· **HOMEWORK...** Questions

p. 468: **Rule of 72...**

#3 (only estimate the doubling time)

#5a & #8

**Compound Interest (Future Value)**

#10 & #12

p. 478: **Compound Interest (Present Value)**

#4, #6, #7, & #9

$t$ ?

$$A = P(1 + \frac{r}{n})^{nt}$$

**8.** Estimate how long it would take for $1000 to grow to $16 000 at each interest rate, compounded annually.

**a)** 6%                              **b)** 12%

$1000 \xrightarrow{\times 2} 2000 \xrightarrow{\times 2} 4000 \xrightarrow{\times 2} 8000 \xrightarrow{\times 2} 16000$

12 yrs

a) $t = \dfrac{72}{\text{Rate}}$

$t = \dfrac{72}{6}$

$t = 12 \text{ yrs}$

TOTAL $\rightarrow$ 12 years $\times 4$

48 years

**10.** Solomon bought a $40 000 corporate bond (an investment in the form of a loan to a company that earns interest). The bond earns 4.8%, compounded semi-annually. After 4 years, the interest rate changed to 6%, compounded annually. Determine the value of Solomon's investment after 6 years.

\* $\mathcal{Reinvest}$

$$A = 40000\left(1 + \frac{0.048}{2}\right)^{2 \times 4}$$

$$\rightarrow A = 48357.03\left(1 + \frac{0.06}{1}\right)^{1 \times 2}$$

$$A = \boxed{48357.03}$$

$$A =$$

4 years          + 2 years

```
48357.03(1+0.06)
^2
         54333.95891
```

$$54333.96$$

P. 428     ↷ A     $P = ?$

**7.** Sasha predicts that she will need $24 000 to remodel her carpentry workshop in 6 years. She has found three investment options to consider:

   **A.** 4.80%, compounded annually

   **B.** 4.75%, compounded semi-annually    RoR = $ earn × 100

   **C.** 4.70%, compounded quarterly             $ Invest

  **a)** Compare the rates of return for these three options. Which option should she choose? Why?

  **b)** How much interest will she earn?

a) $P = \dfrac{A}{\left(1+\frac{r}{n}\right)^{nt}}$

A. $= \dfrac{24000}{\left(1+\frac{0.048}{1}\right)^{1\times 6}}$

$= 18115.22$

B $P = \dfrac{24000}{\left(1+\frac{0.0475}{2}\right)^{2\times 6}}$

$P = 18108.57$

C $P = \dfrac{24000}{\left(1+\frac{0.047}{4}\right)^{4\times 6}}$

$P = 18132.35$

b) A
```
24000-18115.22
          5884.78
```
B/
```
24000-18108.57
          5891.43
```
C/
```
24000-18132.35
          5867.65
```

a) A
RoR %
```
5884.78/24000*10
0
         24.51991667
```
24.52
B/
```
5891.43/24000*10
0
         24.547625
```
24.55
C/
```
5867.65/24000*10
0
         24.44854167
```
24.45

# 8.5 Investments Involving Regular Payments

**GOAL**

Determine the future value of an investment that earns compound interest involving regular payments.
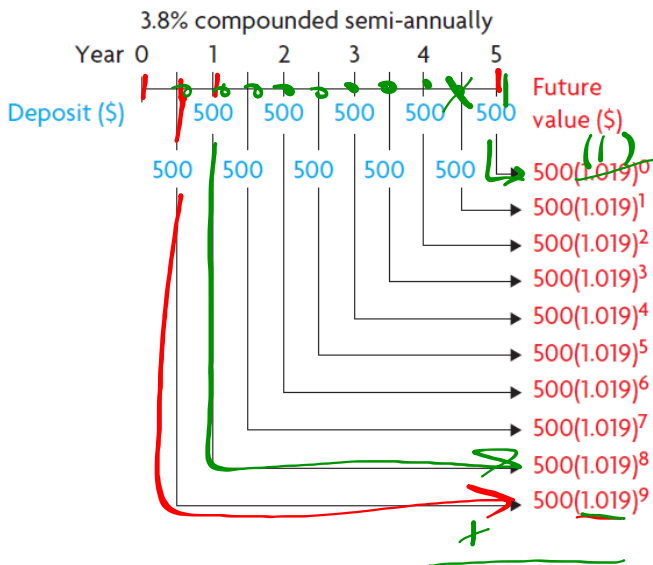
**EXAMPLE 1**
**p. 485**

**Determining the future value of an investment involving regular deposits**

Darva is saving for a trip to Australia in 5 years. She plans to work on a student visa while she is there, so she needs only enough money for a return flight and her expenses until she finds a job. She deposits $500 into her savings account at the end of each 6-month period from what she earns as a server. The account earns 3.8%, compounded semi-annually. How much money will be in the account at the end of 5 years? How much of this money will be earned interest?

