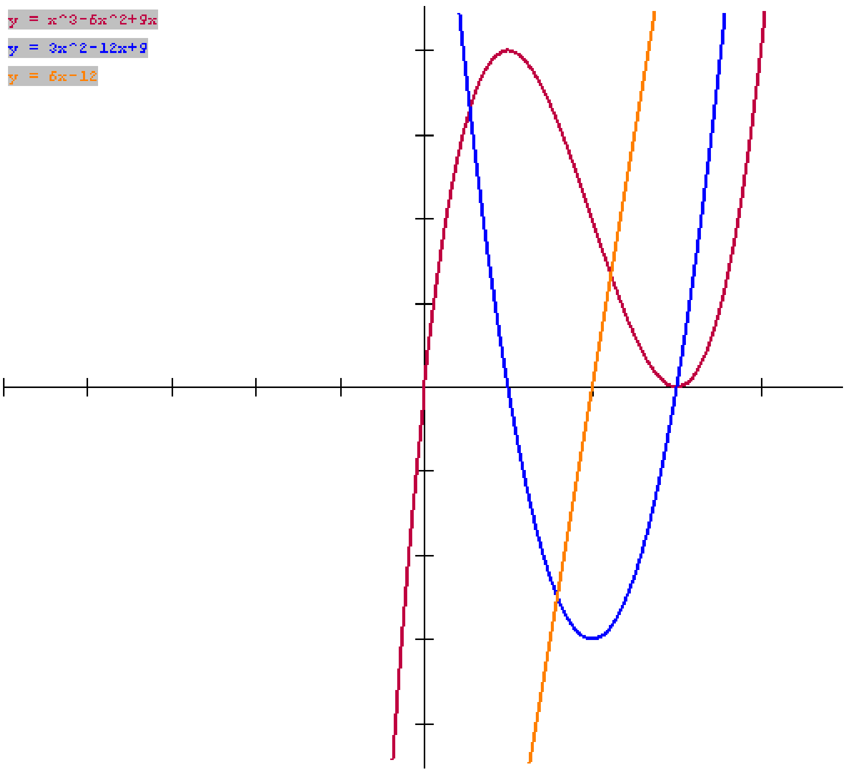
$0 \le t < 1$

vel < 0 & Accel. > 0



$2 < t < 3$

h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.



i) When is the particle speeding up? When is it slowing down?

Time to check your understanding...

A particle moves according to a law of motion $s(t) = 2t^3 - 9t^2 + 12t + 1,\ t \geq 0$.

(a) Determine the velocity of the particle when it has acceleration 6 units/s^2. (0 u/s)

(b) When is this particle moving in a positive direction? $0 \leq t < 1$ or $t > 2$

(c) Sketch the path of this particle, and determine how far it has traveled during the first 8 seconds.

546 units

(a) (velocity) $s'(t) = 6t^2 - 18t + 12$

(Acceleration) $s''(t) = 12t - 18$

$6 = 12t - 18$

$24 = 12t$

$2 = t$

$s'(2) = 6(2)^2 - 18(2) + 12$

$= 24 - 36 + 12$

$= 0$ u/sec

(b) Vel > 0

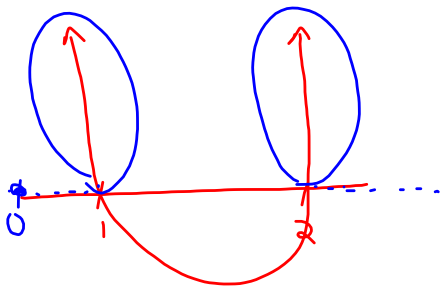
$6t^2 - 18t + 12 > 0$

$\frac{6t^2}{6} - \frac{18t}{6} + \frac{12}{6} = \frac{0}{6}$

$t^2 - 3t + 2 = 0$

$(t-1)(t-2) = 0$

$t = 1$ sec & 2 sec

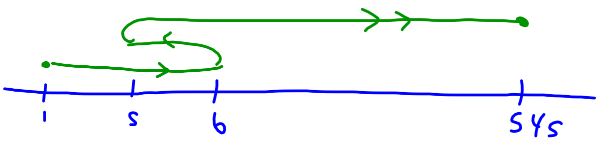


$0 \leq t < 1$ sec or $t > 2$ sec

(c)

t	$s(t)$
0	1
1	6
2	5
8	545

$s(t) = 2t^3 - 9t^2 + 12t + 1$



distance = 5 + 1 + 540

= 546 units

Practice exercises...

Page 125

#3, 4, 5, 8, 9

Page 129

#6, 7, 8

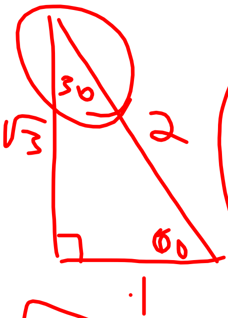
Warm Up

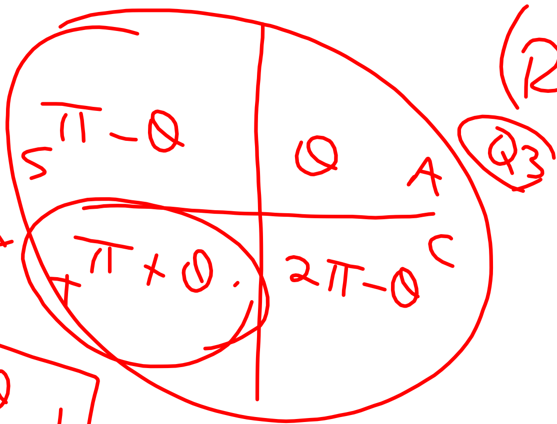
(3) An object is moving back and forth along the x-axis, starting at time $t = 0$. Its position after t seconds is $s(t) = t - 2 - 2\cos t$.
(a) What is the acceleration of the object at time t?
(b) What is the first time at which the velocity will be zero?
(For full credit, you should have no trigonometric functions in your answer; for example, if your answer contains $\sin(\frac{\pi}{4})$ you should know that this is $\frac{1}{\sqrt{2}}$.)

(Binghamton University, 2010)

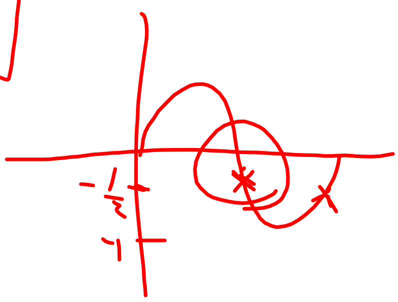
(a) $s'(t) = 1 + 2\sin t$ (Vel)
$s''(t) = 2\cos t$ (Accel)

(b) $0 = 1 + 2\sin t$
$-\frac{1}{2} = \sin t$
(Ref ∡ $\frac{\pi}{6}$, Q3,4)
Q3
$t = \frac{\pi}{1} + \frac{\pi}{6}$
$t = \frac{7\pi}{6}$ sec





$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$



Topics to Review:

- Power rule, product rule, quotient rule, chain rule
- Derivatives of trigonometric functions
- Applications of derivatives...
  *slopes of tangent lines
  *rectilinear motion
- Implicit differentiation
- Higher order derivatives
- Particle Motion Problem

Mar 14-12:53 AM

Review Assignment: Derivatives

Due Monday, May 7

Review Questions...

Page 112 - 114
#1 c, d
#7 b, d
#8 b, d
9 a, b, d, f
#11
#12

Bonus #13

Page 115
#1 (ii)
#3
#4
#5

Page 154
#2
#3