## SOLUTION BY HAND...

I drew a timeline to show the future value of each of the $500 deposits that I made at the end of each 6-month period for 5 years.

3.8% compounded semi-annually

| Year 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|

Deposit ($): 500  500  500  500  500     Future value ($)

500  500  500  500  500 → $500(1.019)^0$
→ $500(1.019)^1$
→ $500(1.019)^2$
→ $500(1.019)^3$
→ $500(1.019)^4$
→ $500(1.019)^5$
→ $500(1.019)^6$
→ $500(1.019)^7$
→ $500(1.019)^8$
→ $500(1.019)^9$

I could see that I needed to do 10 calculations and then determine the sum.

$$A = 500\left(1 + \frac{0.038}{2}\right)$$

```
N=10
I%=3.8
PV=0
PMT=-500
FV=5449.896878
P/Y=2
C/Y=2
PMT:END BEGIN
```

Amount after 5 years
$5449.90

Interest
$I = 5449.90$
$- 500(10)$

$I = \$449.90$

**Notes - TVM Solver.pdf**

# INSTRUCTIONS on using the TVM-Solver...

1.) On the TI-83, press 2$^{nd}$, then FINANCE, then select 1:TVM Solver. On the TI-83 plus and TI-84, press APPS, then 1:FINANCE, then 1:TVM Solver. You should see the screen below:

```
N=█
I%=0
PV=0
PMT=0
FV=0
P/Y=1
C/Y=1
PMT:END BEGIN
```

2.) Now, suppose you are taking out a 5-year loan on $25000 at 6% annual interest compounded monthly and you want to know the monthly payment. Fill in the values on the TVM Solver screen as shown:

```
N=60
I%=6
PV=25000
PMT=█
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

3.) Now, move the cursor to PMT, press the green ALPHA key, then ENTER. Your payment will show up as a negative number:

```
N=60
I%=6
PV=25000
▪PMT=-483.32003...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

NOTE: a **negative** number means that the money is coming 'out of your pocket'

4.) Suppose you know you can afford a $250 per month payment on a 60 month loan at 6% annual interest compounded monthly. Fill out the TVM Solver screen as shown:

```
N=60
I%=6
PV=0
PMT=-250
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

5.) To find how much you can afford to borrow, move the cursor to PV, press the green ALPHA key, then ENTER. The amount you can afford to borrow is shown:

```
N=60
I%=6
▪PV=12931.39019
PMT=-250
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

# SOLUTION WITH TI-84 (Finance APP)...

```
N=■
I%=0
PV=0
PMT=0
FV=0
P/Y=1
C/Y=1
PMT:END BEGIN
```

Total number of payments
Yearly interest rate (as a percent)
Present Value (money invested/borrowed)
Payment
Future Value (money at the end of the term)
{ Number of payments/year
{ Number of times interest gets compounded/year

Payment is given at the beginning/end of pay period

Invest
↓
$$PV = 0$$

$$\left( P = PMT \times N \right) \quad \text{ex } 500 \times 10$$
$$5000$$

Formula          vs        APP
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$                    ↑
↑                                              Payments
ONCE

**EXAMPLE 2**
p. 487

Comparing a regular payment investment with a single payment investment

Adam made a $200 payment at the end of each year into an investment that earned 5%, compounded annually. Blake made a single investment at 5%, compounded annually. At the end of 5 years, their future values were equal.

a) What was their future value?
b) What principal amount did Blake invest 5 years ago?
c) Who earned more interest? Why?

Adam → APP

Blake → formula

a) Adam

```
N=5
I%=5
PV=0
PMT=-200
FV=1105.12625
P/Y=1
C/Y=1
PMT:END BEGIN
```

$FV = \$1105.13$

$P = 200 \times 5$

$P = 1000$

$I = 1105.13 - 1000$

$I = \$105.13$

b) $P = \dfrac{A}{\left(1+\frac{i}{n}\right)^{nt}}$

$P = \dfrac{1105.13}{\left(1+\frac{0.05}{1}\right)^{1\times5}}$

$P = \$865.90$

$I = \dfrac{1105.13 - 865.90}{239.23}$

BEST

# HOMEWORK...

p. 493:  #3, 5, 6, & 9

**NOTE:**  When using the TI-84...

Each question must have the following completed for homework
AND beginning of class tomorrow you will be given time to solve.

```
N=
I%=
PV=
PMT=
FV=
P/Y=
C/Y=
PMT:END BEGIN
```

# Attachments

Notes - TVM Solver.pdf